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BOOK REVIEWS

FREEDEN, W., GERVENS, T. and SCHREINER, M. *Constructive approximation on the sphere with applications to geomathematics* (Numerical Mathematics and Scientific Computation, Clarendon Press, 1998), xv + 427 pp., 0 19 853682 8, £65.

This book provides an excellent modern resource for all workers interested in approximation on spherical geometries. As such it contains a very complete treatment of approximation techniques which take essential account of the special geometrical properties of the sphere, namely that it is a smooth closed manifold in $\mathbb{R}^3$ with special symmetries. The approximation techniques covered by the book include not only classical spherical harmonic approximation but also other techniques such as 'spherical splines' and 'spherical wavelets' which have a more local character, but which still exploit the spherical symmetry. Purely local techniques, such as piecewise polynomial approximation on local charts, are not included here.

Professor Freeden and his colleagues have made essential contributions to the development of this theory over more than two decades and the encyclopaedic list of references at the end of the book reflects these contributions and provides a general resource in this large and active field.

The starting point for the book is the classical theory of spherical harmonic approximation and the first five chapters are essentially devoted to this topic and to various related theoretical issues. Writing as someone who has had to delve into the labyrinthine classical literature on this field on a number of occasions, this reviewer finds the modern unified treatment here extremely appealing. I do not know of another text where one would find in a single place both the classical results for spherical harmonics (e.g. addition theorems and properties of the Laplace series) and the functional analytic development of reproducing kernel Sobolev spaces on the sphere (defined using powers of the Laplace–Beltrami operator). In this book these fundamentals are combined with exciting modern approximation techniques, for example those involving radial basis functions and wavelets.

I was slightly disappointed not to find more discussion of applications to geomathematics in the book, although pointers to the many contributions of the authors in this field are given in the references.

As well as the results mentioned above, Chapter 5 includes a discussion of the important *uncertainty principle*, which identifies the mutual exclusivity of localization in the space domain and in the frequency domain. (Here the transform from space domain to frequency domain is provided by the generalized Fourier expansion in spherical harmonics.) This principle, which is illustrated in Chapter 5 using several types of radial basis function approximation, informs the development of several of the later chapters, particularly Chapter 6 on spherical spline interpolation and Chapters 10 and 11 on spherical wavelet approximation. In between these there are Chapter 7, which treats numerical integration (including rules generated from integrating either polynomial or spline interpolants), Chapter 8 on singular integrals (used for generating certain types of theoretical approximation) and Chapter 9 on the Gabor and Toeplitz transforms (which have applications in signal processing).

After the two substantial chapters on wavelet approximation the authors complete the book by giving four more chapters containing vector and tensor counterparts of some of the scalar results obtained in Chapters 1–11.

This book fills a large gap in the literature by giving a modern treatment of spherical approximation techniques which starts from first principles but reaches modern developments. It will be of interest to researchers engaged in a wide range of practical problems arising from spherical geometries, as well as theoreticians who are interested in the challenging range of approximation theoretic problems (many of which are still open) arising from the special non-Euclidean structure of the spherical geometry.

I. G. GRAHAM

ROBERTS, P. C. *Multiplicities and Chern classes in local algebra* (Cambridge Tracts in Mathematics no. 133, Cambridge, 1998), xi + 303 pp., 0 521 47316 0 (hardback), £37.50 (US$59.95).

In 1957, Serre [**3**] introduced an algebraic measure of intersection multiplicity for two finitely generated modules M and N over a regular local ring $(A, \mathfrak{m})$. If the supports of M and N intersect in the maximal ideal $\mathfrak{m}$, then this multiplicity is given in the form of an Euler characteristic, with the length of $M \otimes_A N$ being adjusted by an alternating sum of torsion terms. In particular, if $M = A/\mathfrak{p}$ and $N = A/\mathfrak{q}$, with $\mathfrak{p}$ and $\mathfrak{q}$ corresponding to the vanishing ideals of subvarieties V and W, respectively, of a smooth variety, which meet in a point P, then this multiplicity gives the multiplicity of the intersection of V and W at P. Serre proved various natural properties of this intersection multiplicity in the case where A contained a field and posed some conjectures about the corresponding situation when A did not. These conjectures, which were homological in nature, were tackled by Peskine and Szpiro, by Hochster, by the author, and others, and soon gave rise to a host of new results and techniques in Commutative Algebra/Algebraic Geometry, and to further conjectures. New algebraic methods (use of the Frobenius, big Cohen–Macaulay modules, Tight Closure, etc.) were boosted by geometrical methods arising from work of Fulton and colleagues (see [**1**]) or from K-theory, and it is in the use of the latter, at the hands of the author and others, where some of the most striking advances have taken place in this general area of the so-called Homological Conjectures. The present book aims to survey these developments, concentrating mainly but not exclusively on the input from geometry, while emphasizing algebraic methods. As such, it is very welcome, especially, but not exclusively, to algebraists.

The book is in two parts. The main, second part is devoted to giving an account of the notion of so-called local Chern characters, first in the case of matrices and then in the more general case of complexes, and the associated notion of the Todd class. This is based on applying the theory of Chern classes of locally free sheaves (on quite general multi-graded rings) to the Grassmannian. The crucial properties of these local characters (an additivity and multiplicativity formula, a Riemann–Roch theorem and local Riemann–Roch formula, mostly all based on a splitting