PAPER 15

THE NEUTRAL POINT DISCHARGE THEORY OF SOLAR FLARES. A REPLY TO COWLING'S CRITICISM

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ABSTRACT

The discharge theory has been criticized by Professor Cowling in §5·3 of *The Sun*, ed. Kuiper (Chicago [1]). A discharge theory requires the thickness of the accelerating layer to be only 5 m, which he finds unacceptable. This criticism is not soundly based, because no lower limit for the layer width is obtained from observation.

Secondly, Cowling invokes Lenz's law: 'the effect of induced currents is always to oppose the change to which they are due.' Lenz's law needs verifying for conducting fluids, however, and though it is usually true, it is shown to be false when there is a neutral point. Since Lenz's law refers to electromagnetic induction, other effects such as the pressure gradient may be omitted in this test, and then a completely rigorous proof is possible: it is shown that in an ideal fluid which is perfectly conducting, perfectly compressible and inviscid, the current density at a neutral point must become infinite.

The pressure gradient is irrelevant to Lenz's law, but it usually opposes any motion and should also be discussed. A physical picture suggests that the pressure gradient will not stop a vortical motion, but merely retard it. The related problem of equilibrium between the pressure gradient and the electromagnetic force may be discussed mathematically for the special case with two-dimensional symmetry. It is found that equilibrium requires an infinite current density at the neutral point. This is a subsidiary argument in favour of discharges at neutral points.

1. INTRODUCTION

The suggestion that a solar flare results from an electrical discharge situated in the neighbourhood of a neutral point of the magnetic field was made by Giovanelli [2]. He had observed a large number of flares, and his suggestion was evoked by their position in the spot groups in which they occurred; he also supported his proposal by the observation that flares are more common in complex groups. Most observers agree that the flare

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phenomenon seems to involve an electric discharge, but Cowling[1] has criticized any such possibility on theoretical grounds and his criticism has been widely accepted. The main purpose of this paper is to answer that criticism.

The defining feature of a discharge in this context is the existence of a large current density. The electrons at least must reach relativisitic energies and the order of magnitude is given by $j \sim nec$; furthermore, constriction may increase n to a value substantially larger than that in the surrounding gas. Then the relation c curl $\mathbf{H} \approx 4\pi j$ provides an estimate of the width b of the discharge, since $\frac{1}{2}b$ |curl $\mathbf{H}$| cannot exceed the value of H outside the discharge. Then $b \sim H/2\pi ne$ and with $H = 300$ gauss and $n = 10^9$ cm^{-3} this is 1 m. The width of a discharge must therefore be minute on the solar scale; at first sight this seems to be fatal to the the discharge theory, and Cowling finds it unacceptable. In section 4, however, it will be shown that this value of the width is not in conflict with any observation. Before discussing the observations, Cowling's theoretical criticism will be answered.

2. LENZ'S LAW

The fundamental difficulty raised by Cowling is Lenz's law: 'the effect of induced currents is always to oppose the changes to which they are due.' If this were true, the necessary large current density could never be built up under solar conditions. This law, however, does not have the same fundamental status as Maxwell's equations and has previously been applied only to electrical machines involving approximately uniform magnetic fields. It therefore needs verification before it can be generalized to hydrodynamics. Since the effects involved are just the induced electric field and the magnetic force density $j \times \mathbf{H}/c$ it is sufficient to consider a perfectly conducting fluid; in such a fluid the field moves with the material. Lenz's law can then be verified for currents which do not flow near any neutral point; for instance, the magnetic force of a twisted field tends to untwist it. It is found, however, that Lenz's law is reversed at a neutral point and this will now be explained.

Consider a neutral point N, where the lines of force in one plane have the form shown in Fig. 1. The limiting lines of force through N form an X and would be perpendicular, if there were no current flowing in the z-direction (normal to the paper). For the field of Fig. 1 there is a roughly uniform current in the z-direction, which contributes a field directed clockwise. The magnetic force therefore has the direction of the short arrows and tends to compress the material and field in the x-direction and stretch them in the y-direction. Since this motion reduces the acute angle between the limiting lines of force at N, it seems probable that it increases the current density. This has been verified mathematically (Dungey [3]). The pressure gradient can be omitted, since it is not involved in Lenz's law. N is chosen to be at rest initially and then does not move. The variables are the spatial gradients at N of $\mathbf{H}$ and of the velocity $\mathbf{u}$ and the mass density μ at N. The time derivatives of these variables are found to depend only on themselves, a very unusual situation in fluid dynamics. Then it is possible to work out what happens when a small perturbation is applied to a static state with $\mathbf{j} = 0$. It is only necessary to consider the signs of the variables to show that they all increase in magnitude indefinitely, and hence Lenz's law is reversed. The rate of growth of the discharge is proportional to the initial spatial gradient of $\mathbf{H}$.

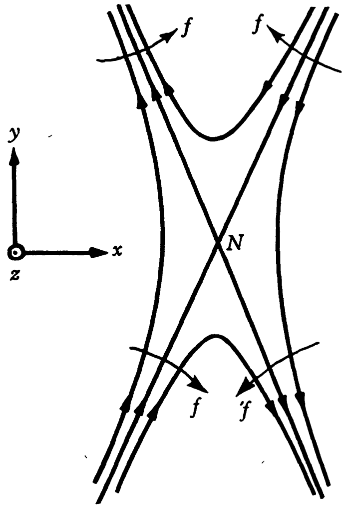

Fig. 1. The direction of the magnetic force, $\mathbf{f} = \frac{1}{c}\,\mathbf{j} \times \mathbf{H}$.

3. THE PRESSURE GRADIENT

Though the pressure gradient is not involved in Lenz's law, it needs to be investigated. No rigorous conclusion has been obtained, and Dr Sweet is at present studying the problem. The pressure gradient has a tendency to oppose the motion which produced it, but it does not by any means follow that it will stop the motion in this case. It would be more likely to stop the motion, if Lenz's law were true, so that the magnetic force were decreased by motion. With Lenz's law reversed there is a race between the two forces to build up fastest.

There is a more important reason why the pressure gradient should be unable to prevent a discharge. Since an increase of pressure must result from compression, a solenoidal motion does not build up a pressure gradient. Now it appears from Fig. 1 that a discharge can result from a solenoidal motion, and hence that it could occur even in an incompressible fluid.

Finally we may consider the possibility of equilibrium between the pressure gradient and the magnetic force. If the pressure gradient could

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prevent a discharge from occurring, a configuration of stable equilibrium should exist. The equation of equilibrium is $\mathbf{j} \times \mathbf{H} = c\nabla p$, which requires that $\mathbf{j} \times \mathbf{H}$ be irrotational. The general problem is complicated, but, when two-dimensional symmetry is imposed, the condition reduces to $(\mathbf{H}.\nabla)\mathbf{j} = 0$, and it is possible for real fields to approximate closely to two-dimensional symmetry. This problem has been studied (Dungey [3]) for a field of the 'figure eight' type shown in Fig. 2. The conclusion was that the condition of equilibrium requires an infinite current density at the neutral point N.

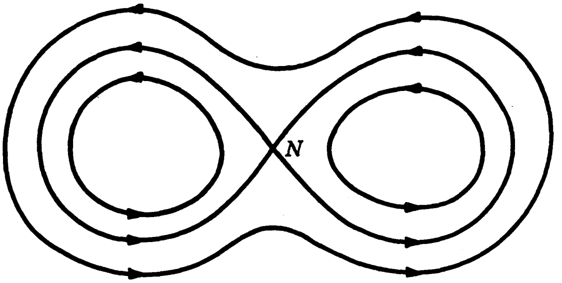

Fig. 2. A 'figure eight' field.

While a rigorous investigation is lacking, then, the indications are that the pressure gradient cannot prevent the discharge.

4. COMPARISON WITH OBSERVATIONS

It is well known that a large voltage can occur in a sunspot group; with a velocity of hundreds of km/sec, the voltage may be estimated as $\sim 10^{10}$ eV, though this could be out by an order of magnitude. It is also known that the energy of a spot field is sufficient to account for all the emissions of a flare (Kiepenheuer [4]). Since the discharge is very thin, it is desirable to check the number of particles accelerated. The discharge is thin in only one dimension and in both other directions extends over distances $\sim a$, determined by the scale of the spot field. The number of particles accelerated per second N is $n \cdot a \cdot b \cdot c$. The previous method of estimating b gives $nb \sim \mathbf{H}/4\pi e$, so that N is independent of n. With $Ha \sim 10^{10}$ gauss cm, $N \sim 5 \cdot 10^{28}$ sec^{-1}, and if these were spread over a hemisphere at the earth's distance, they would be some twenty times as numerous as the normal cosmic rays. A thin discharge is therefore capable of accelerating sufficient particles.

The sudden onset of flares agrees with the discharge theory, because the time of growth for the discharge is not many seconds.

The discharge may be situated above the chromosphere. Some accelerated particles will be shot from the discharge towards the sun, and can produce the visual flare in a manner similar to the production of aurorae in the earth's atmosphere by incoming protons. The beam of accelerated particles is probably narrow in the same direction as the discharge, so that the luminous region should also be narrow; since each particle must have many collisions in the luminous region, however, it cannot be as narrow as the discharge. The true thickness of the luminous region could only be observed, if it were seen end on, and the tendency of flares to a somewhat filamentary form seems to agree with such a model. The brightening of parts of nearby prominences, which has occasionally been observed, could be explained by the impingement of the beam on the prominence, which has a large density; this suggests that the discharge occurs above the chromosphere.

The observed increases of cosmic ray intensity at the earth are compatible with a spectrum that is cut off sharply above a quite low energy. The increase occurs about half an hour after the flare, and increases are observed after most flares of magnitude 2 or more. This behaviour might be explained by a narrow beam of cosmic rays sweeping over a large range of directions, but the correct explanation may be more complicated.

Certain other features of flares may be accounted for by the bulk motion resulting from a discharge at a neutral point. The effect of the discharge is to 'reconnect' the lines of force at the neutral point, and this happens quickly. The 'reconnection' upsets the mechanical equilibrium in the neighbourhood in a way that can be visualized, if the lines of force are seen as strings. Then the mechanical disturbance will spread from the neutral point and may have energy comparable to the energy of the spot field in the solar atmosphere. This disturbance, characterized by a sudden onset, may account for several features: surge prominences and Doppler shifts, which are probably different aspects of the same phenomenon; the emission of a mechanical disturbance responsible for magnetic storms; the activation of prominences; the triggering of other neutral points, in whose neighbourhood the field is weak, resulting in multiple flares.

The emission of radio noise could result from either the high-energy particles or the disturbance in the plasma.

REFERENCES

[1] Cowling, T. G. *The Sun*, ed. Kuiper, (Chicago, 1953), p. 587.
[2] Giovanelli, R. G. *Mon. Not. R. Astr. Soc.* **107**, 338, 1947.
[3] Dungey, J. W. *Phil. Mag.* **44**, 725, 1953.
[4] Kiepenheuer, K. A. *The Sun*, ed. Kuiper, (Chicago, 1953), p. 393.

Cowling: My comments on self-induction were limited to increases in current due to changes in conductivity. Dungey's increased currents were not due to such changes.

May I make three tentative comments on Sweet's paper? First, it seems to provide a neutral line theory rather than a neutral point theory; the field considered as a two-dimensional one. Secondly, a disturbance at photospheric level can effect the field at a higher neutral point only by the transmission of magneto-hydrodynamic waves; the events are not sudden. Would differences in travel of such waves along neighbouring lines of force upset the mechanism proposed by the lack of synchronism resulting? Lastly, in a flare we do get an abnormal temperature which can be seen to rise. A flare would not be visible if it heated the gas up to the point where recombination of hydrogen did not occur; is there not some danger of a flare being invisible if its energy is produced in too narrow a layer?

Sweet: In reply to Professor Cowling's point concerning hydromagnetic waves, the spot pair need only approach each other slowly, and a theory of the forces which are transmitted upwards can be established without the introduction of waves. The limiting pressure to sustain such a quasi-hydrostatic equilibrium is first exceeded at the neutral point. Regarding the temperature in the layer, the density may be much higher, due to the compression at the neutral point, than in the surrounding chromosphere. The cooling by radiation in such a dense gas keeps the temperature low.

Dungey: The acceleration occurs only in a thin region. Giovanelli thinks that the accelerating region is well above the chromosphere and that the visible light is secondary, like that from an aurora.

Ferraro: Does the process not depend upon the rate of approach of the pair of binary sunspots? This rate does not occur in your formulae.

Sweet: The effect does not depend on the velocity of approach of the spots, provided this velocity is small compared to that of hydromagnetic waves. As the spots reach a certain separation the theorem shows that the gas becomes unstable at the neutral point. Equilibrium breaks down and the quasi-steady layer is set up. The spots need not move further.