OPTIMAL REINSURANCE AND DIVIDEND PAYMENT STRATEGIES*

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I. INTRODUCTION AND SUMMARY

This paper presents a normative model for the sequential reinsurance and dividend-payment problem of the Insurance Company (I.C.). Optimal strategies are found in closed form for a class of utility functions. In some sense the model studied can be viewed as an adaptation of Hakansson's investment-consumption model of the individual [3] or a generalization of Frisque's model for the dynamic management of an I.C. [2].

In Section 2 the model is formulated as a discrete time dynamic programming problem. The objective of the I.C. is assumed to be maximization of the expected utility of the dividend streams paid to stock/policy-holders (s/p-holders). The initial reserves level is assumed to be known. The premiums to be collected in each period for selling policies are known in advance. The losses due to claims from policy-holders are random variables independent from period to period. In each period the I.C. must decide on the portion of the reserves to be paid as dividends and on the form and level of reinsurance with a reinsurer that quotes prices for any contract.

Optimal strategies in closed form are found in Section 3 when the utility function of the I.C. is given by the discounted sum of one-period utilities of dividends; and when the one-period utilities belong to the linear risk-tolerance class, which is given by: (Ia) $u(x) = (ax + b)^{c+1}/a(c + 1)$; $ax + b > 0$, $ac < 0$. (Ib) $u(x) = \log(ax + b)$; $ax + b > 0$. (II) $u(x) = -e^{-\gamma x}$; $\gamma > 0$.

The results of Section 3 are discussed and interpreted in Section 4. The optimal dividend payments are found to be linear in the reserves level; while the optimal reinsurance treaty transforms the reserves level (as a function of the losses) in such a way that its *form* is independent of the prereinsurance total wealth of the I.C. It only depends on the I.C.'s utility function, the prices quoted

* This study is based on my Ph.D. thesis submitted to the University of California, Berkeley (1975). I am grateful to Professor W. S. Jewell (Chairman) as well as to Professors Nils Hakansson and David Gale for many helpful comments and criticisms.

by the reinsurer and the probability density function of the prereinsurance losses.

Finally, in Section 5 we discuss a generalization to include expenditures for promotion of sales and an extension to multiplicative utilities.

2. Formulation of the Model

2.1. *The description of the Insurance Company*

The I.C. is faced with a N-period problem. The periods are numbered backwards, thus the interval $(t, t-1)$ is the t^{th} period. We will use the following notation:

p_t: premiums collected by selling policies during period t. They are assumed to be collected at the end of the period for simplicity and they are known in advance.

ξ_t: claims paid to policy-holders during period t — a random variable which takes values on the internal X_t and whose value will be denoted by x_t. For simplicity it is assumed that claims are paid at the end of the period and are independent from period to period.

C_t: dividends paid to s/p-holders at start of period t (decision variable).

R_t: level of reserves at start of period t before dividends are paid.

$\varphi_t(x)$: probability density function of the r.v. ξ_t.

2.2. *The utility function of the I.C.*

We will assume that the utility function of the I.C. over possible streams of dividends $C = C_N, \ldots, C_1, C_0$ is given by one of the three forms: *

(S) Discounted Sum:

$$U(C) = \sum_{k=0}^{N} \alpha^k u(C_{N-k});\ 0 < \alpha < 1$$

(MP) Multiplicative Positive:

$$U(C) = \prod_{k=0}^{N} u(C_{N-k});\ u(.) > 0$$

(MN) Multiplicative Negative:

$$U(C) = -\prod_{k=0}^{N} [-u(C_{N-k})];\ u(.) < 0$$

* For justification and discussion of these forms see [4], [5].

In each case the objective of the I.C. is to maximize the expected value of $U(C)$.

In the following we will concentrate on the form (S). The forms (MP) and (MN) are briefly discussed in Section 5. For more details the interested reader is referred to [6].

2.3 *Reinsurance*

We assume that in each period t there is a reinsurer who accepts any risk for the appropriate premium. The way he quotes premium is the following.

For any claims random variable ξ_t (value denoted by $x_t \varepsilon X_t$) whose probability distribution he knows, the reinsurer assigns a *price function*. $P_{\xi_t}(x_t) > 0$ such that the premium for assuming a contract $Z_t(\xi_t)$, which promises to pay to the cedent \$ $Z_t(x_t)$ at the end of period t depending on the outcome x_t of the random variable ξ_t, is given by:

$$\tilde{P}_t[Z_t(\xi_t)] \equiv \int_{X_t} Z_t(x) P_{\xi_t}(x) dx \tag{1}$$

As a marginal case consider the contract $Z_t(x) = 1; \forall x \varepsilon X_t$ which pays \$1 to the cedent at the end of period t under any event. The premium or present value of \$1 asked by the reinsurer is

$$P_t[1] \equiv \int_{X_t} P_{\xi_t}(x) dx \equiv \pi_t < 1 \tag{2}$$

In other words, $\dfrac{1 - \pi_t}{\pi_t}$ is the interest rate for period t.

The description of the reinsurance process above implies that:

1) There are no transaction costs in reinsuring.
2) Borrowing and lending rates are the same.
3) Reinsurance contracts have a span of one period. That is at the end of each period when the risks realize (the value of ξ is observed) the contracts are fulfilled and then cease to exist.

In the following we will denote by $P_t(x)$ the price function of the claims r.v. ξ_t of period t to avoid the complexity of the notation $P_{\xi_t}(x_t)$.

2.4 *Dynamic Programming formulation*

At the start of period t the I.C.'s reserves level is R_t. It immediately pays dividends C_t thus remaining with $R_t - C_t$ which by the end of the period grow to $(R_t - C_t)/\pi_t$ where

$$\pi_t \equiv \int_{X_t} P_t(x) dx \tag{3}$$

At the end of the period the I.C. collects premiums p_t and pays claims x (the value of ξ_t) and thus, if it conducted no reinsurance, the reserves level for the next period (t — 1) would be $R^{\circ}_{t-1}(x) = \dfrac{R_t - C_t}{\pi_t} + p_t - x$. With reinsurance, however, the I.C. *sells* to the reinsurer $R^{\circ}_{t-1}(x)$ and *buys* $R_{t-1}(x)$ so that the budget constraint

$$\int\limits_{X_t} R_{t-1}(x)P_t(x) = \int\limits_{X_t} \left[\frac{R_t - C_t}{\pi_t} + p_t - x\right] P_t(x)dx \tag{4}$$

is satisfied.

It will be useful to denote the premium demanded by the reinsurer for assuming the risk ξ_t by

$$\rho_t = \int\limits_{X_t} xP_t(x)dx \tag{5}$$

Now let

$f_t(R_t)$: the maximum expected utility for a t-period problem with initial reserves level R_t.

Then the problem of an I.C., whose utility function is of the form (S) above, can be written as a Dynamic Programming problem:

$$f_t(R_t) = \max_{C_t,\, R_{t-1}} \{u(C_t) + \alpha E[f_{t-1}(R_{t-1}(\xi_t))]\};\ 0 < \alpha < 1 \tag{6}$$

subject to the budget constraint (4) and with boundary condition,

$$f_0(R_0) = u(R_0) \tag{7}$$

3. Closed form Solutions

The D.P. problem formulated by (4), (6) and (7) cannot in general be solved analytically. In this section we will find closed form solutions to the problem when we additionally assume that the one-period utility function of the I.C. belongs to the Linear Risk-Tolerance (LRT) class.

The quantity $-\dfrac{u''(x)}{u'(x)}$ is known as the absolute risk aversion index (Pratt [7]). The inverse, $-\dfrac{u'(x)}{u''(x)}$ is known as the risk-toleranre index. The LRT class is then defined as the solutions to the equation

$$\frac{u'(x)}{u''(x)} = \frac{ax + b}{g} \tag{8}$$

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where g, a, b reals and $u''(x) < 0$ and $u'(x) > 0$.

It can be shown that the solutions to (8) are

$$u(x) = \frac{(ax+b)^{c+1}}{a(c+1)}; \ c \neq -1, \ ax+b > 0, \ ac < 0 \qquad \text{(Ia)}$$

$$u(x) = \frac{1}{a} \log(ax+b); \ ax+b > 0, \ a > 0 \qquad \text{(Ib)}$$

$$u(x) = \frac{1}{\gamma}(1 - e^{-\gamma x}); \ -\infty < x < +\infty, \ \gamma > 0 \qquad \text{(II)}$$

It will be useful later to split class Ia into:

$$a > 0, \quad c < -1 \quad \rightarrow u(\cdot) < 0$$

$$a > 0, -1 < c < 0 \rightarrow u(\cdot) > 0 \qquad \text{(Ia}_2\text{)}$$

$$a < 0, \quad c > 0 \quad \rightarrow u(\cdot) < 0 \qquad \text{(Ia}_3\text{)}$$

Theorem Ia (Model Ia)

If $u(x)$ belong to class (Ia) then the solution to the t-period problem as described by (6) subject to (4) and (7) is unique and is given by

$$f_t(R_t) = D_t u(A_t R_t + B_t) \qquad (9)$$

The optimal dividend strategy is

$$C_t^* = A_t R_t + B_t \qquad (10)$$

The optimal reinsurance strategy transforms the wealth of the I.C. to

$$R_{t-1}^*(\xi_t) = \frac{1}{A_{t-1}\alpha^{1/c}}\left[A_t R_t + B_t + \frac{b}{a}\right]\left[\frac{P_t(\xi_t)}{\varphi_t(\xi_t)}\right]^{1/c} + -\frac{b}{aA_{t-1}} - \frac{B_{t-1}}{A_{t-1}} \qquad (11)$$

as long as the initial reserves R_t satisfy the condition:

$$a(A_t R_t + B_t) + b > 0 \qquad (12)$$

where

$$D_t = 1 + D_{t-1}\frac{m_t}{\alpha^{1/c}}, \ D_t \geq 1 \qquad (13)$$

$$A_t = \frac{1}{D_t}, \ 0 \leq A_t \leq 1 \qquad (14)$$

$$B_t = A_t\left[p_t \pi_t - \rho_t + \frac{B_{t-1}\pi_t}{A_{t-1}} + \frac{b\pi_t}{aA_{t-1}} - \frac{b\, m_t}{a\alpha^{1/c}A_{t-1}}\right] \qquad (15)$$

can be calculated recursively starting with

$$D_0 = 1, A_0 = 1, B_0 = 0$$

and

$$m_t = \int_{X_t} \left[\frac{P_t(x)}{\varphi_t(x)}\right]^{1/c} P_t(x)dx \tag{16}$$

Proof: The proof is inductive showing the result to be valid for a 1-period problem and then proving the induction step from $t-1$ to t.

One period problem ($t = 1$)

The DP relation (6) becomes for $t = 1$

$$f_1(R_1) = \max_{C_1, R_0} \{u(C_1) + \alpha E[u(R_0(\xi_1))]\} \tag{17}$$

subject to (4) which for $t = 1$ becomes,

$$\int_{X_1} R_1(x)P_1(x)dx = R_1 - C_1 + p_1\pi_1 - \rho_1 \tag{18}$$

Fix. C_1. To maximize the second term in (17) subject to (18) according to the calculus of variation $R_0^*(\cdot)$ must be chosen so that

$$u'(R_0^*(x)\varphi_1(x) = \lambda P_1(x) \tag{19}$$

where λ is to be determined by substituting in (18).

Using the fact that $u(\cdot)$ belongs to class Ia we solve (19) to find

$$R_0^*(x) = \frac{\lambda^{1/c}}{a}\left[\frac{P_1(x)}{\varphi_1(x)}\right]^{1/c} - \frac{b}{a} \tag{20}$$

Upon substitution of (20) in (18) we find

$$\lambda^{1/c} = \frac{a}{m_1}\left(R_1 - C_1 + p_1\pi_1 - \rho_1 + \frac{b}{a}\pi_1\right) \tag{21}$$

with ρ_1, m_1 defined in (5) and (16) respectively.

Substituting (20) and (21) in (17) we obtain after some algebra:

$$f_1(R_1) = \max_{C_1}\left\{u(C_1) + \frac{\alpha m_1}{a(c+1)}\left[\frac{a}{m_1}\left(R_1 - C_1 + p_1\pi_1 - \rho_1 + \frac{b}{a}\pi_1\right)\right]^{c+1}\right\} \tag{22}$$

where we have used the identity:

$$E\left[\left(\frac{P_1(x)}{\varphi_1(x)}\right)^{1+1/c}\right] \equiv \int_{X_1}\left[\frac{P_1(x)}{\varphi_1(x)}\right]^{1/c} P_1(x)dx \equiv m_1 \tag{23}$$

The second term in the RHS of (22) is strictly concave as long as

$$a(R_1 - C_1 + p_1\pi_1 - \rho_1) + b\,\pi_1 > 0 \tag{24}$$

while the first term, $u(C_1)$, is strictly concave as long as

$$aC_1 + b > 0 \tag{25}$$

Differentiating the maximand in (22) w.r.t. C_1 and equating to zero we obtain the unique optimal dividend strategy

$$C_1^* = A_1R_1 + B_1 \tag{26}$$

with A_1, B_1 as defined in (14) and (15).

Further, when C_1 is given by (26) the conditions (24) and (25) are equivalent and thus the only condition required is

$$a(A_1R_1 + B_1) + b > 0 \tag{27}$$

Finally, substituting (26) in (22) we obtain

$$f_1(R_1) = D_1u(A_1R_1 + B_1)$$

which is in the desired form.

The t-period problem

We assume that the theorem holds for a $(t-1)$-period problem and we show that it holds for a t-period problem. The arguments are similar and we will thus be rather brief (a more detailed proof can be found in [6]).

We first fix C_t and we find that the optimal post-reinsurance wealth $R^*_{t-1}(\xi_t)$ must satisfy

$$R^*_{t-1}(x) = \frac{\lambda^{1/c}}{aA_{t-1}}\left[\frac{P(_tx)}{\varphi_t(x)}\right]^{1/c} - \frac{b}{aA_{t-1}} - \frac{B_{t-1}}{A_{t-1}} \tag{28}$$

where

$$\frac{\lambda^{1/c}}{aA_{t-1}} = \frac{1}{m_t}\left[R_t - C_t + p_t\pi_t - \rho_t + \frac{B_{t-1}}{A_{t-1}}\frac{\pi_t}{m_t} + \frac{b\,\pi_t}{aA_{t-1}}\right] \tag{29}$$

Substitution of (28), (29) in (6) yields

$$f_t(R_t) = \max_{C_t}\left\{u(C_t) + \frac{\alpha D_{t-1}m_t}{a(c+1)}\left[\frac{aA_{t-1}}{m_t}\left(R_t - C_t + p_t\pi_t - \rho_t + \frac{B_{t-1}\pi_t}{A_{t-1}m_t} + \frac{b\,\pi_t}{aA_{t-1}}\right)\right]^{c+1}\right\} \tag{30}$$

Differentiating the maximand w.r.t. C_t and setting equal to zero we find the unique optimal dividend:

$$C_t^* = A_tR_t + B_t \tag{31}$$

as long as R_t is such that

$$a(A_tR_t + B_t) + b > 0 \tag{32}$$

Finally substituting (31) in (29) and using the definitions of A_t, B_t in (14) and (15) we obtain (11) and the Theorem is proved.

Remark: If for a t-period problem the initial reserves R_t are such that $a(A_tR_t + B_t) + b > 0$ and the optimal strategies (10) and (11) are followed, then at the start of period $t - 1$ the reserves R_{t-1} will again satisfy $a(A_{t-1}\, R_{t-1} + B_{t-1}) + b > 0$. To see this we only need to observe (11). This means that following the optimal strategies for a t-period problem we are guaranteed that we will be able to reapply them for a $t - 1$ period problem with no further conditions.

Theorem Ib (Model Ib)

If $u(x)$ belongs to class (Ib) then the solution to the t-period problem as described by (6) subject to (4) and (7) is unique and is given by

$$f_t(R_t) = D_tu(A_tR_t + B_t) + E_t \tag{33}$$

The optimal dividend strategy is

$$C_t^* = A_tR_t + B_t \tag{34}$$

The optimal reinsurance strategy transforms the wealth of the I.C. to

$$R^*_{t-1}(\xi_t) = \frac{\alpha}{A_{t-1}} \left(A_tR_t + B_t + \frac{b}{a}\right) \frac{\varphi_t(\xi_t)}{P_t(\xi_t)} - \frac{b}{aA_{t-1}} - \frac{B_{t-1}}{A_{t-1}} \tag{35}$$

as long as the initial reserves R_t satisfy the condition:

$$a(A_tR_t + B_t) + b > 0 \tag{36}$$

where

$$D_t = 1 + \alpha D_{t-1}, \quad D_t \geq 1 \tag{37}$$

$$A_t = \frac{1}{D_t}, \quad 0 \leq A_t \leq 1 \tag{38}$$

$$B_t = A_t \left[p_t\pi_t - \rho_t + \frac{B_{t-1}}{A_{t-1}}\pi_t + \frac{b\,\pi_t}{aA_{t-1}} - \frac{\alpha}{A_{t-1}}\frac{b}{a}\right] \tag{39}$$

$$E_t = \frac{\alpha}{a} D_{t-1}[\log \alpha + q_t] + \alpha E_{t-1} \tag{40}$$

can be calculated recursively starting with

$$D_0 = 1, \; A_0 = 1, \; B_0 = 0, \; E_0 = 0 \tag{41}$$

and

$$q_t \equiv E\left[\log\left(\frac{\varphi_t(\xi_t)}{P_t(\xi_t)}\right)\right] \tag{42}$$

Proof: is similar to that of Theorem Ia and is deleted. For more details see [6].

Remark 1: Except (33), (40), (42) all the results of Theorem Ib can follow from Theorem Ia by letting $c \to -1$ and $m_t \to 1$.

Remark 2: The Remark at the end of Theorem Ia again holds as it can be checked by observing (35).

Theorem II (*Model II*)

If $u(x)$ belongs to class (II) then the solution to the t-period problem as described by (6) subject to (4) and (7) is unique and is given by

$$f_t(R_t) = D_t u(A_t R_t + B_t) + E_t \tag{43}$$

The optimal dividend strategy is

$$C_t^* = A_t R_t + B_t \tag{44}$$

The optimal reinsurance strategy transforms the wealth of the I.C. to

$$R_{t-1}^*(\xi_t) = \frac{1}{A_{t-1}}[A_t R_t + B_t] - \frac{B_{t-1}}{A_{t-1}} + \frac{\log \alpha}{\gamma A_{t-1}} - \frac{1}{\gamma A_{t-1}} \log\left(\frac{P_t(\xi_t)}{\varphi_t(\xi)}\right) \tag{45}$$

where

$$D_t = 1 + \pi_t D_{t-1}, \quad D_t \geq 1 \tag{46}$$

$$A_t = \frac{1}{D_t}, \; 0 \leq A_t \leq 1 \tag{47}$$

$$B_t = A_t\left[p_t \pi_t - \rho_t + \frac{B_{t-1}}{A_{t-1}}\pi_t + \frac{w_t}{\gamma A_{t-1}} - \frac{\pi_t}{\gamma A_{t-1}} \log \alpha\right] \tag{48}$$

$$E_t = \frac{D_{t-1}}{\gamma}(\alpha - \pi_t) + \alpha E_{t-1} \tag{49}$$

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can be calculated recursively starting with

$$D_0 = 1, \ A_0 = 1, \ B_0 = 0, \ E_0 = 0$$

and

$$w_t = \int_{X_t} \log \left(\frac{P_t(x)}{\varphi_t(x)}\right) P_t(x)\, dx \tag{50}$$

Proof: Similar to that of Theorem Ia. An outline of the proof appears in [6].

4. Interpretation of the Optimal Strategies

4.1 *The dividend strategy*

In all Models the optimal dividend strategy is linear in the reserves level at the start of the period. In our formulation the dividends were not restricted to be positive. Negative dividends would, of course, mean that the s/p-holders agree that an increase in the reserves *now* is desirable for better profits in the *future*. If, however, we insist that dividends should be non-negative we can easily achieve it by restricting to Models Ia_1, Ia_2, Ib with $-b/a > 0$. In the case of Model II, a sufficient condition to guarantee the non-negativity of dividends for a N-period problem is $A_N R_N + B_N \geq 0$ and $\alpha \geq \dfrac{P_t(x)}{\varphi_t(x)}$; $x \varepsilon X_t$, $t = N, \ldots, 1$. This can be seen by looking at (45). A necessary condition for the latter is $\alpha \geq \pi_t$ for all t.

4.2 *The reinsurance strategy*

We can interpret $\left(\dfrac{P_t(\phi_t}{\varphi_t(\xi_t)}\right)^{1/c}$ as a *unit* of *post-reinsurance risky asset* for Model Ia. The name is suggested by observing (11) since $\left(\dfrac{P_t(\xi_t)}{\varphi_t(\xi_t)}\right)^{1/c}$ is the only quantity which is a function of the outcome of the random variable ξ_t and its *form* is independent of the initial wealth of the I.C. In this sense, m_t can be interpreted as the *cost of a unit of post-reinsurance risky asset*. Similarly, in Model Ib (35) the unit of post-reinsurance risky asset is $\dfrac{\varphi_t(\xi_t)}{P_t(\xi_t)}$ and its cost is 1. In Model II (45) the unit of risky asset is $\log \dfrac{P_t(\xi_t)}{\varphi_t(\xi_t)}$ and its cost is w_t.

In Models Ia, Ib the amount of risky asset increases linearly with the initial reserves level, while in Model II the amount of risky asset is fixed independent of the reserves level.

https://doi.org/10.1017/S0515036100006322 Published online by Cambridge University Press

If $\dfrac{P_t(x)}{\varphi_t(x)}$ is non-decreasing in x then the post-reinsurance wealth of the I.C. is non-increasing in x in all Models. This of course means that the I.C. participates positively in the risk. That is, the larger the claims x paid to the policy-holders, the less the wealth of the I.C. after reinsurance. We can think of $\dfrac{P_t(x)}{\varphi_t(x)}$ as the *loading factor*. An increasing loading factor then means that the reinsurer asks for a greater loading to a certificate that guarantees final reserves of \$1 to the cedent when the claims x paid to the policy-holders are large than when they are small.

Further, in Models Ia and Ib the post-reinsurance wealth $R_{t-1}(\xi_t)$

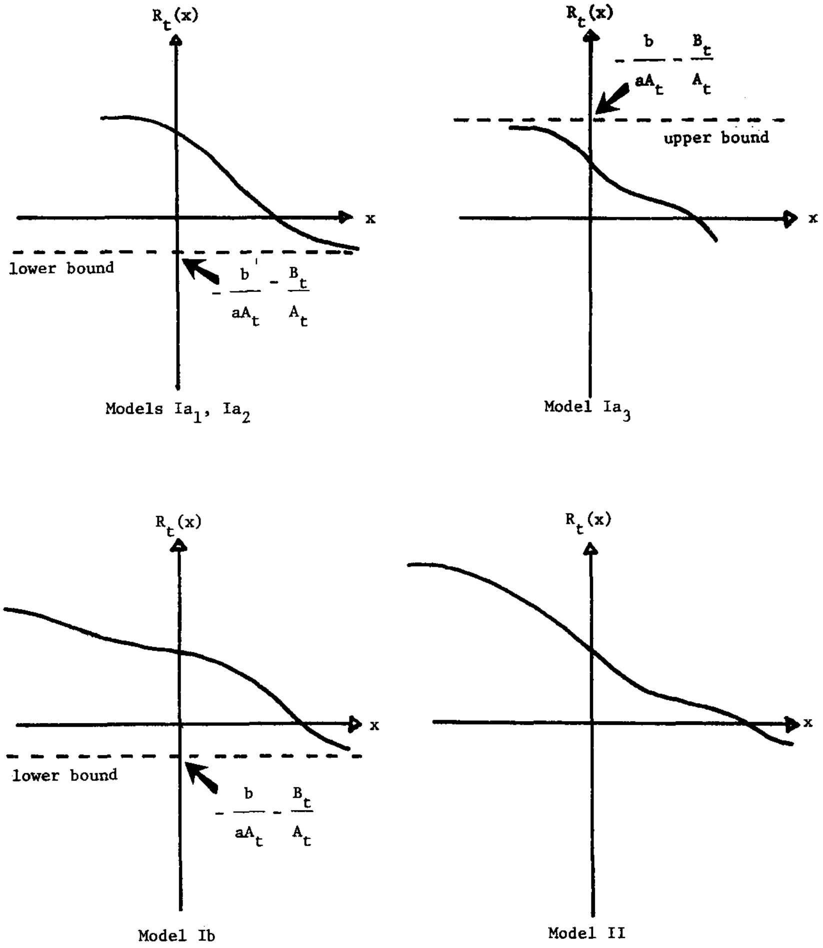

Figure 1. The post-reinsurance wealth $R_t(x)$ as a function of the claims x.

of the I.C. satisfies the condition $a(A_{t-1}R_{t-1}(\xi_t) + B_{t-1}) + b > 0$. This condition imposes upper or lower bounds on $R_{t-1}(\xi_t)$ depending on the sign of a which is negative for Model Ia_3 and positive for Models Ia_1, Ia_2 and Ib (see Figure 1).

The negativity of a makes Class Ia_3 the only one with an increasing risk-aversion index (Classes Ia_1, Ia_2, Ib have decreasing while Class II has a constant risk-aversion index). Thus Class Ia_3 (to which also the quadratic utility function belongs) must be applied with caution as it is doubtful whether it has meaning in real life (for a discussion of this point see Arrow [1]).

5. Generalizations - Extensions

(a) All Models can be easily extended to an infinite horizon by simply letting the number of periods N tend to infinity. The optimal strategies remain qualitatively the same.

(b) All Models can be generalized to include a decision on expenditures to promote sales if we assume that the sales volume is a concave function of the money spent. The optimal dividend and reinsurance strategies remain essentially the same. This is intuitively expected by observing that the quantity p_t (premiums collected from policy-holders) appears only in the constant B_t and not in A_t or D_t or E_t.

(c) Multiplicative Utilities. If instead of the form (S) we assume that the I.C.'s utility over dividend streams is given by (MN) or (MP) (Section 2.2) we can again find closed form solutions but only when (MN) is coupled with the Class Ia_1 of utility functions or (MP) with Class Ia_2. The results are similar in nature with those of Section 3. Again the optimal dividend strategy is linear in the reserves while the *form* of the post-reinsurance wealth of the I.C. is independent of its initial wealth. It only depends on the price function, the probability density function of the claims, the one-period utility function of the I.C. and the number of periods remaining.

These extensions-generalizations are treated in detail in [6].

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