12

Neutral currents in semileptonic reactions

12.1 Neutrino–hadron neutral-current interactions

The first experimental support for the electroweak theory came from the observation of neutral currents in semileptonic reactions. Neutral currents appear because the product SU(2) × U(1) contains two neutral generators. We have shown that one linear superposition of generators is the electromagnetic current and the second is a neutral current. In Chapter 8 we discussed leptonic neutral-current reactions. In this chapter we deal with the observation of neutral currents in semileptonic reactions and, in particular, neutrino–hadron interactions.

The coupling of the Z_μ boson to leptons was given in Eq. (8.11) and that to quarks in Eq. (9.20). The neutral-current neutrino–hadron interactions have the general form

$$\begin{aligned} H_{\text{eff}} &= \frac{G}{\sqrt{2}}\big[\bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu\big] \sum_{i=1}^{3} \big(\bar{q}_i\, \tau_3\gamma_\mu q_i - \sin^2\theta_{\text{W}}\, \bar{q}_i\, Q\gamma_\mu q_i\big) \\ &= \frac{G}{\sqrt{2}}\big[\bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu\big]\big(x V_\mu^3 + y A_\mu^3 + \gamma V_\mu^0 + \delta A_\mu^0\big), \end{aligned} \tag{12.1}$$

where V_μ^3 and A_μ^3 are the isospin partners of the charged currents. V_μ^0 and A_μ^0 are isoscalar currents for which there are several possibilities. For comparison we give the normalization of V_μ^3 in terms of quarks:

$$\begin{aligned} V_\mu^3 &= \frac{1}{2}\big(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\big), \\ A_\mu^3 &= \frac{1}{2}\big(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\big). \end{aligned} \tag{12.2}$$

We could have defined V_μ^3 abstractly, in terms of its isospin transformation properties, but, now that quarks permeate our daily language, this notation is appropriate. The interested reader can always revert to the transformation properties. Similarly, we define the isoscalar currents

$$
\begin{aligned}
V_\mu^0 &= \frac{1}{2}\left(\bar{u}\,\gamma_\mu u + \bar{d}\gamma_\mu\, d\right) + \cdots, \\
A_\mu^0 &= \frac{1}{2}\left(\bar{u}\,\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5\, d\right) + \cdots,
\end{aligned}
\tag{12.3}
$$

where $\cdots$ involve $\bar{s}s$ and $\bar{c}c$ terms. With this normalization, the isoscalar piece of the electromagnetic current is $\frac{1}{3}V_\mu^0$. In the electroweak theory

$$
\begin{aligned}
x &= 1 - 2\sin^2\theta_\mathrm{W}, \qquad y = -1, \\
\gamma &= -\frac{2}{3}\sin^2\theta_\mathrm{W}, \qquad \delta = 0.
\end{aligned}
\tag{12.4}
$$

The vanishing of δ is a specific property of the standard model when we consider only up and down quarks. It is non-zero as soon as strange and heavier quarks or higher-order corrections are introduced.

The weak mixing angle θ_W is the same angle as that introduced in the leptonic sector. The first issue was the existence of the neutral currents. This was a difficult experimental problem because neutral-current interactions were new and the experiments had a large neutron background.

After the discovery of neutral currents, there was still interest in establishing that they belonged to the standard model. As problems, there remained

(i) verification of the Lorentz structure of neutral currents as vector and axial-vector operators, and
(ii) verification of the internal symmetry structure as a superposition of isovector and isoscalar operators in terms of a mixing parameter: $\sin^2\theta_\mathrm{W}$.

In analyzing these issues there are two separate kinematic regions where we know the hadronic matrix elements of the currents. One region is deep inelastic scattering, where the structure functions have been measured and have been explained successfully in terms of quark-parton distribution functions. The other region involves low-energy experiments, for which form factors for elastic scattering and the excitation of the $\Delta(1232)$ resonance are already known. In the next few sections we study reactions that allow us to decipher the couplings of neutral currents to hadrons.

When the standard model became popular, it appeared very important to discover neutral currents. It was also fortunate that experiments with the capability of searching for them were running or were beginning to run. It was not clear, however, how large neutral-current cross sections should be. There was a need for theoretical predictions. At that time the quark-parton model was in its infancy and its predictions were frequently questioned.

Thus theoretical predictions were carried out at two levels. One approach was through the symmetry properties of the currents relating V_μ^3 and V_μ^0 to the charged and electromagnetic currents. The other approach was to calculate cross sections in the quark-parton model. Nowadays we know that both approaches are correct.

12.2 Model-independent predictions

The simplest processes to consider are those involving total cross sections on isospin neutral targets. We define

$$\sigma_- = \frac{1}{2}\left[\sigma\,(\nu + \mathrm{p} \to \mu^- + \mathrm{X}_1) + \sigma(\nu + \mathrm{n} \to \mu^- + \mathrm{X}_2)\right]$$

and

$$\sigma_0 = \frac{1}{2}\left[\sigma(\nu + \mathrm{p} \to \nu + \mathrm{X}_3) + \sigma(\nu + \mathrm{n} \to \nu + \mathrm{X}_4)\right]. \tag{12.5}$$

An incoherent sum over all possible final states that yields an isoscalar final state is assumed. For the charged-current cross section we write

$$\sigma_- = V + A + I, \tag{12.6}$$

where V comes from the vector current alone, A from the axial current alone, and I is the interference term. We can represent them as follows:

$$V = \sum |\langle X|\,\varepsilon \cdot V|p\rangle|^2, \qquad A = \sum_{x,\varepsilon} |\langle X|\,\varepsilon \cdot A|p\rangle|^2,$$

and

$$I = 2\sum \mathrm{Re}(\langle X|\varepsilon \cdot V|p\rangle^* \langle X|\,\varepsilon \cdot A\,|p\rangle), \tag{12.7}$$

with the sums running over all final states and polarizations of the W boson. In Eqs. (12.6) and (12.7) an average over protons and neutrons is understood.

The vector currents are isovector quantities that are related to the isovector part of the neutral current through an isospin rotation. The neutral current contains in addition an isoscalar term, but, since we consider isoscalar target and isoscalar final states, the isoscalar–isovector interference drops out. It follows now that

$$\sigma_0 = \frac{1}{2}\left(x^2 V + xI + A + y^2 S\right), \tag{12.8}$$

where S is the contribution of the isoscalar current. The overall factor of $1/2$ follows from the fact that the charged current transforms like the generator $\sqrt{2}\tau^+$ of SU(2)

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and the neutral current like τ^3. Since $y^2 \geq 0$,

$$R = \frac{\sigma_0}{\sigma_-} \geq \frac{1}{2}\frac{A + xI + x^2 V}{A + I + V}. \tag{12.9}$$

Furthermore, Schwarz's inequality implies

$$4AV \geq I^2. \tag{12.10}$$

On combining the two inequalities (see Problem 1), we arrive at

$$R \geq \frac{1}{2}\left[1 - (1 - x)\left(\frac{V}{A + I + V}\right)^{\frac{1}{2}}\right]^2. \tag{12.11}$$

The term V can be deduced from knowledge of the isovector contribution to the electroproduction cross section

$$\sigma_{\text{em}} = \frac{1}{2}\big[\sigma(\mathrm{e} + \mathrm{p} \to \mathrm{e} + \mathrm{x}_1) + \sigma(\mathrm{e} + \mathrm{n} \to \mathrm{e} + \mathrm{x}_2)\big]. \tag{12.12}$$

Not knowing the isoscalar contribution, we use again inequalities,

$$V \leq \frac{G}{\pi}\frac{Q^4}{4\pi\alpha^2}\sigma_{\text{em}} = V_{\text{em}}, \tag{12.13}$$

which gives the final result

$$R \geq \frac{1}{2}\left[1 - 2\sin^2\theta_{\mathrm{W}}\left(\frac{V_{\text{em}}}{\sigma_-}\right)^{\frac{1}{2}}\right]^2. \tag{12.14}$$

This derivation makes judicious use of inequalities. Within the electroweak theory the method is model-independent and holds for many physical processes. When we plot R versus $\sin^2\theta_{\mathrm{W}}$ there is a minimum for the ratio; for similar inequalities see Pais and Treiman (1972).

One may also use reactions induced by antineutrinos to obtain additional relations. On going over to antineutrinos one must change the sign of the interference term I. The charged- and neutral-current cross sections on isoscalar targets are, respectively,

$$\sigma_+ = (V + A - I), \tag{12.15}$$

$$\bar{\sigma}_0 = \frac{1}{2}\big(A + x^2 V - xI + y^2 S\big). \tag{12.16}$$

On combining Eqs. (12.6), (12.8), (12.15), and (12.16), we obtain (Paschos and Wolfenstein, 1973)

$$R_- = \frac{\sigma_0 - \bar{\sigma}_0}{\sigma_- - \sigma_+} = \frac{1}{2}\big(1 - 2\sin^2\theta_{\rm W}\big), \tag{12.17}$$

$$R_+ = \frac{\sigma_0 + \bar{\sigma}_0}{\sigma_- + \sigma_+} = \left(\frac{1}{2} - \sin^2\theta_{\rm W} + \frac{10}{9}\sin^4\theta_{\rm W}\right). \tag{12.18}$$

These relations are truly independent of any details of scaling violations and eliminate some theoretical corrections inherent in the quark-parton method. They are frequently used to determine the mixing angle $\sin^2\theta_{\rm W}$.

In the above derivations we set the parameter

$$\rho = \frac{M_{\rm W}^2}{M_{\rm Z}^2\cos^2\theta_{\rm W}} \tag{12.19}$$

equal to unity. This is the lowest-order value, which appears also in Eq. (8.19), but radiative corrections will modify it. Extensive analyses of the data including radiative corrections from the top quark and the Higgs meson gave the value

$$\rho = 0.9998\ ^{+0.0034}_{-0.0012} \qquad \text{and} \qquad M_{\rm H} < 1002\ \text{GeV},$$

which is indeed very close to unity. This is a confirmation of the SU(2) structure of the theory and we will continue giving ρ the value unity.

12.3 Neutral-current cross sections

It is perhaps more transparent to discuss the various cross sections in the parton model (Sehgal, 1973; Kim *et al.*, 1981). The effective Lagrangian density was written, at the beginning of this chaper, in terms of the first generation of quarks. We re-express the effective interaction in terms of chiral couplings,

$$\begin{aligned}\mathcal{L} = -\frac{G}{\sqrt{2}}\bar{\nu}\gamma^\mu(1-\gamma_5)\nu\big\{&\bar{u}\gamma_\mu[u_{\rm L}(1-\gamma_5) + u_{\rm R}(1+\gamma_5)]u \\ &+ \bar{d}\gamma^\mu[d_{\rm L}(1-\gamma_5) + d_{\rm R}(1+\gamma_5)]d + \cdots\big\},\end{aligned} \tag{12.20}$$

with $u_{\rm L}$ and $u_{\rm R}$ the couplings of the left- and right-handed up quarks and with a similar definition for $d_{\rm L}$ and $d_{\rm R}$. The ellipses indicate again contributions from higher generations. We adopted this notation because it is convenient to write down the elementary cross sections as they were classified in Section 8.3 in terms of the chiralities of the leptonic and hadronic vertices. The new couplings are related to

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those defined at the beginning of this chapter as follows:

$$\begin{aligned} u_{\mathrm{L}} &= \frac{1}{4}(x+y+\gamma+\delta), \\ u_{\mathrm{R}} &= \frac{1}{4}(x-y+\gamma-\delta), \\ d_{\mathrm{L}} &= \frac{1}{4}(-x-y+\gamma+\delta), \\ d_{\mathrm{R}} &= \frac{1}{4}(-x+y+\gamma-\delta). \end{aligned} \tag{12.21}$$

The neutrino experiments determined combinations of $u_{\mathrm{L}}, \ldots, d_{\mathrm{R}}$, which then were translated into x, y, γ, and δ, thus testing the isospin and parity content of the current. Finally, they were all determined in terms of a single mixing angle $\sin^2\theta_{\mathrm{W}}$. The expressions become rather long and it is convenient to introduce a shorter notation. We denote generically by f_q and $f_{\bar{q}}$ the parton distribution functions for q and $\bar{q}$ and their left-handed or right-handed couplings by q_{L} and q_{R}, respectively. One easily finds cross sections for the elementary processes

$$\begin{aligned} \frac{\mathrm{d}\sigma_{\mathrm{NC}}(\nu\mathrm{q})}{\mathrm{d}x\,\mathrm{d}y} &= \frac{2G^2ME}{\pi} x f_q(x)\left[q_{\mathrm{L}}^2 + q_{\mathrm{R}}^2(1-y)^2\right], \\ \frac{\mathrm{d}\sigma_{\mathrm{NC}}(\bar{\nu}\mathrm{q})}{\mathrm{d}x\,\mathrm{d}y} &= \frac{2G^2ME}{\pi} x f_q(x)\left[q_{\mathrm{R}}^2 + q_{\mathrm{L}}^2(1-y)^2\right], \\ \frac{\mathrm{d}\sigma_{\mathrm{NC}}(\nu\bar{\mathrm{q}})}{\mathrm{d}x\,\mathrm{d}y} &= \frac{2G^2ME}{\pi} x f_{\bar{q}}\left[q_{\mathrm{R}}^2 + q_{\mathrm{L}}^2(1-y)^2\right], \\ \frac{\mathrm{d}\sigma_{\mathrm{NC}}(\bar{\nu}\bar{\mathrm{q}})}{\mathrm{d}x\mathrm{d}y} &= \frac{2G^2ME}{\pi} x f_{\bar{q}}\left[q_{\mathrm{L}}^2 + q_{\mathrm{R}}^2(1-y)^2\right]. \end{aligned} \tag{12.22}$$

There are various ways to combine these cross sections and isolate the coupling constants. In experiments with isoscalar targets,

$$f_{\mathrm{u}}(x) = f_{\mathrm{d}}(x) = f(x) \qquad \text{and} \qquad f_{\bar{\mathrm{u}}}(x) = f_{\bar{\mathrm{d}}}(x) \equiv \bar{f}(x).$$

Furthermore, we can integrate over y and set $K = 2G^2ME/\pi$ to obtain

$$\begin{aligned} \frac{\mathrm{d}\sigma_{\mathrm{NC}}(\nu\mathrm{N})}{\mathrm{d}x} &= Kx\left[\left(f + \frac{1}{3}\bar{f}\right)\left(u_{\mathrm{L}}^2 + d_{\mathrm{L}}^2\right) + \left(\frac{1}{3}f + \bar{f}\right)\left(u_{\mathrm{R}}^2 + u_{\mathrm{R}}^2\right)\right], \\ \frac{\mathrm{d}\sigma_{\mathrm{NC}}(\bar{\nu}\mathrm{N})}{\mathrm{d}x} &= Kx\left[\left(\frac{1}{3}f + \bar{f}\right)\left(u_{\mathrm{L}}^2 + d_{\mathrm{L}}^2\right) + \left(f + \frac{1}{3}\bar{f}\right)\left(u_{\mathrm{R}}^2 + d_{\mathrm{R}}^2\right)\right], \\ \frac{\mathrm{d}\sigma_{\mathrm{CC}}(\nu\mathrm{N})}{\mathrm{d}x} &= Kx\left(f(x) + \frac{1}{3}\bar{f}\right), \\ \frac{\mathrm{d}\sigma_{\mathrm{CC}}(\bar{\nu}\mathrm{N})}{\mathrm{d}x} &= Kx\left(\frac{1}{3}f + \bar{f}\right). \end{aligned} \tag{12.23}$$

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Most experiments measure ratios of cross sections, where the flux of the neutrinos drops out. A popular ratio is

$$R_\nu = \frac{\sigma_{NC}(\nu N)}{\sigma_{CC}(\nu N)} = \left(u_L^2 + d_L^2\right) + \frac{2-B}{2+B}\left(u_R^2 + d_R^2\right), \tag{12.24}$$

with

$$B = \frac{\int_0^1 dx\, x\left[f(x) - \bar{f}(x)\right]}{\int_0^1 dx\, x\left[f(x) + \bar{f}(x)\right]} \tag{12.25}$$

measuring the relative strength of the valence- and the sea-quark contributions. For instance, $B = 1$ corresponds to vanishing sea contribution. For the experimental value $B = 0.8$ the ratio becomes

$$R_\nu = \frac{1}{2} - \sin^2\theta_W + \frac{50}{63}\sin^4\theta_W. \tag{12.26}$$

The experimental values for R_ν and $R_{\bar{\nu}}$ are

$$R_\nu = 0.29 \pm 0.01 \qquad \text{and} \qquad R_{\bar{\nu}} = 0.34 \pm 0.03.$$

In order to compare them with the prediction of Eq. (12.23) it is necessary to include precise quark distribution functions. They include contributions from sea-quark (s, $\bar{s}$) and (c, $\bar{c}$) pairs of the target. In addition, scaling violations, which have been established and analyzed in charged-current reactions, must be included (Kim *et al.*, 1981). The analysis yields

$$u_L^2 + d_L^2 = 0.29 \pm 0.01 \qquad \text{and} \qquad u_R^2 + d_R^2 = 0.03 \pm 0.01,$$

which lead to the value $\sin^2\theta_W = 0.228 \pm 0.001$.

12.4 Parity violation in electron scattering

Effects of weak neutral currents in low-energy ($Q^2 \ll M_Z^2$) electron–hadron reactions are submerged in the dominant electromagnetic interaction. For these reactions we must search for a clear signature of weak origin, such as parity violation. Experiments of this type have been carried out in deep inelastic electron–hadron scattering and in atomic physics (see Problem 4). The couplings of the Z boson to electrons and quarks have been discussed already.

A parity-violating observable is the difference of cross sections for right- and left-handed polarized electrons. These are electrons polarized along their direction of motion, i.e. electrons with definite helicity. Since helicity changes sign under spatial reflection, a difference between the two cross sections is an indication of parity violation. We denote the left-handed and right-handed electrons by the spinors

$$e_{\mathrm{L,R}} = \frac{1}{2}(1 \mp \gamma_5)u(k), \tag{12.27}$$

respectively. Their interactions at high energies with protons and neutrons are described with sufficient accuracy by the parton model. Consequently, we can write the hadronic neutral current as

$$J_\mu(x) = \bar{u}\gamma_\mu[u_{\mathrm{L}}(1-\gamma_5) + u_{\mathrm{R}}(1+\gamma_5)]u + \bar{d}\gamma_\mu[d_{\mathrm{L}}(1-\gamma_5) + d_{\mathrm{R}}(1+\gamma_5)]d \tag{12.28}$$

as it appears in Eq. (12.20). The interaction of the electrons with hadrons now involves the exchange of a photon and a Z boson. In cross sections there are contributions from the electromagnetic amplitude and weak terms. The latter contribution is responsible for the asymmetry. The amplitudes are given as

$$m_\gamma = -\frac{\mathrm{i}e^2}{q^2}\bar{e}\gamma_\mu e\,(e_{\mathrm{u}}\bar{u}\gamma^\mu u + \cdots), \tag{12.29}$$

and

$$\begin{aligned} m_{\mathrm{Z}} = -&\frac{\mathrm{i}g^2}{\cos^2\theta_{\mathrm{W}}\left(q^2 - M_Z^2\right)}\left(g_{\mathrm{L}}\bar{e}_{\mathrm{L}}\gamma_\mu e_{\mathrm{L}} + g_{\mathrm{R}}\,\bar{e}_{\mathrm{R}}\gamma_\mu e_{\mathrm{R}}\right) \\ &\times\left[u_{\mathrm{L}}\bar{u}\gamma^\mu(1-\gamma_5)u + u_{\mathrm{R}}\bar{u}\gamma^\mu(1+\gamma_5)u + \cdots\right], \end{aligned} \tag{12.30}$$

where e_{u} is the charge of the up quark and the ellipses indicate contributions from other quarks. For the Z-boson couplings, g_{L} and g_{R} are the helicity couplings to electrons while u_{L} and u_{R} are the corresponding couplings to the up quark. For the computation of the interference term we follow the presentation of Section 8.3, where it was shown that, in the squared amplitude, the following conditions hold.

(i) The electron bilinears are left-handed or right-handed. The same is true for the quarks.
(ii) When left-handed leptonic couplings combine with left-handed quark couplings then $\mathrm{d}\sigma/\mathrm{d}y$ is independent of y. The same holds for right-handed leptonic couplings with right-handed hadronic combinations.
(iii) When left-handed leptonic couplings combine with right-handed hadronic couplings, then the dependence is $(1-y)^2$.

The form of the interference terms now follows:

$$\frac{\mathrm{d}\sigma_{\mathrm{L}}}{\mathrm{d}x\,\mathrm{d}y} \propto \left[g_{\mathrm{L}}u_{\mathrm{L}} + g_{\mathrm{L}}u_{\mathrm{R}}(1-y)^2\right]u(x) + \cdots, \tag{12.31}$$

$$\frac{\mathrm{d}\sigma_{\mathrm{R}}}{\mathrm{d}x\,\mathrm{d}y} \propto \left[g_{\mathrm{R}}u_{\mathrm{L}}(1-y)^2 + g_{\mathrm{R}}\,u_{\mathrm{R}}\right]u(x) + \cdots, \tag{12.32}$$

with the subscripts L and R in the cross section denoting left- and right-handed polarized electrons and the ellipses indicating contributions from down quarks.

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The parity-violating observable is built into the asymmetry

$$A = \frac{\mathrm{d}\sigma_\mathrm{R} - \mathrm{d}\sigma_\mathrm{L}}{\mathrm{d}\sigma_\mathrm{R} + \mathrm{d}\sigma_\mathrm{L}}, \tag{12.33}$$

which is easy to derive from (12.31) and (12.32). To arrive at the final result, we must include the down quarks. In an isoscalar target, such as deuterium or carbon, there are equal numbers of up and down quarks, so only the combination $u(x) + d(x)$ appears in the cross sections, which drops out in the asymmetry. On collecting the various terms together, the asymmetry is expected to be

$$A = \frac{GQ^2}{\sqrt{2}4\pi\alpha}\frac{9}{5}\left[a_1 + a_2\frac{1-(1-y)^2}{1+(1+y)^2}\right], \tag{12.34}$$

with $a_1 = 1 - (20/9)\sin^2\theta_\mathrm{W}$ and $a_2 = 1 - 4\sin^2\theta_\mathrm{W}$. I have given several steps of the derivation so that the interested reader can reproduce it using the various coupling constants given in the book. The magnitude of the asymmetry for $Q^2 = 1\ \mathrm{GeV}^2$ is

$$A \approx -1.6 \times 10^{-4}.$$

The effect was observed at the Stanford Linear Accelerator Center (SLAC) (Prescott *et al.*, 1978, 1979). Electron–proton and positron–proton collisions have been extended at HERA to very large values of $Q^2 = 400$–$40\,000\ \mathrm{GeV}^2$, at which the effects of the Z propagator are also observable.

Problems for Chapter 12

1. Make judicious use of the Schwarz inequality to prove Eqs. (12.11) and (12.14).
2. Select the Feynman rules for the electron–hadron reaction and show that the effective interaction has the form

$$H_\mathrm{eff}^\mathrm{ep} = \frac{G}{\sqrt{2}}\bar{e}\gamma_\mu(g_\mathrm{V} - g_\mathrm{A}\gamma_5)e\left[\bar{u}\gamma_1^\mu(V_\mathrm{u} + a_\mathrm{u}\gamma_5)u + \bar{d}\gamma^\mu(V_\mathrm{d} + a_\mathrm{d})d + \cdots\right],$$

   where

$$\begin{aligned} g_\mathrm{V} &= \frac{1}{2} - 2\sin^2\theta_\mathrm{W}, \qquad g_\mathrm{A} = -\frac{1}{2}; \\ V_\mathrm{u} &= \left(1 - \frac{8}{3}\sin^2\theta_\mathrm{W}\right), \qquad a_\mathrm{u} = -1; \\ V_\mathrm{d} &= -\left(1 - \frac{4}{3}\sin^2\theta_\mathrm{W}\right), \qquad a_\mathrm{d} = 1; \end{aligned} \tag{12.35}$$

   and the ellipses stand for strange and heavier quarks.
3. Combine the results of the previous problem with the outline of Section 12.4 and obtain the final form of the asymmetry.

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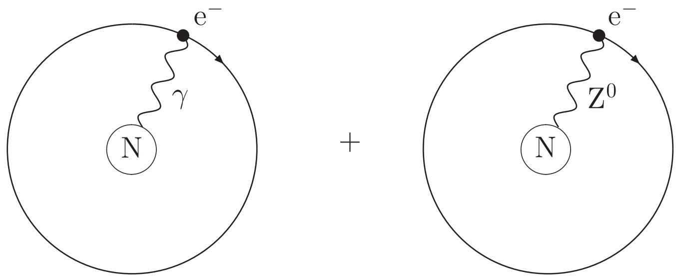

Figure 12.1. A schematic drawing of a Z exchange in an atom.

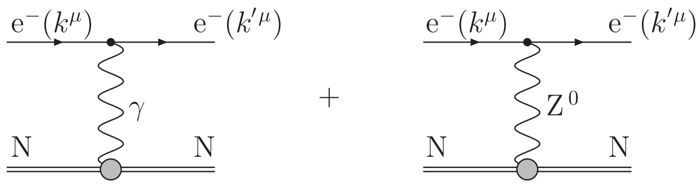

Figure 12.2. Feynman diagrams for electron–nucleus interaction.

4. Another manifestation of neutral-current interactions appears as parity violation in atoms. The neutral current introduces a new interaction between the orbiting electron and the nucleus. The total force in the atom is the sum of electromagnetic and weak diagrams (Fig. 12.1) or can be expressed in terms of Feynman diagrams (Fig. 12.2).

   The sum of the amplitudes contributes

$$m = \bar{e}(k')\gamma^\mu e(k)\frac{e^2}{q^2}\langle N|J_\mu^{\text{em}}|N\rangle + \frac{g^2}{8\cos^2\theta_{\text{W}}}\,\bar{e}(k')\gamma^\mu(g_{\text{V}} + g_{\text{A}}\gamma_5)e(k)\frac{1}{q^2 - M_{\text{Z}}^2}\langle N|J_\mu^{\text{NC}}|N\rangle,$$

   where $q_\mu = k_\mu - k'_\mu = p'_\mu - p_\mu$. In order to identify weak effects, a signal with parity violation is required.

   Two parity-violating amplitudes are

$$M_1 = \frac{G}{\sqrt{2}}\bar{e}(k')g_{\text{A}}\gamma^\mu\gamma_5 e(k)\left\{\langle N|V_\mu^3|N\rangle - 2\sin^2\theta_{\text{W}}\langle N|J_\mu^{\text{em}}|N\rangle\right\}$$

   and

$$M_2 = \frac{G}{\sqrt{2}}\bar{e}(k')g_{\text{V}}\gamma^\mu e(k)\langle N|A_\mu|N\rangle.$$

   For $\sin^2\theta_{\text{W}} = 0.25$ (which is close to the experimental value), only the M_1 amplitude survives. For this reason and because of the suppression of the hadronic matrix element in M_2, we discuss below only M_1.

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The momenta involved in atomic experiments are small, so it is convenient to obtain a non-relativistic limit of the weak interaction.

(i) For $|\vec{k}| \ll m_e$ and $|\vec{k}'| \ll m_e$, show that

$$\frac{1}{q^2 - M_Z^2} \approx \frac{1}{4\pi} \int d^3r \, e^{i\vec{q}\cdot\vec{r}} \, \frac{e^{-M_Z r}}{r}.$$

The weak interaction is of short range and, in the limit of large M_Z, it acts at the origin, where the nucleus is located. Later on we replace the Yukawa potential by a three-dimensional δ-function.

(ii) For reduction of the hadronic matrix element to the non-relativistic limit, consider

$$\langle N, p'|J_\mu^{em}|N, p\rangle = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}\frac{(p-p')^\nu}{2M}F_2(q^2)\right]u(p).$$

Show that, in the non-relativistic limit, only the $\mu = 0$ component survives and gives the charge Q of the nucleus. Similar arguments for the V_μ^3 matrix element give

$$\langle N, p'|V_\mu^3|N, p\rangle = g_{\mu 0}\frac{1}{2}(Z - N),$$

with Z the number of protons and N on the right-hand side the number of neutrons in the nucleus. On combining the results from steps (i) and (ii), we find that the nucleus generates, through the M_1 amplitude, the potential

$$Z_\mu(r) = g_{\mu 0}\frac{G}{\sqrt{2}}\delta^3(\vec{r})Q_W(Z, N),$$

with $Q_W(Z, N) = \frac{1}{2}[Z(1 - 4\sin^2\theta_W) + N]$.

(iii) The transition-matrix element of the electron is

$$\langle e_f|H_{PV}|e_i\rangle = \int \bar{e}_f(r)\gamma_\mu\gamma_5 Z_\mu e_i(r) d^3r.$$

For atomic physics the electron wave function can be written as

$$e_i(r) = \begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{\nabla}}{2m_e} \end{pmatrix}\psi_i(r),$$

with $\psi_i(r)$ the space wave function of level i, m_e the mass of the electron and $\vec{\nabla}$ the momentum operator. Show that the matrix element reduces to

$$\langle e_f|H_{PV}|e_i\rangle = \frac{G}{\sqrt{2}}\frac{1}{2m_e}Q_W \int d^3r \, \psi_f^*(r)\left[\vec{\sigma}\cdot\vec{\nabla}\delta^3(r) + \delta^3(r)\sigma\cdot\vec{\nabla}\right]\psi_i(r).$$

The effect of the neutral current on an atomic level is to induce the mixing of levels with opposite parities. Thus the absorption rates of beams of monochromatic light with various polarizations by atoms differ. One effect of parity violation in heavy atoms is the rotation of the plane of polarization of laser light passing through atomic vapors. Such experiments have been performed and effects of the neutral current have been observed (Bouchiat and Bouchiat, 1974).

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