

Appendix 11

The Feynman rules for QCD

We present here a list of the Feynman rules for QCD that are valid in two classes of gauge:

- the *covariant* gauges labelled by a parameter ‘ a ’ ($a = 1$ is the Feynman gauge; $a = 0$ the Landau gauge) in which the subsidiary condition, at least at the classical level, is $\partial^\mu A_\mu^c = 0$ for all values of the colour label c , and the gauge-fixing term in the lagrangian is $\frac{1}{2a} \sum_c (\partial^\mu A_\mu^c)^2$;
- an *axial* gauge, one of a family again labelled by ‘ a ’, in which the subsidiary condition is $n^\mu A_\mu^c = 0$ for all c , where n^μ is a fixed space-like or null 4-vector, and where the gauge-fixing term in the lagrangian is $\frac{1}{2a} \sum_c (n^\mu A_\mu^c)^2$.

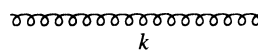
We allow the quarks to have a mass parameter m , which should be put to zero when working with massless quarks.

(a) *The propagators*

$$\begin{array}{l}
 \text{lepton} \quad \xrightarrow{\quad p \quad} \quad \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} \\
 \text{quark } j \quad \xrightarrow{\quad p \quad} \quad l \quad \delta_{jl} \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}
 \end{array}$$

In the above the arrow indicates the flow of fermion number and p is the 4-momentum in that direction. (Note: j, l are quark colour labels, b, c

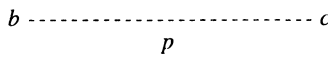
gluon and ghost colour labels.)

gluon: b, β  c, γ
 k

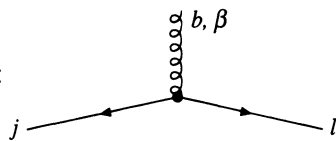
$$\delta_{bc} \frac{i}{k^2 + i\epsilon} \times \begin{cases} \text{covariant gauges:} \\ \left[-g_{\beta\gamma} + (1-a) \frac{k_\beta k_\gamma}{k^2 + i\epsilon} \right] \\ \text{axial gauges with } a = 0: \\ \left[-g_{\beta\gamma} + \frac{n_\beta k_\gamma + n_\gamma k_\beta}{n \cdot k} - \frac{n^2 k_\beta k_\gamma}{(n \cdot k)^2} \right] \end{cases}$$

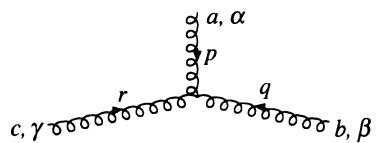
Note that in the above axial gauges the propagator is orthogonal to n^β , and it is orthogonal to k^β when $k^2 = 0$.

ghost:

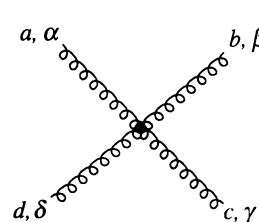
b  c $\delta_{bc} \frac{i}{p^2 + i\epsilon}$ (covariant gauges only).

(b) The vertices

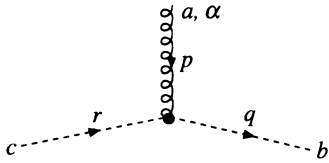
quark–gluon vertex:  $ig(\mathbf{t}^b)_{lj} \gamma^\beta$

triple-gluon vertex:  $gf_{abc} \left[g^{\alpha\beta} (p - q)^\gamma + g^{\beta\gamma} (q - r)^\alpha + g^{\gamma\alpha} (q - r)^\beta \right]$

where p, q, r are momenta, with $p + q + r = 0$.

quartic gluon vertex:  $-ig^2 \left[f_{eac} f_{ebd} (g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\delta} g^{\beta\gamma}) + f_{ead} f_{ebc} (g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\gamma} g^{\beta\delta}) + f_{eab} f_{ecd} (g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\gamma}) \right]$

gluon-ghost vertex:



$$-gf_{abc}q^\alpha \quad (\text{covariant gauges only})$$

$$(p + r = q)$$

Note that the ghosts are scalar fields, but a factor -1 must be included for each closed loop, as in the case for fermions.