'The Distribution of Navigational Errors'

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1. INTRODUCTION. The navigational probability distribution proposed by Čumbelić[1] is not satisfactory as an error distribution because in its general form it is not smooth, having a gradient discontinuity at its mid-point. The distribution is extremely simple, its probability density function consisting of two straight lines, from $(0, (4y-1)2\lambda)$ to $(\pm\lambda,(3-4y)/2\lambda)$ in Čumbelić's notation. The discontinuity is at the intersection of these lines and arises because in proposing a parabolic form for the cumulative distribution function of his distribution the author has failed to notice the requirement that a cumulative distribution function must have zero curvature at the point of symmetry if the resulting probability distribution is to be smooth there.

2. AN ALTERNATIVE DISTRIBUTION. The requirement is for a symmetrical probability distribution confined to the interval $(-\lambda,\lambda)$ with one further adjustable parameter to accommodate a variety of shapes from approximately normal, through the uniform distribution, to U-shaped. The natural choice is the symmetrical β-distribution $f(x) = (\lambda^2 - x^2)^a/k$ where a is the parameter and k the constant of integration $\lambda^{2a+1} B(\frac{1}{2}, a+1)$. This is a standard distribution (Pearson's Type II), the details of which may be found in any textbook of mathematical statistics.[2]

REFERENCES

[1] Čumbelić, P. (1987). The distribution of navigational errors. This *Journal*, **40**, 30–41.

[2] Stuart, A. and Ord, J. K. (1987). *Kendall's Advanced Theory of Statistics*, Vol. 1: *Distribution Theory*. London: Griffin.

The Author Replies

It is agreed that the proposed navigational probability distribution in its general form is not smooth, but it may also be noted that the sea is not generally smooth either but navigation has to continue.

It is not agreed, however, that the distribution is unsatisfactory. It was intended as, and has been successfully used as, an approximation for the overall random error in astro-navigation. It is based on sampling data rather than on intuition.

It may help to clarify the matter if we return to the first sentence of my paper. 'Practical navigation is far from an exact science because the navigator in his daily routine uses many approximations which would be unacceptable in careful scientific work.' The proposed distribution should therefore be treated as just another approximation which will substitute a number of less satisfactory approximations.

The simplicity of the proposed distribution, which Dr Edwards mentions, is felt to be a point in its favour. Also the distribution was included in a mathematical model for computer simulation in astro-navigation[1] and tested in real navigational practice with great success.

As regards the alternative symmetrical β-distribution suggested by Dr Edwards, it would be of interest to know how the unknown parameter, a, should be estimated in navigational practice.

REFERENCE

[1] Čumbelić, P. (1986). Mathematical models and their simulations in astro-navigation. *Doctorate Dissertation*. University of Rijeka.