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Some existence problems in differential equations approached through functional analysis

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We study three related problems. Let a and b be real numbers with $a < b$. Let f be a continuous function on $(a, b) \times R^4$. In the first two chapters we consider existence and properties of solutions of the generalized boundary value problem

$$y'' = f\big(x, y, y', y'(a), y'(b)\big) \text{ , for } a < x < b \text{ ,}$$
$$y(a) = 0 \text{ , and } y(b) = 0 \text{ .}$$

Let g be a function satisfying the Carathéodory conditions on $[a, b] \times R^2$. In Chapter 3 we consider existence and properties of solutions of the boundary value problem under 'measurability' assumptions

$$y'' = g(x, y, y') \text{ , almost all } x \text{ in } [a, b] \text{ ,}$$
$$y(a) = 0 \text{ , and } y(b) = 0 \text{ .}$$

We turn the differential equations into integral equations, obtain *a priori* bounds for the solutions and their derivatives, and obtain existence results by application of Schauder's fixed point theorem to the appropriate subset of $C'[a, b]$. We obtain *a priori* bounds by associating with the right hand side of the differential equation auxiliary functions satisfying appropriate inequalities. In the generalized problem these auxiliary functions are generalizations of those introduced by Akô [2] in the boundary value problem; we give conditions on f which ensure their

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existence. We consider the question of existence of maximum solutions (see Akô [1]) for the problems. We obtain uniqueness results for the generalized problem which extend known results for the ordinary problem.

Let A, B, y, and λ belong to R^n and $f(x, y, \lambda)$ be a function from $[a, b] \times R^{2n}$ to R^n satisfying the Carathéodory conditions. In the last two chapters we consider the problem

$$y' = f(x, y, \lambda) \text{ , almost all } x \text{ in } [a, b] \text{ ,}$$
$$y(a) = A \text{ , and } y(b) = B \text{ .}$$

Following Kibenko and Perov [3] we use the method of shooting with the initial value problem and variable λ to prove existence. We apply these results to prove an existence result for the generalized boundary value problem.

References

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