

## PLANETARY EPHEMERIDES

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### ABSTRACT

In the past twenty years there has been a great amount of growth in radiometric observing methods, as well as in classical optical observations. Through radar ranging and Doppler observations of the planets and spacecraft, we have been able to improve our knowledge of the location and motion of the planets by several orders of magnitude and have succeeded in planning and executing space missions which would have been difficult if not impossible to plan and to perform utilizing the classical ephemerides. We will outline the goals and methods employed by the Jet Propulsion Laboratory in its effort to develop improved ephemerides which accurately reflect the motions of planets in an inertial system.

We will demonstrate that in the ideal situation a radar observation is largely independent of the problems usually associated with the precessional motion of the earth's axis and is, in fact, a reliable method for obtaining inertial mean motions. Based upon our hypothesis that the modern JPL ephemerides are valid in a fixed (i.e., non-rotating) coordinate system, we will explore the implications concerning optical observations of the Sun, precession, equinox drift and the relationship between dynamical and universal time scales, as well as comparisons with Newcomb (1898) ephemerides.

### I. RANGE OBSERVATIONS

It may be shown that interplanetary ranging data by itself is sufficient to determine mean motions with respect to an inertial system. Starting with a very simplified example, we consider two planets in coplanar, circular orbits. We observe the round-trip time delay  $\tau$  and by using the relationship  $\rho = c\tau/2$  which relates the round-trip time delay to the one-way distance  $\rho$  we observe that at conjunction  $\rho_c = a + a_e$  and at

opposition  $\rho_o = a - a_e$ , where  $a$  and  $a_e$  represent the semi-major axes of the planet and the Earth. We can then obtain ratios of semi-major axes to remove dependence upon laboratory units and we can employ Kepler's law to obtain

$$n_e/n = (a/a_e)^{3/2} . \quad (1)$$

Furthermore, from the observed synodic period,  $T_{\text{syn}}$ , we can determine both mean motions from

$$n_e = 2\pi T_{\text{syn}}^{-1} (1 - n/n_e)^{-1} , \quad (2)$$

with a similar equation for the other planet - which shows that one is indeed able to determine the inertial mean motions of the planets from the observations at conjunction and opposition and from the synodic period.

In practice, of course, the situation is more complicated. A more complete expression for the actual one-way distance would be

$$\rho = | \vec{r} - \vec{R} - \vec{R}_e + \vec{R}_p | \quad (3)$$

where  $\vec{r}$  and  $\vec{R}$  are the heliocentric vectors to the planet and the Earth, where  $\vec{R}_e$  is the geocentric position of the observer, and where  $\vec{R}_p$  is the planetocentric point from where the signal is returned. In Eq. (3) there is only slight dependence upon precession, since  $\vec{r}$  and  $\vec{R}$  are referred to the inertial coordinate system (B1950.0) in which the planetary ephemerides are numerically integrated. Hence precession only enters into the calculation of  $\vec{R}_e$  when one relates the geocentric body-fixed position of the observer to a space-fixed system. An additional source of error enters into Eq. (3) through the calculation of sidereal time since the sidereal time as calculated with respect to the FK4 equinox will differ from that calculated with respect to the dynamical equinox by  $E$ , the equinox error. In this paper we will only be interested in the rate of motion of an equinox and we will ignore any constant angular offset between the fixed dynamical equinox and one contained in the JPL ephemerides that may differ from it by a constant angle. Hence, the total error committed in the reduction of the station coordinates to a fixed equinox is given by

$$T \Delta k = (\Delta p \cos \epsilon - \dot{E}) T \quad (4)$$

where  $\Delta k = -0.3$ . This effect is extremely small for radar observations, amounting to only 0.1 m/y, the effect being proportional to  $R_e \Delta k$ . Thus, for practical purposes we can say that radar data are not affected by errors in precession and equinox motion, the effects being absorbed in station locations since they are of a diurnal nature.

Of greater consequence are problems in the determination of  $\vec{R}_p$ , which contains errors from various sources such as (a) topographical variations in the case when a radar signal is reflected from a surface

feature of the planet, (b) orbit determination uncertainties when the transponder is located on board an orbiting spacecraft (e.g., Mariner IX), or (c) uncertainties in the physical ephemeris of the planet and the location of the landed spacecraft when the transponder is on the surface of a planet (e.g., Viking Lander). Formal covariance analysis predicts that we can determine inertial mean motions of the Earth and Mars to better than 0!01 per century, although a more realistic value for this number might be 0!03 per century. Simple error estimates from Eq. (2) indicate that the Earth's mean motion can be determined to

$$\sigma(n_e) = 0!001 \text{ T } q(m) \quad (5)$$

where  $q(m)$  is the one-way range uncertainty in meters. Thus, if  $q = 30m$  - a very conservative value - we can determine the Earth's mean motion to 0!03 per century. On the other hand, an error of 1" per century in  $\Delta n$  could not optically be observed in the semi-major axis since the error is only 1000m or 0!001.

## II. TIME SCALES

For radar observations atomic clocks are always employed, and since Universal Time only enters the problem through the diurnal rotation of the Earth, we will assume that our observations are effectively on a dynamical time system. In actual practice, if we let  $t_d$  represent dynamical time and let  $t_u$  represent Universal Time, then we have

$$t_d = t_u + \Delta T = t_u + (t_e - t_u) + (t_d - t_e) = t_u + \Delta T_e + \delta T \quad (6)$$

where we have divided  $\Delta T$  into two parts:

$$\Delta T = \Delta T_e + \delta T \quad (7)$$

with  $\Delta T_e$  signifying "Ephemeris Time" minus Universal Time and  $\delta T$  representing dynamical time minus ephemeris time. The time scale  $t_e$  is that appropriate to some ephemeris (Newcomb's for example) and was generally tabulated in the astronomical ephemerides before atomic time systems were developed. Values of  $\Delta T$  are ultimately obtained from the Bureau International de l'Heure (BIH) and, assuming that the stars are on the FK4 system, the corresponding Universal Times are correct and will not change with the adoption of the FK5.

In our processing of older optical observations we employ the values of  $\Delta T_e$  given by Brouwer (1952), rather than the generally unknown  $\Delta T$  of Eq. (7). The Brouwer values of  $\Delta T_e$  are consistent with Brown's lunar theory containing a quadratic term in the Moon's mean longitude of  $\dot{n}(\lambda/2) = -11!22$ . The term  $\delta T$  in Eq. (7) is present in order to allow one to employ another, perhaps more realistic, motion for the Moon's secular acceleration. If one adopts a different secular acceleration of the Moon, the approximate value of  $\delta T$  is

$$\delta T = -(\dot{n}_\zeta + 22''.44) T^2 \quad (\text{sec}). \quad (8)$$

If Morrison's (1979) value of  $\dot{n}_\zeta = -26''$  is correct, then we would obtain  $\delta T = 3.86 T^2$  as one estimate of  $\delta T$ . On the other hand, use of the value  $\dot{n}_\zeta = -38''.3$  given by Duncombe et al. (1975) would produce a value  $\delta T = 16.8 T^2$ . A recent determination by Williams (1980), based upon lunar laser ranging data, yields  $\dot{n}_\zeta = -23''.1 \pm 2''$ , so he would obtain  $\delta T = 0.87 T^2$ . For very old observations (which are not processed by JPL) the difference can be appreciable. Whatever the true value of  $\delta T$  may be, we should investigate its influence on the meridian circle data.

### III. OPTICAL OBSERVATIONS

For optical observations, the Universal Time of transit is calculated as follows. By definition, at transit the right ascension of the planet referred to the equinox of date is equal to the sidereal time. We therefore determine the Universal Time  $t_u$  at which transit occurs, as predicted by the ephemeris. If the sidereal time definition is  $ST(t_u)$  on the FK4 system or  $ST'(t_u)$  on the FK5 (dynamical equinox) system with (in an of-date system)

$$\alpha_{\text{dyn}} = \alpha_{\text{cat}} + E, \quad (9)$$

then by international agreement  $ST'(t_u) = ST(t_u) + E$ , so that one obtains the identical Universal Time on the FK4 system as well as on the FK5 system.

Let a  $\tilde{\nu}$  (tilde) refer to a right ascension with respect to the equinox of date and let a subscript F refer to a right ascension with respect to a fixed equinox. Let an apostrophe (') denote a correct value, so that  $\tilde{\alpha}'$  is the correct right ascension with respect to the dynamical equinox of date and  $\alpha'_F$  is the correct right ascension with respect to the fixed equinox of B1950. If  $\tilde{\alpha}'(t')$  represents the correct right ascension referred to the real equinox of date at the real dynamical time  $t'$ , then  $\tilde{\alpha}'(t') = \alpha'_F(t') + p_\alpha T$ , where  $p_\alpha$  is the correct (not Newcomb) precession in right ascension. Then the real transit occurs at  $t'_u$  when

$$\tilde{\alpha}'(t'_u + \Delta T) = ST'(t'_u) = ST(t'_u) + E \quad (10)$$

if one has an ephemeris which is valid with respect to the real equinox. At JPL, however, we always use the Newcomb precession  $p^0$  which differs from the correct value due to Fricke (1971) by  $p = p^0 + 1''.1T$ . Hence, at JPL, we compute the time of transit from

$$\tilde{\alpha}(t_u + \Delta T_e) = ST(t_u) \quad (11)$$

where  $\tilde{\alpha}$  is the right ascension referred to the equinox of date using Newcomb's precession  $p^0$ :  $\tilde{\alpha}(t) = \tilde{\alpha}'(t) - \Delta p_\alpha T$ . Thus, if  $C'$  represents the true RA of transit and if  $C$  represents the RA calculated via Eq. (11),

then we make the error  $O' - C = \Delta p_{\alpha} T + 0.04 \delta T$ . In addition to the error made by JPL in computing the meridian transit, the observed right ascension ( $O$ ) is in error (if it is on the FK4 system) by the amount given in Eq. (9). Hence,

$$O - C = (O' - C') + (\Delta p_{\alpha} T - \dot{E} T + 0.04 \delta T), \quad (12)$$

and it is the quantity  $\Delta k = \Delta m - \dot{E} + 0.04 \delta T/T$  which JPL estimates from the optical data (Standish 1976).

The preceding, then, is a short summary of the manner in which we process both radar and optical observations. In the case of optical transits it is seen that we make the error  $O' - C$ , while the data contain the equinox error of Eq. (9). Since the parameter  $\Delta k$  is estimated (along with a similar equation  $\Delta n$  for declination) in the analysis, no harm is done and one obtains from the optical observations the values of  $\Delta k$  and  $\Delta n$ .

## V. EPHEMERIS DATA

Before comparing the ephemerides with Newcomb, we would like to outline the types of data which have been employed. The so-called "Export Ephemerides" such as DE-69 (O'Handley et al. 1969) and DE-96 (Standish et al. 1976) are generally available to all interested scientists. They are better documented than our "interim" ephemerides which are either (a) export ephemerides which have not yet been thoroughly documented or (b) ephemerides which represent temporary milestones (e.g., for a specific space mission). In recent years the interim ephemerides (DE-102, DE-108, DE-111) have seen rather widespread use by scientists and consequently it is appropriate for us also to discuss them here. The data upon which the ephemerides are based are summarized in Table 1.

The U.S. Naval Observatory optical observations were transformed to the FK4 system using USNO tables for Ephemerides DE-96 to DE-108. For DE-111 the transformations of Schwan (1977) were employed. Limb corrections for Mercury and Venus were applied through DE-96, while phase modeling was employed later.

Radar bounce data (radar signal being reflected from a planet) reached its greatest influence in DE-96 and DE-102, subsequently being superseded by Viking observations. Radar observations prior to 1967 are weighted at approximately 15km while subsequent data are weighted at 1.5km. Spacecraft ranging and normal points were employed in DE-102 and in subsequent ephemerides. Mariner IX data were weighted at 40m away from conjunction and 400m near conjunction. Viking orbiters and landers were weighted at 50m and 15m in DE-108 and DE-111. Mars radar closure observations (whereby one employs two observations of Mars spaced one synodic period apart to remove effects of topographic variations), employed in DE-96 through 108, were weighted at 150m, while the Mars radar-occultation (employment of spacecraft-determined

occultation radii to eliminate the radius dependence of the radar range) comparisons used in DE-108 were weighted at 500m. Finally, in DE-111, a combined planetary-lunar ephemeris solution was obtained in which lunar laser ranging data (weighted at 34cm) were employed for the first time to obtain a simultaneous solution for the Moon and planets.

In Table 2 we summarize some of the constants employed in the ephemerides. Prior to DE-96 a different speed of light was employed, which accounts for the major changes in the astronomical unit. The values in Columns 4 ( $\Delta k$ ) and 5 ( $\Delta n$ ) are the parameters which result from the optical data only in estimating optical (and precession) drifts. The interpretation of  $\Delta k$  and  $\Delta n$  should be viewed with caution. They are quantities which relate the presumed inertial ephemeris to the apparent observations made with a meridian circle. Only if the solar observations are indeed on the FK4 can the corrections be interpreted in a manner equivalent to the results obtained by Fricke (1980). It should be remembered that the term  $E$  results from the observational errors while the  $\Delta p$  terms are present because of inconsistencies in calculating the events. The values of  $\dot{n}$  given in Column 6 are primarily for outside users of JPL ephemerides. They should only be considered by users of the ephemerides who analyze lunar observations and who derive their own values of  $\delta T$ .

## V. COMPARISON OF EPHEMERIDES

We now come to a discussion of the modern JPL ephemerides in comparison with the classical ones of Newcomb. Several scientists such as Stumpff (1977, 1979, 1980), Kristensen (1980), van Flandern (1980), Schubart (1980) and others have compared the JPL ephemerides with subroutines or tapes which are generally described as being "Newcomb" ephemerides. Schubart, in comparing DE-102 with Herget's (1953) evaluation of Newcomb's tables in a B1950 coordinate system, finds "no significant secular deviation" of the JPL longitude from that of Newcomb. Kristensen, however, finds that DE-108 minus Newcomb theory in an of-date system is about  $+0^{\circ}83T$  and van Flandern obtains  $1^{\circ}01T$  in an of-date comparison of DE-102 minus Newcomb, even though the mean motion differences between DE-102 and DE-108 are very slight. In both cases the authors employ different computer subroutines to evaluate Newcomb's theory with respect to the equinox of date and then use the new IAU precession ( $\Delta p = 1^{\circ}1T$ ) to precess the JPL ephemerides from B1950 to the equinox of date, while Schubart did not employ any precession since both DE-102 and Herget's evaluation of the Newcomb Tables are in a fixed B1950 system. Stumpff obtains a value based on a DE-96 comparison (which has a poorer determination of the sun's mean motion), but which when reduced to the DE-108 system is in general agreement with that of Kristensen if Stumpff employs the new IAU precession.

If, in fact, the JPL ephemerides are correct and are relative to a fixed equinox in the B1950 system (it may be rotated slightly from the dynamical equinox), then one must employ the new precession in

calculating correct positions with respect to the dynamical equinox of date. Hence, the use of the new precession by Kristensen, van Flandern and Stumpff is proper. If the Newcomb ephemerides are valid with respect to the equinox of date, then the Kristensen - van Flandern - Stumpff analyses should show no significant secular deviations for DE-102 and subsequent JPL ephemerides (evaluated in an of-date system with the new precession). On the other hand, one might expect the Schubart comparison of DE-102 with Herget's evaluation of Newcomb in a B1950 coordinate system to show a residual trend of 1" T since we believe the Newcomb precession (which Herget employed to precess the of-date Newcomb Tables positions to B1950) to be in error.

These apparently contradictory (to our hypothesis that the JPL ephemerides are inertial) results may still have an explanation, however. That possible answer lies in questioning the time-honored assumption that Newcomb's solar ephemeris is accurate in representing the real solar motion with respect to the dynamical equinox of date. We would, in fact, suggest that Newcomb's theory is not accurate with respect to the dynamical equinox of date.

In developing the FK4 catalogue, Fricke and Kopff (1963) employed the same equinox N1 (apart from a constant offset) as Newcomb employed (1872, 1882) in Newcomb's analysis of inner planet observations which were used to produce the solar ephemeris (Newcomb 1895). Hence, let us make the following three assumptions: (1) the FK4 and N1 catalogues have the same equinox motion (Fricke and Kopff, 1963); (2) Newcomb's solar ephemeris (Newcomb 1898) is with respect to "Newcomb's dynamical equinox"  $\alpha_{DN}$  where

$$\tilde{\alpha}_{DN} = \tilde{\alpha}_{N1} + 0.3 T \quad (13)$$

(Newcomb 1895, pp. 88, 126); (3) the real dynamical equinox  $\tilde{\alpha}'$  is related to the FK4 equinox by Fricke's recent (1980) determination

$$\tilde{\alpha}' = \tilde{\alpha}_{FK4} + 1.27 T. \quad (14)$$

From these three assumptions, then, we can derive

$$\dot{\tilde{\alpha}}' - \dot{\tilde{\alpha}}_{DN} = +0.97 \quad (15)$$

where a dot signifies a time derivative (per century). Hence, since Newcomb's solar ephemeris was with respect to  $\tilde{\alpha}_{DN}$  it follows that the sun's real mean motion  $\tilde{n}'$  with respect to the  $\tilde{\alpha}_{DN}$  equinox of date differs from that calculated by Newcomb ( $\tilde{n}_N$ ) in the amount

$$\tilde{n}' - \tilde{n}_N = 0.97 \cos \epsilon = 0.89 \text{ per century.} \quad (16)$$

Thus, we should expect that an accurate solar ephemeris would have a mean motion 0.89 per century larger than Newcomb's mean motion, when measured in an of-date system. This result is in accord with the findings of Stumpff, Kristensen and of van Flandern given earlier. A comparison

of an inertial mean motion  $n'$  with a Newcomb ephemeris mean motion  $n_N$  in a fixed system should then show

$$n' - n_N = 0''.89 - \Delta p = -0''.21 \text{ per century} \quad (17)$$

in agreement with the finding of Schubart.

Hence, it seems that the hypothesis that the solar data of Newcomb are affected by an error of approximately  $0''.9$  in an of-date system may be the key to explaining the results noted earlier.

## VI. THE LATEST DATA

So far, then, we have put forth some possible, if not plausible, reasons for asserting that the modern ephemerides are indeed valid representations in an inertial system. One final example will serve to support that belief. In its efforts to develop a Very Long Baseline Interferometry (VLBI) system for use in navigating future space missions, JPL has conducted some radio-interferometric experiments measuring the angular separation between radio sources and the Mars Viking Orbiter. Employing orbit determination methods to reduce the orbiter data, we can effectively "observe" the radio source relative to Mars.

In these experiments a radio source catalogue is employed and the angular separations are calculated in an of-date system. The residuals, calculated by Newhall (1980) on DE-108 exhibit fairly large means, being  $1''.76$  in right ascension and  $1''.22$  in declination. However, we may assume that the radio source catalogue is valid at a current epoch and equinox (quite similar to a star catalogue), while the Mars ephemeris is valid in a fixed B1950 frame. If one then applies the precession corrections to reduce the Mars ephemeris to a current equinox and if one assumes that the Newcomb precession is in error by  $1''.1$ , then the declination residuals become smaller by a factor of 3. The right ascension residuals, originally calculated on DE-108, become smaller by a factor of 5 if DE-111 is used with the new precession. Thus, we again have some evidence that the latest ephemerides, produced from radiometric and optical data, are indeed valid representations of the planetary motions in an inertial system.

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Table 1  
JPL Ephemerides - Data Sets

Observations	Dates	No. of Obs.	Ephemeris
USNO Transits			
Wash. system	1911-1967	34 304	69
FK4 system	1911-1971	37 583	96,102
	1911-1976	38 942	108
	1911-1976	39 396	111
Radar (Bounce)			
Mercury, Venus, Mars	1964-1968	704	69
	1964-1973	5 052	96,102
Mercury, Venus	1964-1977	1 199	108
	1964-1977	1 307	111
Spacecraft Ranging			
Mariner V (Venus)	1967	214	69
Mariner IX (Mars)	1971-1972	804	96
	1971-1972	803	102,108,111
Viking Orbiter	1976-1977	4 463	102
	1976-1977	2 892	108
Viking Lander	1976-1977	147	108
	1976-1978	665	111
Pioneer X, XI	1973-1974	2	96,102,108,111
Mars Radar, Closure			
	1971-1973	291	96
	1971-1976	306	102
	1971-1978	321	108
Mars Radar, Occult.	1967-1978	2 890	108
Saturn Sat. Astrometry	1973-1979	4 790	111
Lunar Laser Ranging	1969-1978	2 531	111

Table 2  
Planetary Ephemerides - Miscellaneous Constants

Ephemeris	Astr. Unit 149 597 800+	E/M 81.+	$\Delta k$	$\Delta n$	$\dot{n}_c$	c(km/s) 299792.+
69	93.0 km	.301	---	---	-38"3	.5
96	71.411...	.3007	-1"19	+0"15	-38.3	.458
102	70.684...	.3007	-0.76	+0.29	-27	.458
108	70.705...	.300492	-0.57	+0.42	-38	.458
111	70.653...	.300587	-0.78	+0.44	-23	.458

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