SNOW-LOAD EXCITATION OF THE EARTH'S ANNUAL WOBBLE

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ABSTRACT. A global, monthly snow depth data set has been generated from weather satellite (Nimbus 7) observations using passive microwave remote-sensing techniques. In this paper we analyzed five years of data, 1980-1984, to compute the snow-load excitation of the annual wobble of the Earth's rotation axis. A uniform sea-level decrease has been assumed in order to conserve water mass. The result shows dominant seasonal cycles. The prograde component of the annual excitation is Ψ^+ = (5.0 milliarcsec, -110°) and the retrograde component Ψ^- = (5.0 milliarcsec, -31°). These computed values are compared with previous groundwater estimates, as well as the inferred values from ILS and LAGEOS polar motion measurements. The importance of accurate data is stressed and future plans proposed.

1. INTRODUCTION

The seasonal mass redistribution of air and water over the Earth that excites its annual wobble has been the subject of a great deal of research since the start of this century. The annual wobble is a major component of the Earth's polar motion; the greatest contribution to its excitation is the redistribution of atmospheric mass. A relatively minor but significant contribution comes from the water storage variations on the continents; and now it is this contribution to the annual wobble excitation that is the least accurately determined (e.g., Wahr, 1983).

Generally, the continental water storage consists of (i) snow, (ii) soil moisture (stored in the root zone and available for evapotranspiration), (iii) underground storage (water percolated down below the root zone to the groundwater table), and (iv) river runoff (that has not yet found its way back to the ocean). The first, rudimentary calculation of continental water-storage excitation of the annual wobble was made by Jeffreys in 1916. Systematic methods of calculating continental water storage started with the work of Thornthwaite by mid century, and later by Van Hylckama in 1956 who obtained monthly values of the water storage

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for 10° by 10° squares of longitude and latitude. These data were used to calculate the continental water-storage wobble excitation by Munk & MacDonald (1960, p.120). The global water budget and subsequent wobble excitation calculations were later revised by Van Hylckama (1970). A complete history and discussion of this research can be found in Munk & MacDonald (1960, Chap.9) and Lambeck (1980, Chap.7).

Recently, a global, multi-year snow depth data set was developed at the Goddard Space Flight Center using satellite remote sensing techniques. This has allowed us to conduct a detailed study of the snow-load excitation of the annual wobble.

2. DATA: SATELLITE SNOW-DEPTH DETERMINATIONS

The snow depth data set under study is derived from passive microwave observations made by the SMMR (scanning multichannel microwave radiometer) on board the Nimbus 7 satellite (for a review of the physical principles behind the technique, see Foster et al., 1984). Nimbus 7 was launched on October 24, 1978, into a sun synchronous polar orbit (inclination 80°) with local noon and local midnight equatorial crossings. Here we have used 5 years of data spanning the period 1980 through 1984.

In its present form, the data set consists of monthly snow depths, each obtained by averaging data from approximately 200 night orbits. The spatial resolution is 0.5° by 0.5° in longitude and latitude, covering the Northern Hemisphere land area from the Equator to 85°N, except Greenland where the ice sheet is too thick for our present technique to sense the bottom of the snow cover. No data have been reduced for the Southern Hemisphere at the present time. The lack of data north of 85°N, where no land exists, does not concern us since (hydrostatically) floating ice does not excite polar motion. The lack of data from the Southern Hemisphere is not critical because there is very little snow except in Antarctica, which is at high latitudes and somewhat symmetric with respect to the pole, so that its contribution to wobble excitation should be relatively unimportant (c.f. Equation 1). The lack of Greenland data, however, is a deficiency which we hope to remedy in the future (see Section 5). Fortunately, Greenland is also at high latitude and so the exclusion of its snow should not be critical.

3. FORMULATION AND COMPUTATION

The basic formula for the excitation of the Earth's polar motion due to a surface loading is given by Munk & MacDonald (1960, p.106):

$$\Psi(t) = - \frac{a^4 \rho_s}{C - A} \int_{\mathrm{Land}} \Delta h(\theta,\lambda,t) \sin\theta \cos\theta \exp(i\lambda) \, dS. \qquad (1)$$

The complex polar motion excitation function $\Psi(t) = \Psi_x(t) + i\Psi_y(t)$ is expressed in units of milliarcseconds (mas), where the x- and y-axes point to the Greenwich meridian and the 90°E longitude, respectively.

The polar and equatorial principal moments of inertia are denoted by C and A, respectively, and the mean radius of the Earth by a. The density of the snow, ρ_s, is taken to be 0.3 g cm^{-3} as an average value, and $\Delta h(\theta,\lambda,t)$ is the monthly departure from the mean snow depth at colatitude θ and longitude λ as a function of time t. The integration is over the entire land area with surface element $dS = \sin\theta \, d\theta \, d\lambda$, and is to be evaluated through a summation over the 0.5° by 0.5° data grid. Note that Equation (1) has already taken into account the elastic deformation in the Earth resulting from the snow loading (c.f. Munk & MacDonald, 1960, p.41). In practice, it is not necessary to remove the mean snow depth from the monthly data. An eqivalent but more economical procedure is to compute the excitations using the snow depth data, and then remove their time averages.

In order to conserve water mass, we make the assumption that the water stored as continental snow originates in the ocean. We further assume, for simplicity, that this results in a uniform decrease in the sea level over the entire ocean, given by $\Delta d(t)$ = (total snow mass)/ (ρ_w•area of ocean), where ρ_w = 1 g cm^{-3} is the water density. This variation in sea level, in turn, provides additional excitation of the polar motion. This ocean contribution can be readily evaluated from an expression similar to (1) except that now ρ_w is substituted for ρ_s, $\Delta d(t)$ for $\Delta h(\theta,\lambda,t)$, and the integration is over the ocean. Because of the orthogonality of spherical harmonic functions, this integration depends only on the spherical harmonic coefficients of degree 2 and order 1 of the "ocean function" (which assumes value 1 over the ocean and 0 over the land, see e.g., Lambeck, 1980, p.51-53). The ocean contribution to the polar motion excitation is thus found to be

$$\Psi(t) = -(1.70 + 2.16i)\, \Delta d(t) \quad \text{mas} \tag{2}$$

where the sea level decrease Δd is in centimeters. The net snow-load excitation function is then the sum of (1) and (2).

4. RESULTS

In a typical year (during 1980-1984), the snow accumulation culminates in February-March with a total mass of 3.2 X 10^{18} g, and dwindles to less than 0.02 X 10^{18} g in July-August. The corresponding sea-level variation has an amplitude of about 0.9 cm.

The net snow-load excitation function $\Psi(t)$ obtained by summing (1) and (2) is presented in Figure 1. The seasonal cycle is clearly evident, and Ψ_y exhibits the greatest variation due to the geographical distribution of the continents along this axis. The Fourier power spectrum (not shown) of the complex $\Psi(t)$ clearly shows the dominant annual signal as well as almost every higher harmonic in both prograde (positive) and retrograde (negative) frequencies.

Next we shall compare our computed $\Psi(t)$ with that inferred from polar motion measurements. Here we shall only concentrate on the annual wobble because the higher harmonics, being distant in frequency from the Earth's natural Chandler frequency, are dynamically suppressed and have rather low signal-to-noise level.

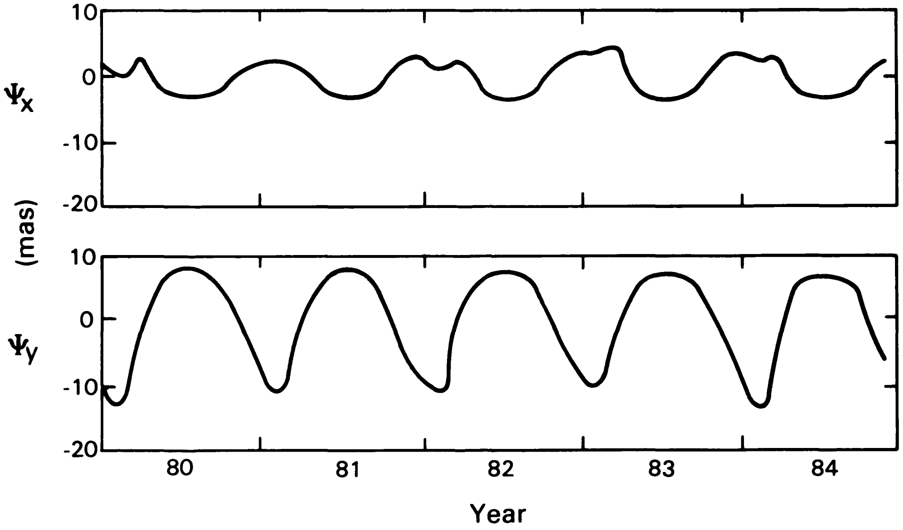

Figure 1. The excitation function, in milliarcseconds, of the polar motion due to continental snow load for the period 1980-1984. The x- and y-axes point to the Greenwich meridian and the 90°E longitude, respectively.

To extract the annual signal from $\Psi(t)$, we performed a least-squares fit of an annual sinusoid to $\Psi_x(t)$ and $\Psi_y(t)$ individually. The result, in mas, is $\Psi_x(t) \simeq 1.397 \sin\omega t + 3.047 \cos\omega t$, and $\Psi_y(t) \simeq -3.857 \sin\omega t - 8.561 \cos\omega t$, where $\omega = 2\pi/(365\text{days})$. The nominal origin of time t=0 is at mid-January. We then convert the above annual fit to the prograde component $\Psi^+\exp(i\omega t)$ and the retrograde component $\Psi^-\exp(-i\omega t)$ (see Munk & MacDonald, 1960, p.47). The complex coefficient "vectors" Ψ^+ and Ψ^- are then rotated "backwards" in the complex plane (retrograde or clockwise for Ψ^+ and prograde or counterclockwise for Ψ^-) by 15° to bring the origin time t=0 to January 1. The final result is

$$\Psi^+ = -1.7 - 4.7i \text{ mas}, \quad \text{or} \quad (5.0 \text{ mas}, -110^\circ) \tag{3a}$$

$$\Psi^- = 4.3 - 2.6i \text{ mas}, \quad \text{or} \quad (5.0 \text{ mas}, -31^\circ). \tag{3b}$$

These "vectors" are plotted respectively in Figures (2a) and (2b), labelled C.

In Figure 2 we have also plotted other $\Psi^\pm$ estimates for reference. The vector labelled ILS indicates the inferred values from the International Latitude Service measurements for the period 1900-1979 (e.g., Wilson & Vicente, 1980), while LAGEOS indicates those from the LAGEOS satellite laser ranging measurements for the period 1977-1985 (determined by R. Gross, personal communication, 1986). These are the astrometric observations for which we are seeking complete geophysical explanations. Presumably (see Section 1) a major part has its origin in the atmosphere; but it is evident from Figure 2 that the snow-load contribution is indeed considerable.

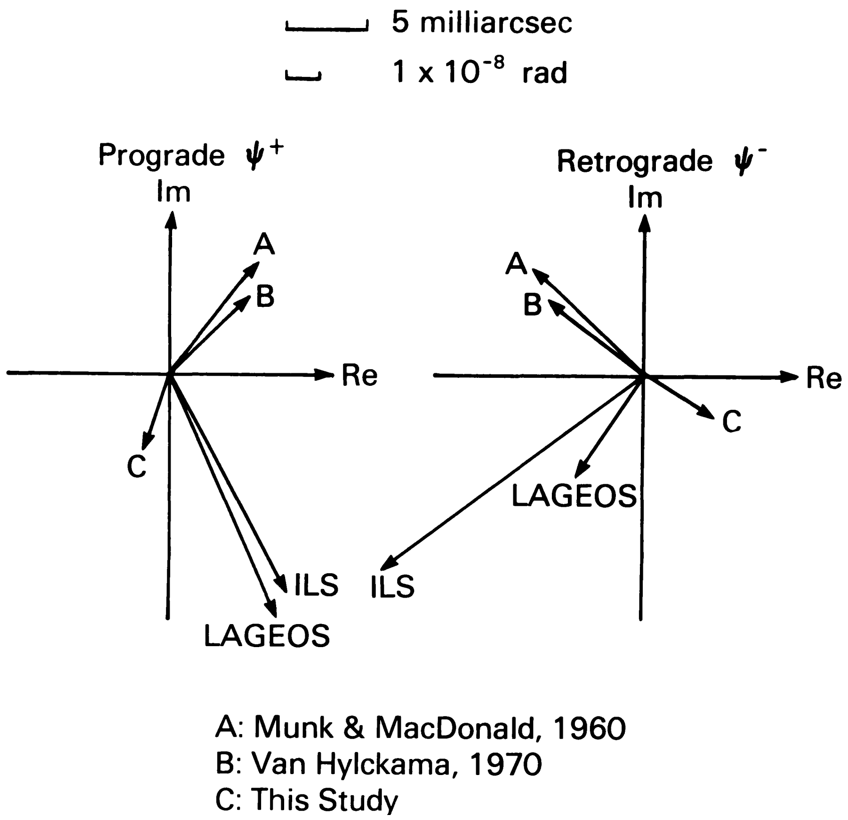

Figure 2. The prograde and retrograde components of the annual wobble excitation function as of January 1: 'C' indicates the snow-load excitation derived in this study from satellite snow depth data; 'A' indicates Munk & MacDonald's (1960) value for the total water-storage excitation based on Van Hylckama's (1956) data; and 'B' indicates Van Hylckama's (1970) estimate based on his revised data. 'ILS' and 'LAGEOS' are, respectively, the inferred values from the ILS and LAGEOS polar motion measurements.

The vectors labelled A are those obtained by Munk & MacDonald (1960, based on Van Hylckama's 1956 data) for continental water storage (of which the snow is a component). The vectors labelled B are those determined by Van Hylckama (1970) for the continental water storage using his revised data. It is interesting to note that in both prograde and retrograde cases our vector C is in almost the opposite direction to the vectors A and B. This will be discussed in the next Section.

5. DISCUSSIONS AND FUTURE PLANS

We have also analyzed a global snow depth data set gathered by conventional climatological means -- the data set published by the Rand Corporation (and distributed by the World Data Center A for Glaciology). It consists of 12 mean-monthly snow depth values for each 4° of latitude by 5° of longitude grid, obtained from empirical evaluations and special Rand methodology. Unfortunately, we were not able to locate any detailed description of the procedure.

We computed the continental snow-load excitation (Equation 1) for the Northern Hemisphere Rand data. We found that the result, unlike the above satellite result, does not exhibit any seasonal behavior and appears unrealistic. The problem was traced to the following observation embodied in Equation (1): as a result of the Northern Hemisphere "bimodal" geography, the wobble excitation is essentially the result of the snow-load difference between Eurasia and North America (including Greenland). As far as the polar motion excitation is concerned, the two continental snow loads act to cancel each other out (a "tug-of-war" analogy). A small error in the snow estimation can thus be greatly magnified. We believe this is indeed the case with the Rand data set: for example, according to the Rand data, the snow load in Eurasia is nearly the same as that in North America in the month of January, so much so that there is a more than 99% cancellation between the contributions to Ψ_y from each continent. Furthermore, the Rand data dictates that, during the yearly cycle, only in February and October does Eurasia have more snow load than North America. In light of these problems, we decided not to look into the Rand data any further.

The moral is that the importance of accurate data cannot be overemphasized; and this is where satellite data can be of great value. A corollary is that one should really be critical of the validity of Van Hylckama's estimations presented in Figure 2 (the vectors A and B). Thus whether the snow contribution to the annual wobble excitation (the vector C, Figure 2) is truly opposite to (or, for that matter, in any other relations with) the total water storage contribution remains to be determined. At any rate, we believe that our determination of the vector C represents one step closer to the solution of the annual wobble excitation problem.

In addition to the polar motion excitation Ψ, we have also computed the variations in the length of day, ΔLOD, and the Earth's dynamic oblateness, ΔJ_2, due to the seasonal snow load. For the calculation of ΔJ_2 and ΔLOD the incompleteness of the satellite snow data at high latitudes becomes a limitation because of its lack of Antarctic data -- while Antarctica is unimportant for Ψ (see Section 2), whether it is so for ΔJ_2 and ΔLOD is not clear. These results will be reported in detail elsewhere; here we only mention in passing that the annual peak-to-peak variation in ΔJ_2 due to snow load is about 2×10^{-10}, and that in ΔLOD is about 40 microseconds.

In the near future we intend to improve our satellite snow data in several ways. We will improve the calibration accuracy, especially for Greenland and Antarctica, as well as introduce a more realistic snow density model (giving ρ_s as a function of depth, for instance). We will also improve the temporal resolution of the data from a month to perhaps a few days. The latter, together with a longer time series which we will have as more data accumulate, are essential for a study of the snow-load contribution to the excitation of the Chandler wobble in the polar motion.

In the long run, we intend to look into other satellite remote-sensing data sets that bear importance to the water-storage excitation of the polar motion. For example, global rainfall, soil moisture, and the seasonal biomass variation (which presumably determines the amount of evapotranspiration) may all be deducible from existing satellite observations. Finally, we stress that a long term, accurate altimetric monitoring of the ice sheets on Greenland and Antarctica will also prove valuable in the study of polar motion excitations.

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DISCUSSION

Shapiro: It is surprising that the total ground water value is 180° out of phase with snow load values, and also that it is nearly 90° out of phase with the ILS/Lageos value. Even if the former relation is a coincidence (based on "bad" ground water data), do you have any physical explanation, at least, for the latter nearly orthogonal relation?

Reply by Chao: Not at present. The observed ILS/Lageos annual wobble is, of course, primarily caused by the atmospheric variation. Any further discrepancy between our snow load values and the ILS/Lageos values is to be accounted for by other components in the seasonal continental water storage, notably the soil moisture.

Herring: Could not the southern hemisphere be important because the snow masses will not be compensated by snow on continents in the other hemisphere?

Reply by Chao: This is an interesting observation. The only substantial snow field outside Antarctica is in South America and its snow mass is only a few percent of the northern hemisphere snow mass. Its contribution is in phase with and probably comparable to that of Greenland. For completeness we intend to include the southern hemisphere in our future calculation.

Rochester: I enjoyed your description of the 'cold war' in monthly snow loads between Eurasia and North America. But your transparency seemed to suggest that the contribution of sea level decrease to wobble excitation is to be multiplied by the density of snow rather than of sea water! Comments?

Reply by Chao: The sea level contribution was actually calculated according to the total snow mass and the ocean function, and the density of water (1 gm cm^{-3}) was used.

Dickey: Your lower ψ curve is approximately sinusoidal, while your upper ψ plot is much more irregular. Is there an explanation for this?

Reply by Chao: The reason that ψ_x appears more irregular than ψ_y is because of its smaller amplitude since there is less snow load along the Greenwich Meridian–International Date Line.