of practical utility and that at which the Schlömilch series becomes applicable.

The statement quoted by Prof. Insolera in (2) refers to his formula IV′. This formula is given for $a_{x_1x_2\ldots x_m}$ without any explicit limitation of its applicability, and the words at the beginning of the investigation "quando si abbia da fare con gruppi di pochi elementi, cosi che q_m sia abbastanza piccolo" (when one has to do with combinations of a few lives so that q_m is sufficiently small) would not, we think, lead the ordinary reader to suppose that the formula does not apply to the calculation of a joint-life annuity on three lives of 60.

With regard to the approximation to K given in (3) it should be borne in mind that the expression is derived from formula IV′ and is of limited application. It appears to give $a_{65\,.\,65}=6{\cdot}375$, the true value being 5·486. The approximate formula $a_{xy}=a_w+\log_e s(\text{I}a)_w$, gives (without using tables at more than one rate of interest) the correct result 5·486. —Eds. *J.I.A.*].

MORTALITY AMONG NEUTRALS IN WAR-TIME.

To the Editors of the Journal of the Institute of Actuaries.

Dear Sirs,—Those Members of the Institute who read Professor Hersch's paper "La Mortalité chez les Neutres en Temps de Guerre", reviewed in *J.I.A.*, vol. l, p. 72, will remember that in this paper the author endeavoured to answer the question: "Which classes "of a population are most seriously affected by the indirect effect "of a War?"

The method adopted by the author was to consider the increase of mortality due to a War as the *absolute difference* between the mortality experienced in a time of War and the normal mortality of a time of peace, and to compare the results thus obtained for the different age groups. The method was, in fact, equivalent to a comparison of $q_{ng}-q_n$ age-group by age-group, where q_{ng} represents the mortality from all causes, including the indirect effect of a War, and q_n the normal mortality.

The same subject was dealt with by Mr. J. W. Nixon in his paper "War and National Vital Statistics with Special Reference "to the Franco-Prussian War"—Journal of the Royal Statistical Society, vol. lxxix, part 4. In this paper the author contended that the proper method of comparison was to compare, not the absolute, but the percentage increase in mortality, *i.e.*, not $q_{ng}-q_n$, but

$$\frac{q_{ng}-q_n}{q_n}$$

By adopting this method Mr. Nixon arrives at the following conclusions:

"It will be seen that the highest age-groups generally show the "smallest increases in mortality, the highest increases being shown "at periods of young and middle life. This, I think, is contrary "to expectation. It in also contrary to certain definite conclusions "based on the same figures recently made in a Swiss publication. "This pamphlet states that the effects of War on mortality are the "greatest at the two extremes of life, and are lowest at the ages "10 to 14. . . . . . The method by which these conclusions are "reached is fallacious."

Mr. Nixon then discusses Prof. Hersch's method in a foot-note:

"As showing the incidence of mortality on the two sexes the "author seems quite sound, but in dealing with the incidence of "mortality at different ages of life his analysis is statistically quite "unsound. . . . . . The author takes the mean number of deaths "in the years 1870–71, and finds the excess in these two years "over the year 1869 per 1,000 of the population living at each age- "group. He thus obtains a curve which reproduces in general "outline the usual U-shaped mortality curve for ages, namely, a "very high death-rate at the two extremes of life, and a low death- "rate for the intervening ages. As persons over 65 had a high "death-rate in 1869, and also a high death-rate in the years "1870–71, the difference between these two is likely to be much "higher than the difference between the death-rates of a young "age-group, *e.g.*, 15–20. By using this method of absolute increase "in death-rates the Author, in effect, says that a rise in the death- "rate from (say) 10 to 15 per 1,000—*i.e.*, a rise of 5 per 1,000— "is much more serious than a rise in the death-rate from (say) "3 to 6 per 1,000—*i.e.*, one of 3 per 1,000. The former, however, "is a rise of 50 per-cent, the later of 100 per-cent, and these "figures are the ones which should be compared."

These remarks of Mr. Nixon's appear to have aroused considerable interest in Italy, and have given rise to two papers, the first of which, by Corrado Gini, appeared in the September–December 1916 number of the "Rivista Italiana di Sociologia", while the other by F. P. Cantelli, entitled "Sull 'Aumento di "Mortalità dovuto alla guerra. Riflessioni critiche di metodologia "statistica", originally appeared in the November 1917 number of the "Giornale degli Economisti e Rivista di Statistica", and has now been published in pamphlet form. Both of these writers take exception to the criterion proposed by Mr. Nixon, and put forward alternative solutions of the problem.

The course adopted by Signor Gini is, in effect, to compare the increase in mortality with the probable error, so that if in two sections A and B of the populations, the rates of mortality are increased from m_1 and m_3 respectively, to m_2 and m_4, then the increase in A must be regarded as $>$ or $<$ the increase in B according as

$$\frac{m_2 - m_1}{\sqrt{m_1(1-m_1)}} \text{ is } > \text{ or } < \frac{m_4 - m_3}{\sqrt{m_3(1-m_3)}}$$

https://doi.org/10.1017/S0020268100028274 Published online by Cambridge University Press

The objections to the criterion proposed by Gini are, I think, obvious. They are put with great force by Prof. Cantelli on pages 13–15 of his pamphlet, to which I would refer anyone who is interested in the matter.

Prof. Cantelli points out that the criterion proposed by Nixon cannot give a satisfactory answer to the question: "Which classes of a population are the most seriously affected by the indirect effects of a War"? For "Consider a case—fictitious or not does not matter—which exposes the weakness of this criterion. Two populations, A and B, have suffered severely from the effects of a War. In the population A the mortality has increased from 10 per thousand to 100 per thousand, while in the population B it has increased from 10 per hundred to 100 per 100. Which of the two populations has been more severely hit by the War? That is, which population has suffered the heavier mortality? Clearly, it seems to me, population B, which has been completely wiped out by the effects of the War. But the expression used by Nixon gives the same result for the two populations, namely:

$$\frac{100-10}{10}$$

"But even excluding the above example, it does not seem to me that the expression

$$\frac{q_{ng}-q_n}{q_n} \text{ or } \frac{m_{ng}-m_n}{m_n}$$

can answer questions of the kind asked above, since it, as we see from its very nature, ignores the question of the exposed to risk."

Prof. Cantelli goes on to show that this question can only be answered by a comparison, age-group by age-group, of the values of q_g, where q_g is the probability of death on the assumption that the exposed to risk are only subject to mortality caused by the indirect effects of the war. He finds the value of q_g as follows:

"Let us suppose that out of l persons exposed to risk m die in one year from all causes, including the indirect effects of war. Let us further suppose that we know that if no deaths were due to the effects of war and that only the ordinary rate of mortality was in operation n persons would die in the year. We can then say that during the whole year of risk, $l-n$ persons are not liable to die from normal causes. Hence it follows that the $m-n$ persons, who die out of the $l-n$ under consideration, die from causes due solely to the war, *i.e.*,

$$q_g = \frac{m-n}{l-n}$$

"But

$$\frac{m}{l} = q_{ng}; \quad \frac{n}{l} = q_n$$

$$\therefore \quad q_g = \frac{q_{ng}-q_n}{1-q_n}$$ "

https://doi.org/10.1017/S0020268100028274 Published online by Cambridge University Press

In a foot-note the author points out that this formula can be derived directly from Karup's theorem

$$p_{ng} = p_n \times p_g \,.$$

In order to compare the results given by his formula with those given by the formulæ of Hersch and Nixon, Gini calculated values of q_n from the mean annual number of deaths in the decennium 1876–85 and the census of 1880, and gives a series of tables, for the age-groups used by Prof. Hersch, for Switzerland, Holland and Belgium, distinguishing in each case between males and females.

I reproduce here Gini's tables for Swiss males, and have added, for purposes of comparison, a table of q_g calculated according to the formula given by Prof. Cantelli.

The indirect effect of the Franco-Prussian War, 1870–71, *on the Mortality of the Swiss Male Population.*

Age Group	The absolute excess of Mortality, namely, $10{,}000\,(q_{ng}-q_n)$ Prof. Hersch's method (1)	Normal Mortality, namely, $10{,}000\,q_n$ as calculated by Signor Gini (see above) (2)	Percentage increase in Mortality $\frac{q_{ng}-q_n}{q_n}$ Mr. Nixon's method (3)	Relative increase in Mortality $\frac{m_2-m_1}{\sqrt{m_1(1-m_1)}}$ Gini's method (4)	$q_g = \frac{q_{ng}-q_n}{1-q_n}$ Prof. Cantelli's method (5)
0–1	424	2,583	16	·097	·05716
1–5	74	205	36	·052	·00755
5–10	36	61	59	·046	·00362
10–15	11	35	31	·018	·00110
15–20	13	49	27	·018	·00130
20–25	65	73	89	·077	·00653
25–30	69	87	79	·074	·00696
30–35	38	99	38	·038	·00384
35–45	40	125	32	·036	·00405
45–55	51	198	26	·037	·00520
55–65	70	369	19	·037	·00726
65–75	132	774	17	·049	·01430
75–	252	1,722	15	·067	·03044

It will be noticed that this latter table supports the conclusion drawn by Prof. Hersch, and militates against those drawn by Mr. Nixon.

Yours faithfully,

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