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SPECTRAL MULTIPLIER THEOREMS FOR ABSTRACT DIFFERENTIAL OPERATORS

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We establish new spectral multiplier theorems for abstract differential operators by working on various functional calculus methods obtained using different reproducing formulae. Here we have put together the results from two articles. One of them is published [2] and the other [3], which is a joint work with van Neerven and Portal, is accepted for publication. Apart from this, we also give an alternative approach to a result of Bailey and Sikora [1] on square function estimates for the Laplace operator on certain nondoubling manifolds and extend it to a more abstract setting.

In the results from the published article, we consider specific group generators which are abstractions of first-order differential operators. We show spectral multiplier estimates similar to one available for d/dx on $L^p(\mathbb{R})$ assuming only that the group is bounded on L^2 rather than L^p. That is, we only assume that the operator iD generates a group that has finite propagation speed on L^2 and satisfies some Sobolev embedding property. In fact, we show R-bounded Hörmander calculus results for the square of a perturbed Hodge–Dirac operator (recovering, in particular, the result for uniformly elliptic divergence form operators with L^∞ coefficients).

For the results taken from the second article, we consider operators acting on a UMD Banach lattice X that have the same algebraic structure as the position and momentum operators associated with the harmonic oscillator $-\frac{1}{2}\Delta + \frac{1}{2}|x|^2$ acting on $L^2(\mathbb{R}^d)$. More precisely, we consider abstract harmonic oscillators of the form $\frac{1}{2}\sum_{j=1}^d (A_j^2 + B_j^2)$ for tuples of operators $A = (A_j)_{j=1}^d$ and $B = (B_k)_{k=1}^d$, where iA_j and iB_k are assumed to generate C_0 groups and to satisfy the canonical commutator relations. We prove functional calculus results for these abstract harmonic oscillators that

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match classical Hörmander spectral multiplier estimates for the harmonic oscillator $-\frac{1}{2}\Delta + \frac{1}{2}|x|^2$ on $L^p(\mathbb{R}^d)$. This covers situations where the underlying metric measure space is not doubling and the use of function spaces that are not particularly well suited to extrapolation arguments. For instance, as an application, we treat the harmonic oscillator on mixed norm Bargmann–Fock spaces. In addition to this, we show, for future application, an example of Weyl pairs on certain noncommutative L^p spaces, indicating that proving spectral multiplier estimates for the harmonic oscillators in this setting should be possible.

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