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Testing Reynolds stress model in solar interior

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Abstract. The Reynolds stress model (RSM) for turbulent convection motion is compared to the MLT in solar model. The free parameters involved in the RSM are also tested with the aid of helioseismology. It is found that, the structure of solar convection zone is differ from the MLT when using the RSM, especially for the Reynolds correlations and the temperature gradient. Both the local and non-local RSM can improve the calculated solar p-mode oscillation frequencies with the appropriate choice of the parameters' value.

Keywords. Convection, turbulence, sun: interior, sun: oscillations

1. Basic equations and solar models

Reynolds stress model (RSM) based on fully hydrodynamic moment equations provides us a better way to study convective motions in stars than the MLT. The second-order moment equations for turbulent convection can be find in many lectures (see for example Canuto 1997, Canuto & Dubovikov 1998, Xiong 1977, Li and Yang 2001). With some closure approximations, we can apply these equations to the convection zone of a spherically symmetric star. The final equations we used in the work are:

$$\frac{1}{\overline{\rho}r^2}\frac{\partial}{\partial r}\left(C_s\overline{\rho}r^2\frac{k}{\varepsilon}\overline{u_r'u_r'}\frac{\partial\overline{u_r'u_r'}}{\partial r}\right)=\frac{2\beta g_r}{\overline{T}}\overline{u_r'T'}+\frac{2}{3}\varepsilon+C_k\frac{\varepsilon}{k}\left(\overline{u_r'u_r'}-\frac{2}{3}k\right), \tag{1.1}$$

$$\frac{1}{\overline{\rho}r^2}\frac{\partial}{\partial r}\left(C_s\overline{\rho}r^2\frac{k}{\varepsilon}\overline{u_r'u_r'}\frac{\partial k}{\partial r}\right)=\frac{\beta g_r}{\overline{T}}\overline{u_r'T'}+\varepsilon, \tag{1.2}$$

$$\frac{2}{\overline{\rho}r^2}\frac{\partial}{\partial r}\left(C_{t1}\overline{\rho}r^2\frac{k}{\varepsilon}\overline{u_r'u_r'}\frac{\partial\overline{u_r'T'}}{\partial r}\right)=\frac{\overline{T}}{c_p}\frac{\partial\overline{s}}{\partial r}\overline{u_r'u_r'}+\frac{\beta g_r}{\overline{T}}\overline{T'^2}+C_t\left(\frac{\varepsilon}{k}+\frac{\lambda}{\overline{\rho}c_p}\frac{\varepsilon^2}{k^3}\right)\overline{u_r'T'} \tag{1.3}$$

$$\frac{1}{\overline{\rho}r^2}\frac{\partial}{\partial r}\left(C_{e1}\overline{\rho}r^2\frac{k}{\varepsilon}\overline{u_r'u_r'}\frac{\partial\overline{T'^2}}{\partial r}\right)=\frac{2\overline{T}}{c_p}\frac{\partial\overline{s}}{\partial r}\overline{u_r'T'}+2C_e\left(\frac{\varepsilon}{k}+\frac{\lambda}{\overline{\rho}c_p}\frac{\varepsilon^2}{k^3}\right)\overline{T'^2}. \tag{1.4}$$

As we seen, our RSM introduced 6 new parameters C_t, C_e, C_k, C_{t1}, C_{e1}, and C_s. Each parameter characterizes a specific process of turbulence and has clear physical meaning. C_t and C_e measures the dissipation of Reynolds correlation $\overline{u_r'T'}$ and $\overline{T'^2}$, C_k determines the anisotropic degree of turbulence. C_{t1}, C_{e1} and C_s described the non-local effects of the turbulence. They are the diffusion parameters of the Reynolds correlations.

Three series of solar models are calculated: the standard solar model (SSM) using the MLT, local convection solar models (LSMs) using our local RSM, and non-local convection solar models (NLSMs) using our non-local RSM. The detailed information of the value of parameters is shown in Table 1.

Table 1. The information of model parameters

Model	C_t	C_e	C_k	C_{t1}	C_{e1}	C_s	α	Y_0
SSM							1.64017	0.275063
LSM	3.0	1.25	2.5				0.90580	0.275062
LSMa1	1.0	1.25	2.5				0.415325	0.275064
LSMa2	9.0	1.25	2.5				2.01495	0.275064
LSMb1	3.0	0.25	2.5				0.46681	0.275064
LSMb2	3.0	6.25	2.5				1.19033	0.275061
LSMc1	3.0	1.25	1.0				0.70914	0.275063
LSMc2	3.0	1.25	6.25				1.02898	0.275062
NLSM	3.0	1.25	2.5	0.08	0.2	0.05	0.88541	0.275060
NLSMa1	3.0	1.25	2.5	0.05	0.2	0.05	0.87977	0.275063
NLSMa2	3.0	1.25	2.5	0.11	0.2	0.05	0.89214	0.275060
NLSMb1	3.0	1.25	2.5	0.08	0.1	0.05	0.89450	0.275060
NLSMb2	3.0	1.25	2.5	0.08	0.3	0.05	0.87760	0.275062
NLSMc1	3.0	1.25	2.5	0.08	0.2	0.02	0.88495	0.275061
NLSMc2	3.0	1.25	2.5	0.08	0.2	0.08	0.88549	0.275060
LSMn	1.0	0.25	1.01				0.210778	0.275070
NLSMn	1.0	0.25	1.01	0.1	0.15	0.04	0.21024	0.275067

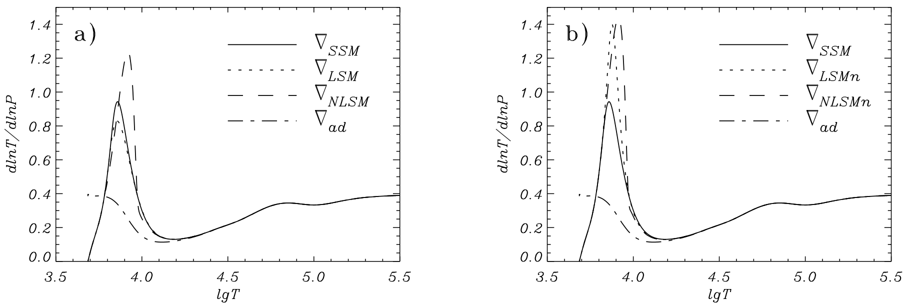

Figure 1. Distributions of the temperature gradient of some solar models in the solar convection zone. ∇_{ad} is the adiabatic temperature gradient of SSM.

2. Results

In our present work, all 17 solar models showed in Table 1 were evolved from the zero-age main sequence to reach the present solar age. We investigate the structure of solar convection zone carefully. It is found that, compared with the SSM, our solar models with RSM have small but meaningful change in the structure of the convection zone, especially the temperature gradient. Figure 1 shows the distribution of the temperature gradient in some solar models. We notice the temperature gradient of different solar models are different in the superadiabatic convection zone (SCZ). Xiong & Chen (1992) pointed out that their non-local RSM gave a larger temperature gradient than the MLT did in the upper SCZ. Our result shows that, with appropriate choices of the free parameters, even the local RSM can give the larger temperature gradient than the MLT does, almost as large as the non-local RSM gives (see Fig. 1b). Baturin & Mironova (1998) suggested that assume higher temperature gradient in the solar SCZ will help to lower the calculated p-mode frequencies. We will see this effect in the following.

In order to test the values of the free parameters in stellar condition, we calculated the p-mode oscillation frequencies of all 17 solar models showed in Table1, and comparing them with the observed ones obtained by GONG on November 8, 2004. It is found that, with appropriate choices of the turbulent parameters, both LSMs and NLSMs can

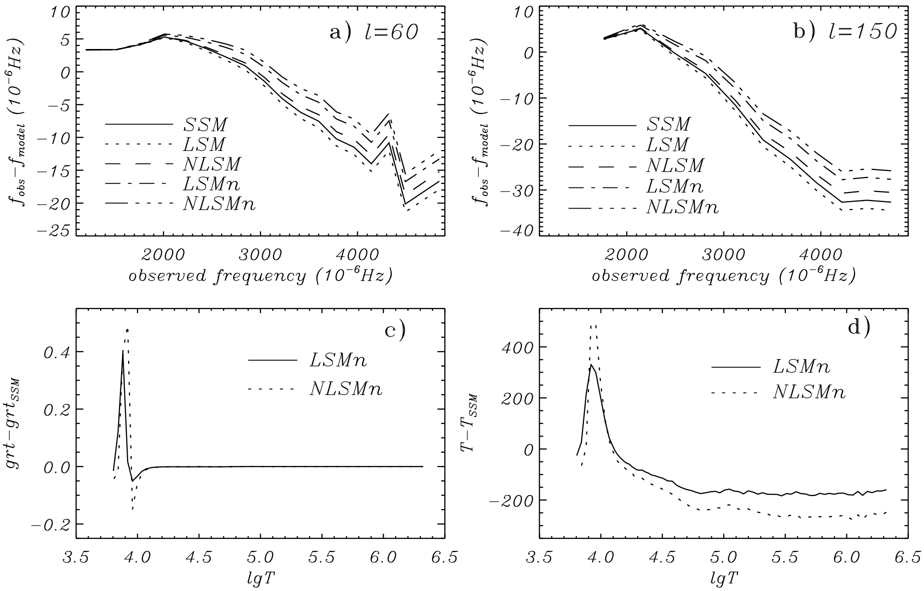

Figure 2. (a) and (b): The frequency differences of some solar models compared with observations for mode l=60 and l=150. (c) and (d): The differences of temperature gradient and temperature for solar models LSMn and NLSMn compared with SSM in the convection zone.

give the calculated p-mode frequencies more consistent with the observations than the SSM does. Decreasing the values of parameters C_t and C_e, or increasing the values of parameters C_{t1} and C_{e1} helps to reduce the calculated frequencies. The values of parameters C_k and C_s has no obvious effect on the calculated p-mode frequencies. Based on these results, we built two solar models LSMn and NLSMn by combined better values of the free parameters. As we can see in Figure 2a and 2b, LSMn and NLSMn can reduce the frequency differences between the calculations and the observations as much as 25 percent for the modes of middle and high l. The reason can be found in Fig. 2c and 2d. It can be seen from these figures that the temperature gradient of our improved solar models LSMn and NLSMn is different from that of SSM and thus the temperature is lowered by about 150-250K in the inner convection zone. It is due to such a temperature depression that makes the sound speed lowered and the calculated frequencies decreased in comparison with those of SSM. This result can give a clue to solar modelling, that is, including turbulence in solar model is help to reproduce the observed solar p-mode oscillation frequencies. This work is sponsored by the NSFC through project number 10303007, and utilizes solar p-mode frequency data obtained by the GONG Program.

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