Part IX

BELIEF, CAUSE AND INDUCTION

https://doi.org/10.1086/psaprocbienmeetp.1990.1.192727 Published online by Cambridge University Press

Conditional Probabilities, Conditionalization, and Dutch Books

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Relations between conditional probabilities, revisions of probabilities in the light of new information, and conditions of ideal rationality are discussed herein. The formal character of conditional probabilities, and their significance for epistemic states of agents is taken up. Then principles are considered that would, under certain conditions, equate rationally revised probabilities on new information with probabilities reached by conditionalizing on this information. And lastly the possibility of kinds of 'books' against known non-conditionalizers is explored, and the question is taken up, What, if anything, would be wrong with a person against whom such a book could be made?

I. Conditional Probability

1. The standard mathematical treatment takes probability functions as basic, and conditional probabilities defined. If P(-) is a probability function, then P(-/••) is its conditional probability function if and only if P(h/e) is defined only for e such that P(e) > 0, and is then equal to P(e & h)/P(e).

What is the significance of these ratios? A widely held view relates them to the epistemic states of persons who satisfy certain very demanding simplifying conditions, persons each of whom: (i) is highly opinionated and has for every proposition a completely definite degree of confidence; (ii) is logically omniscient and absolutely sure of every necessary proposition, while placing no credence at all in any impossible one; and (iii) has credences for propositions that sum his credences for world-propositions that entail them. Credences of such persons are represented by unique probability functions. The question before us, more precisely put, is, What is the significance of the conditional probabilities of such persons? What, about their epistemic states, do these ratios of probabilities represent?

The widely held view, now made definite in my terms, is that these ratios have to do with what I call possible evidential bearings of propositions on one another, so that P(h/e) measures what is for this person the possible evidential bearing of e on h, which I identify with the maximum possible potential evidential bearing for him of e on h. To explain, I say that the *potential evidential bearing* for a person of a positively probable proposition e on a proposition h is measured by,

P(h/e) - P(h);

PSA 1990, Volume 1, pp. 503-515

https://doi.org/10.1086/psaprocbienmeetp.1990.1.192727 Published online by Cambridge University Press

so that (i) e would be evidence for or against h depending on whether or not [P(h/e) - P(h)] is positive or negative, and (ii) e would be more evidence for, or less evidence against, h, than e' would be for or against h', if and only if [P(h/e) - P(h)] > [P(h'/e') - P(h')]. I say that *possible evidential bearing* of e on h' — P(h/e) — is either 0 or the limit of the potential evidential bearing e on h — [P(h/e) - P(h)] — as P(h) descends toward without reaching 0 while P(h/e) is held fixed. (I note that unless P(h/e) is 0 it could not be held fixed when P(h) reached 0.) Unqualified references below to evidential bearings and relevances are always to potential evidential bearings.

This common view of the significance of conditional probabilities is bold and simple. According to it, the evidential bearing of e on h, is, when a person has some confidence in e, bound up with his degrees of confidence in e, and in the conjunction of h with e. It can seem surprising and implausible that there is no more to possible evidential bearings than that, and that a measure for them is already implicit in a measure for degrees of unconditional confidence. But there is less surprise in the fully equivalent suggestion that credences for conjunctions must reflect the evidential bearings that it is felt the conjuncts have for one another. The identity P(h/e) = P(e & h)/P(e) is of course equivalent to the identity P(e & h) = P(e)P(h/e).[1]

2. According to the standard treatment, P(h/e) is defined only if P(e) > 0. But it seems that even if someone is sure that e is not true, if e is logically possible it can be for this person possible evidence, so that if he learned that e, he would not be at a complete loss what increments in credences for other propositions to make of this lesson. It seems that we must choose, and say either that not all possible evidential bearings are measured by conditional probabilities, or that conditional probabilities are to be understood in a way that extends them beyond the region to which they are standardly confined.

My proposal is to view extended conditional probabilities as measuring all possible evidential bearings, even ones of certainties. For this view I let an extended probability function, P, be a real-valued function defined for all propositions, *and* (here come extended conditional probabilities) for all ordered pairs of propositions, such that not only, as is standard,

(1) $P(p) \geq 0$,

(2) $\Box p \rightarrow P(p) = 1$,

(3) $\sim\Diamond (p \,\&\, q) \rightarrow P(p \,\&\, q) = P(p) + P(q)$,

but also,

(4) $P(q/p) \geq 0$,

(5) $\Box\ (p \rightarrow q) \rightarrow P(q/p) = 1$,

(6) $\Box\ [p \rightarrow \sim(q \,\&\, r)] \rightarrow P[(q \vee r)/p] = P(q,p) + P(r/p)$,

(7) $\Box\ (p \leftrightarrow q) \rightarrow P(r/p) = P(r/q)$,

(8) $P[(q \,\&\, r)/p] = P(q/p)P[r/(q \,\&\, p)]$,

and finally, to connect the two parts of P,

(9) $P(p) > 0 \rightarrow P(q/p) = P(p \,\&\, q)/P(p)$.

https://doi.org/10.1086/psaprocbienmeetp.1990.1.192727 Published online by Cambridge University Press

Axioms (4) through (7) seem correct as constraints on consistent evidential bearings. And axiom (8), though perhaps too complicated to be intuitive, is only marginally stronger than,

$$P(q \& p) > 0 \rightarrow P[(q \& r)/p] = P(q/p)P[r/(q \& p)].$$

This is a theorem in virtue of axioms other than (8).

Axioms (4) through (8) are the axioms of Arthur Burks' Calculus of Inductive Probability. He defines unconditional probabilities by a rule that is equivalent to the principle,

$$\Box\ p \rightarrow P(q) = P(q/p),$$

and observes that

$$P(p) > 0 \ \rightarrow P(q/p) = P(p \& q)/P(p),$$

(my axiom (9)) is a theorem of his calculus. His definition of unconditional probability is a theorem of my theory.

I concentrate (as in (Sobel 1987)) on persons each of whose credences are measured by the unconditional part of an extended probability function, and for whom possible evidential bearings among propositions are measured by the conditional part of that function. I assume that only contingent propositions have possible evidential bearings, and that they have such bearings on only contingent propositions. This means that, for present purposes, axioms (4) through (9) could be weaker and more complicated: they could have restrictive antecedents that left P(q/p) quite wild for non-contingent p and q.

While no evidence-related significance is attached to P(q/p) when p is either impossible or necessary, I admit such significance for P(q/p) even when p is certainly false, and also even when it is certainly true. This last case is special, however, for suppose P(p) = 1. Then since, for every q, P(q/p) = P(q), the potential evidential bearing of p on q, [P(q/p) - P(q)], is zero. But I want the possible evidential bearing of p on q to remain equal to P(q/p), and still to be, by definition, either 0 or the limit of the potential evidential bearing of p on q as P(q) descends toward without reaching zero while P(q/p) remains unchanged. For this to work, I must suppose a change in P(p), since otherwise, if P(p) remained unchanged at 1, the potential bearing of p on q would be constantly zero as P(q) descended. I suppose an infinitesimal change in P(p), and propose to understand possible evidential bearings certainties in terms of the possible evidential bearings of corresponding near-certainties. The fancy is that credences be represented by parts of a family of infinitesimally close extended probability functions, that is, by parts of extended probability functions that assign non-standard real numbers to propositions and ordered pairs of propositions, wherein the several assignments by members of the family to any proposition or ordered pair of propositions are infinitesimally close to one another. (This paragraph makes a change from a position taken in (Sobel 1987), where I say that "possible evidential bearings... seem to be restricted to propositions a person is not certain of" (p. 59). My main reason for thinking that was that certainties, if they have possible evidential bearings, have identical bearings, since, for any p and p' both of which are certain, for any q, both P(q/p) and P(q/p') equal P(q). Though I there took this as a reason for denying that certainties have possible evidential bearings, I now consider it only a noteworthy feature of their bearings.)

II. Conditionalization

1. Two formal considerations — two theorems — suggest that at least under certain conditions, revisions of credences in the light of new-found certainties must be by conditionalization understood thus:

> Where P and P' are extended probability functions, P' comes from P by conditionalization on p if and only if for every q, $P'(q) = P(q/p)$. (I note that this rule determines $P'(s/r)$ when $P'(r) > 0$, but not when $P'(r) = 0$, and this whether or not $P(r) = 0$.)

Let P be an initial probability function, and let P' be a probability function that comes by revisions executed upon elevation to a certainty of some initially somewhat probable proposition p, and let Pp be the probability function that comes from P by conditionalization on p. Here are the two theorems of both standard and non-standard theory:

Theorem 1. $[P(p) > 0 \ \& \ P'(p) = 1] \rightarrow [P'(q/p) = P_p(q/p) \rightarrow P'(q/p) = P(q/p)]$

Theorem 2. $[P(p) > 0 \ \& \ P'(p) = 1] \rightarrow [P'(q/p) = P(q/p)] \rightarrow P'(q/p) = P_p(q/p)]$

Assuming that conditional probabilities measure possible evidential bearings, Theorem 1 says that revisions by conditionalization upon propositions of which one has come to be certain, preserve what were the possible evidential bearings of these propositions when they were still uncertain; and Theorem 2 says that, of possible revisions upon new-found certainties, only revisions by conditionalization do this. (Cf., Skyrms 1986, p. 191.) Each theorem is a very easy corollary of the principle that conditionalization upon a positively probable condition preserves probabilities conditional on the condition:

$$P(p) > 0 \rightarrow P(q/p) = P_p(q/p).$$[2]

For simpler theorems of only non-standard probability theory, the condition $P(p) > 0$ can be dropped.

2. Before proceeding to several progressively more plausible principles for conditions under which it is exclusively rational to revise probabilities in the light of new information by conditionalizing on it, several remarks are in order concerning 'learning' and 'information'. I assume for present purposes that a person has learned something if and only if he has been moved to alter his confidence in this thing *directly* by the impingements upon him of reality 'outer' and 'inner'. I think of learning-alterations as unmediated by cerebral processing, and contrast them with changes mediated by conscious and unconscious exercise of mechanisms for revisions of confidence consequent on learning, and by (conscious or unconscious) reflection on what has been learned. Following standard, albeit somewhat awkward, usage for present subjects, 'learning' is here intended in a way that allows persons to learn untruths. 'Information' learned can likewise be disinformation.

Here is a simple and unqualified conditionalization rule for revisions in the light of a new-found certainty:

> CR1: For any person whose probability function for t is P and whose probability for a subsequent time t' is P': if this person learns for sure that p in the interval (t,t') so that $P'(p) = 1$, then, if this person is rational, for every q, $P'(q) = P(q/p)$.

Against this principle is the consideration that in the time between t and t' the subject may have learned many things other than p—that is, this subject's experiences may have been directly responsible for shifts in degrees of confidence for many propositions other than p, including shifts that have elevated other propositions to certainties. Richard Jeffrey, commenting on an equivalent rule, states that "[e]vidently, the rule is correct only if [p] conveys all of the information that prompted your change of judgement" (Jeffrey 1985, p. 116). He supposes that a rule to conditionalize on p needs to be confined at least to cases in which "your beliefs... change from those characterized by...prob...to those characterized by...PROB... simply because you have come to fully believe [p]" (Jeffrey 1983, p. 90 [exercise 13 of Chapter 5]). And this condition will be satisfied only when the change is not prompted by any information other than that conveyed by p.

One response to deficiencies of CR1 would be to insure that p includes everything that the person has learned.

> CR2: For any person whose probability function for t is P and whose probability for a subsequent time t' is P': if this person learns p for sure in the interval (t,t') so that P'(p) = 1, and *this person learns nothing other than p in the interval (t,t');* then if this person is rational, for every q, P'(q) = P(q/p).

There are several problems with this restricted principle. Perhaps the most important one is that in the interval (t,t') this learner of p may have changed his mind about the possible evidential bearings of what he has learned. Such a change might result from reflection on his system of credences either before or after becoming certain of p. Even when occasioned by learning p, as they can be since learning a thing can lead to reviews of the presumed evidential significance of the thing learned, such changes are not necessarily unreasonable for ordinary intellects. It seems clear that no principle for rational revisions by conditionalization can be right unless it either addresses itself only to cases in which such changes do not take place, or would not be reasonable. CR2 does not meet this condition.

Another problem with CR2, is that though this principle is concerned with cases in which elevations of confidence to certainties are products of learning rather than impatience, dyspepsia, or browbeating, there is no requirement in the principle that the person will be convinced that the new-found certainties at issue are by learning. But if a person thinks that he has come to a certainty in one of those other ways then, whether or not he is right in this suspicion, it can be reasonable for him not to conditionalize on this certainty.

Finally, for any p, it can seem strange for it to be true that a person had learned p for sure, and learned nothing more. Whatever proposition p is, one supposes that if a person learns it and changes his confidence in it, then, if he is paying attention, he learns at least this additional thing, that his confidence in p has changed. The emphasized part of the antecedent of CR2, if satisfiable, is so only because ordinary persons are limited in the attention they can pay to themselves.

These difficulties that CR2 runs into can suggest the following rather different and more complicated principle.

> CR3: For any person whose probability function for t is P and whose probability for a subsequent time t' is P': if this person learned p for sure in the interval (t,t') so that P'(p) = 1, and is sure at t' that he learned p for sure in that interval; then if this person is rational, for every q such that *he has not changed his mind since t about the possible evidential bearing of p on q,* P'(q) = P(q/p).

CR3, in contrast with CR2, does not require that p be everything that the subject has learned in the interval (t,t'). Also, CR3 calls, not for the revision of the whole of P, but only for that part with respect to which the evidential bearing of p has not changed in the interval (t,t'). I note that regarding the propositions of this part, given the assumption of rationality, *it seems that things other than p that the person learns in the* interval cannot be of relevance independent of p, and that though p may not be all that has been learned it can stand for everything that has been learned that is of relevance to the propositions of this part of P. Indeed, a principle of equivalent intent results if the underlined constancy of the possible evidential bearings condition is replaced by the condition that *this person has not learned anything other than p that is of possible evidential relevance to q independent of the possible evidential relevance to q of p*. For one may assume that under that condition, for a rational person the possible evidential bearing of p on q would not change. (Let r be of possible evidential relevance to q *independent of* the possible evidential relevance to q of p if and only if $P[q/((r \& p)] \neq P(q/p)$.

Here, for comparison with CR3, is a closely related formal theorem.

Theorem 3. For probability functions P and P', any p such that $P'(p) = 1$, and any q: if $P(q/p) = P'(q/p)$, then $P'(q) = P(q/p)$.

This theorem 'says' that if (a) a person's credences at both time t and time t' are represented probability functions, (b) this person is at t' certain of p (whether or not by learning in the interval (t,t'), supposing $P(p) < 1$ and t' is later than t), and (c) this person is of one mind at t and t' about the possible evidential bearing of p on q'; then, supposing $P(p) < 1$ and t' is later than t, this person *has* by t' revised his credence at t for q by conditionalizing on p. (Note that this gloss on the above theorem depends on allowing certainties to have possible evidential bearings, since p, even if not certain at t, is to be certain at t'.)

III. Dutch Books against Non-conditionalizers

Bets — simple, conditional, and fair — have been explained in (Sobel 1987). If a simple bet is fair for a person, then its expected return for this person is nil. Similarly for fair conditional bets: if a conditional bet is fair for a person, then when it is 'on' its expected return is zero. Furthermore, fair conditional bets based on positively probable conditions are equivalent to books of fair and simple bets ((Sobel 1987), pp. 58-9). In what follows a case is found in which a non-conditionalizer would be in a certain kind of 'diachronic Dutch Book' trouble. A diachronic Dutch Book or Dutch Strategy that is spelled out in (Sobel 1987) is here developed informally. Then comes the consideration of the question of how the non-conditionalizer, by falling short of an ideal for intellects, is at fault for his trouble.

1. What we want here is a case in which a non-conditionalizer who was prepared at any time to accept any bets that were fair in terms of his probabilities at that time would be, because of his non-conditionalizing, at the mercy of a really clever bookie who always knew this agent's current probabilities, and knew everything that the agent knew (though no more) about his possible future possible probabilities.

Case 1. Suppose that Timothy Nayes, an unbayesian free thinker, places some credence in p so that $P(q) > 0$, but that he would not, if he were to learn p for sure by a certain future time t', revise his probability for q by conditionalizing on p. Suppose, furthermore, that both he and a clever bookie know all this.

https://doi.org/10.1086/psaprocbienmeetp.1990.1.192727 Published online by Cambridge University Press

Can Nayes be 'had' by a really clever bookie? Not necessarily. Let P be Nayes's probability function now, at t, and P' be his probability function at t' if he learns for sure by then that p. It is true (i) that bets on q conditional on p that are fair will be at odds based on P(q/p); (ii) that, if he by t' learns for sure that p, fair simple bets on q will be at odds based on P'(q); and (iii) that by hypothesis, if he by t' learns p for sure, $P(q/p) \neq P(q)$ so that these are different odds. But though it has been assumed that Nayes would not conditionalize on p, it has not been assumed that either he or the clever bookie knows now, at t, how, that is, in what direction, and by what amount, he would revise his probability for q were he to learn for sure that p by t'. And so even a really clever bookie may not be in a position at t to place bets that, when combined with possible bets at t', take advantage of the possible differential in odds described.

Case 2. Let us *add* to our suppositions that Nayes would, if he were to learn p for sure by t', revise his probability to P'(q), *and* that both Nayes and a clever bookie know this at t.

Can Nayes now be 'had' by this clever bookie? Not necessarily, still. Suppose a bookie at t secured a bet on q conditional on p. And suppose that Nayes does not learn p for sure by t'. The revision to P'(q) that the bookie would need to have in some sense been banking on may well not take place — if it does, that it does is just luck. But even so the bet on q conditional on p may be '*on*', for it *is* 'on' if and only if p is *true*, as it can be whether or not it is *learned.*

Case 3. Let us therefore add again to our suppositions. Let it be that p is true if and only if Nayes will learn it for sure by t', and that Nayes and a clever bookie both know *this.*

Is Nayes now in trouble? Yes he is. A clever bookie can at t set a bet on or against q conditional on p at odds based on P(q/p), and set a small simple bet against p at odds based on P(q) (which I recall exceeds 0) so that if that conditional bet is not 'on', that is, if not p, then he wins this small side-bet. And he at t can *plan*, if that conditional bet is 'on', at t' to place a simple bet against or on q at odds based on P'(q). He can make this plan with *confidence*, for he knows that that conditional bet is 'on' if only if p is true, and so in *this* case, if and only if Nayes learns p for sure by t' and revises to P'(q). The bookie places a conditional bet and plans for a simple one that, when 'on' and placed respectively, take advantage of the swing that will have occurred from fair odds at t on q conditional on p, to fair odds at t' on q simply. He can place and plan bets that insure a net win whether or not q, and a net win bigger than the small loss he will have incurred on the side-bet against p. He has available at t a schedule of conditional and simple bets at t, and a possible bet at t', that guarantees him a net win, and guarantees Nayes a net loss, whether or not p, and whether or not q.

2. I think that Dutch Books are signs of faults — that only somewhat imperfect intellects should be vulnerable to them. Dutch Books against non-conditionalizers want to be explained in terms of independently plausible conditions of intellectual perfection that are not satisfied by their victims. The problem posed by Case 3 is that it is not obvious that there must be something wrong with Nayes, and that in a sense he must bring this trouble on himself. I think that at least under a certain assumption he cannot be a paragon of rationality and intellectually ideal. The assumption is that 'the certainty model' is right for him, that he is a 'certainty-model learner' who learns things only with certainty — that for him all "knowledge [of contingencies] originates in observation...that observation makes particular sentences (observation reports) cer-

tain and that the probability of other sentences is attributable to the certainty of these" (Skyrms 1986, p. 18).[4] The assumption is that for Nayes impingements of reality on his intellect generate in the first instance nothing less than subjective certainties.

In what follows I approach and then, making use of this assumption, reach the conclusion that under the assumption that he is a certainty-model learner Nayes must be something less than intellectually ideal in Case 3.

For a preliminary conclusion, we have that if Nayes were intellectually ideal in Case 3, then he would be sure that if by t' he learned for sure that p, he would have changed his mind about its possible evidential bearing on q.

Proof: Nayes is sure in Case 3, as he is even in Case 1, that if he learns p for sure he will not conditionalize on it when revising his probability for q:

$$\text{(i) } P'(p) = 1 \rightarrow P'(q) \neq P(q/p)$$

Suppose now that he were intellectually ideal. Then he would also be sure that his credences would be coherent at t' and represented by a probability function, and that in particular, if he were then certain of p, his probability for q would then equal his probability for q conditional on p:

$$\text{(ii) } P'(p) = 1 \rightarrow P'(q) = P'(q/p)$$

But then, from these two things of which he would be sure it follows that, as was to be shown, if intellectually ideal he would be sure that if by t' he learned for sure that p, he would change his mind about the possible evidential bearing of p and q.

$$P'(p) = 1 \rightarrow P'(q/p) \neq P(q,p).$$

But this consequence, namely that Nayes if intellectually ideal would be sure in Case 3 that he would change his mind about the possible evidential bearing of p on q were he to learn for sure that p, does not by itself show that there is anything wrong with him intellectually. For he could be sure that he would have good reasons for such a change of mind. He could be sure that if by t' he learned p for sure he would have learned other things of further, independent-of-p, evidential relevance to q. For example, he could be sure that he would have learned that he had learned p for sure, and he could realize that this would be further, and independently of p, relevant to q. As indeed it could be if, for example, q were the proposition that he learns p for sure.[5]

It has not been shown that it is implicit in Case 3 that Nayes is not intellectually ideal. But this is not surprising since so far I have used nothing that is not also a feature of Case 1, and Case 1 does not yet harbour a Dutch Book. I have observed that though were Nayes intellectually ideal he would be sure that were he to learn that p, he would then change his mind about its possible evidential bearing on q, this is consistent with his being intellectually ideal For he might be sure he would learn other things of independent relevance to q. I offered as an example that he might be sure that he would learn that he had learned that p, and that that might for him be further, and independent of p, relevant to q. But it can be seen that this example is ruled out by what is distinctive about Case 3. Let p* be the proposition that as of t' Nayes learns p for sure. Then it is distinctive of Case 3 that in it Nayes is at t sure that $p \leftrightarrow p^*$,

$$\text{(iii) } P(p \leftrightarrow p^*) = 1.$$

https://doi.org/10.1086/psaprocbienmeetp.1990.1.192727 Published online by Cambridge University Press

This means that, if he were intellectually ideal, he could not think that p* would be further, and independently of p, relevant to anything. It follows in particular from (iii) that,

$$\text{(iv)}\ \ P[q/(p^* \,\&\, p)] \neq P(q/p).$$

What would be welcome would be a general argument that, given the assumption that Nayes is intellectually ideal, ruled out all such examples, and that explained why, in Case 3, Nayes could not be sure of anything r that would be further, and independently of p, relevant to q, i.e.,

$$P[q/(r \,\&\, p)] \neq P(q/p),$$

that, if he were to learn for sure that p, he would also learn for sure that r,

$$P(p^* \rightarrow r^*) = 1.$$

That is, what would be most welcome would be a general argument that, assuming Nayes to be an ideal learner, showed that,

$$P(p^* \rightarrow r^*) = 1 \rightarrow P(q/r \,\&\, p) = P(q/p).$$

Such an argument would show that it was not consistent with his being intellectually ideal that he should be sure in this case that, were he to learn that p, he would change his mind about its evidential relevance to q. For given such an argument, if he were sure of this, he would be equally sure that he would, upon learning that p, change his mind about its possible evidential bearing on q, and change his mind about this *for no reason* not already in his possession before learning that p. Satisfied as he would be with the grounds and reasons for his views concerning possible evidential bearings, he would need to anticipate a corruption of these views were he to learn that p. But his viewing as possible that personal development would not be consistent with his being intellectually ideal. For an ideal intellect would be satisfied not only with his present credences and views of possible evidential bearings, but with all his past and possible future ones as well. Such self-satisfaction is, I think, "a condition of an enduring intellect's full integration and self-possession" ((Sobel 1987), p. 72) and perfection.

As a preliminary to such a general argument, I note the immediate availability of a conclusion related to the one we seek — specifically, the related conclusion that in Case 3, if Nayes were intellectually ideal, he could not be sure that if he were to learn for sure that p, he would learn something r such that r*, the proposition that he had learned r, would be of independent relevance to q. In Case 3 we have $P(p \leftrightarrow p^*) = 1$. So in this case we have that,

$$P(p^* \rightarrow r^*) = 1 \rightarrow P(q/r^* \,\&\, p) = P(q/p).$$

What still needs to be proved (what is wanted for the argument described in the previous paragraph) is that, given that, in Case 3, $P(p \leftrightarrow p^*) = 1$,

$$P(p^* \rightarrow r^*) = 1 \rightarrow P(q/r \,\&\, p) = P(q/p).$$

But this follows if, for any r such that $P(p^* \rightarrow r^*) = 1$,

$$\mathbf{P(r^* \leftrightarrow r) = 1.}$$

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That is, what is wanted for the argument described in the previous paragraph does obtain if about anything that he is sure he would learn along with p, Nayes is sure that he would learn it if and only if it was the case.[6] And this condition can be maintained. For it is assumed that the certainty model is right for Nayes, and it seems that when this model is right for an ideal intellect, this intellect should be sure not only that it learns things only with certainty, but sure that it learns and becomes certain only of things that are both true and such that they cannot escape his notice. For an ideal certainty-model learner it should be both that $P(r^* \rightarrow r) = 1$ and that $P(r \rightarrow r^*) = 1$, and therefore that $P(r^* \leftrightarrow r) = 1$.[7]

So Nayes, *if* intellectually ideal, would realize in Case 3 that he would, were he to learn p, change his mind about its possible evidential bearing on q, and change his mind about this possible bearing for no reason. He would realize that the change would not be due to reflection on other things learned. But then he could not be intellectually ideal. For *were* he intellectually ideal, then he would *not* be, as any ideal intellect *would* be, fully satisfied with his views of possible evidential bearings, both with his actual views of possible bearings of things and with views that he realizes would be his consequent to learning certain things.

IV. Conclusions

Conditional probabilities of a highly opinionated and logically omniscient person would represent what for him were possible evidential bearings of contingent propositions on one another. So, under certain conditions, such a person, if rational, would revise probabilities in the light of new information by conditionalizing on this information. In particular, such a person, would if rational revise his probability for q by conditionalizing on p, upon becoming certain of p without any change of mind about its possible evidential bearing on q. If, contrary to this condition of rationality, such a one were a known non-conditionalizer, then, at least in connection with things that he would learn for sure if and only if they were true, books could be made. And if this known non-conditionalizer was a certainty-model learner, the possibility of these books would, in a sense, be his fault. For a certainty-model learner would, if an ideal and faultless learner, be sure that he learned only true things that could not escape his notice. And that, we have seen, entails that such a learner, if a non-conditionalizer in such cases, could not be an ideal and faultless intellect. Such a one could not know the satisfaction with his views of possible evidential bearings that an ideal and faultless intellect would enjoy.

I have not claimed that conformity to the certainty model is itself part of the ideal for an intellect. But there is plausibility in this idea, and thus plausibility in stronger conclusions than those so far maintained. Reviewing limitations of the certainty model, Skyrms writes:

> [W]hat I take to be the heart of the matter is this: no matter what language we use to describe our observations, the act of observation and the act of believing a sentence attributing a certain character to that observation are distinct....The link between them is causal....If I am of sound mind and body, adopt a modest observation language, and am proficient in its use, this causal process may be highly reliable as a means for generating true beliefs. But there is no reason whatsoever to believe that it is infallible. (Skyrms 1986, p. 194)

My suggestion, adapted to this way of distinguishing observations (experiences) from beliefs, is that it is plausible that any intellect should aspire to language adequate to its experiences and aspire to certain and complete observational beliefs. The process is never infallible, but it seems a proper goal to which every intellect should aspire that its observation-to-beliefs process be infallible (i.e., unerring), and that it should issue observational reports that are not only invariably accurate and true, but that are complete with respect to all details of experiences. And if to be this kind of ideal certainty-model learner is a proper part of the ideal for intellects then the conclusion of this paper can be strengthened. It is then properly not just imagined certainty-model learners but absolutely all learners who, if vulnerable to the kind of book discussed, would so be only because they were in one way or another less than perfect, and thus at fault for their possible victimizations.

Notes

[1]I contend that (i), if $P(e) > 0$, e is potential evidence for h if and only if, the difference between the posterior to e probability of h and the prior probability of h is positive:

$$P(h/e) - P(h) > 0.$$

For a nearly equivalent condition it may be noted that, if not only $P(e) > 0$ but also $P(h) > 0$, then $[P(h/e) - P(h) > 0]$ if and only if the relevance quotient of e for h exceeds one,

$$P((h/e)/p(h) > 1.$$

For another nearly equivalent condition, it may be that, if $P(e \& \sim h) > 0$, then $[P(h/e) - P(h) > 0$ if and only if the difference between the posterior to e odds on h and the prior odds on h is positive,

$$P(h,e)/P(\sim h,e) - P(h)/P(\sim h) > 0.$$

However, there are not analogous (near) equivalents to my contention (ii) that e would be more evidence for, or less evidence against, h, than e' would be for or against h', if and only if $[P(h/e) - P(h)] > [P(h'/e') - P(h')]$. To see that it is not true that always, $P(h/e) - P(h) > P(h'/e') - P(h')$ if and only if $P(h/e)/P(h) > P(h'/e')/P(h')$, consider that $3/4 - 2/4 > 3/6 - 2/6$, but $3/2 = 3/2$. To see that it is not true that always, $P(h/e) - P(h) > P(h'/e') - P(h')$ if and only if $P(h/e)/P(\sim h/e) - P(h)/P(\sim h) > P(h'/e')/P(\sim h'/e') - P(h')/P(\sim h')$, suppose that $P(h/e) = 3/5$, $P(h) = 1/5$, $P(h'/e') = 3/4$, and $P(h') = 2/4$. These probability and conditional probability assignments also show that it is not true that always $P(h/e)/P(h) > P(h'/e')/P(h)$ if and only if $P(h/e)/P(\sim h/e) - P(h)/P(\sim h) > P(h'/e')/P(\sim h'/e') - P(h')/P(\sim h')$.

Notwithstanding the observations of the previous paragraph, when comparing possible potential evidences e and e' for a single hypothesis h, the condition for a positively probable e being more potential evidence for h that e', specifically, that

$$P(h/e) - P(h) > P(h/e') - P(h),$$

is equivalent to

$$P(h/e) > P(h/e'),$$

and thus, if $P(h) > 0$, to

$$P(h/e)'/P(h) > P(h/e')/P(h),$$

https://doi.org/10.1086/psaprocbienmeetp.1990.1.192727 Published online by Cambridge University Press

as well as (compare this with Jeffrey 1985, p. 108),

$$P(h/e)/[1 - P(h/e)] > P(h/e')/[1 - P(h/e')],$$

when both [1 - P(h/e)] and [1 - P(h/e')] are positive, and so, as well as,

$$P(h/e)/P(\sim h/e)] - P(h)/P\sim h) > P(h/e')/P(\sim h/e')] - P(h)/P(\sim h),$$

when both P(e & ~h) and P(e' & ~h) are positive.

There are alternative-to-condition-(ii)-of-the-text explications of comparative potential evidential bearings. See the previous but one paragraph. I have nothing decisive to say against these alternatives. To be definite, I settle for condition (ii). I favour this difference-between-posterior-and-prior-probabilities measure, because, thinking now of simple cases in which revisions of probabilities by conditionalization on e would be appropriate, 'the difference that learning e would make to the probability of h' sounds right as a measure of e's potential evidential bearing on h. Also this measure is defined for a somewhat wider range of cases than the alternatives to it considered above.

[2]Compare,

$$P(p) > 0 \;\; P(p \,\&\, q)/P(p) = P_p(p \,\&\, q)/P_p(p),$$

which is entailed by the principle that "unlike all other revisions of P to make p certain, [conditionalizing P on p] does not distort the profile of probability ratios...among sentences that imply p" (Lewis 1976, p. 311) .

[3]There are other cases in which Nayes could be 'had' by a clever bookie. For example, he could be 'had' by a clever bookie even if it was not true that p would be true if and only if he learned p for sure by t', if it was true that Nayes was sure that p would be true if and only if he was certain of p at t' (v. (Sobel 1987), p. 77ff). Indeed it would sufficient that the bookie know that Nayes, though not sure, was sufficiently confident that p would be true if and only if he was certain of p at t' (ibid., p. 79). Also Nayes can be 'had' even if, though he does not have such opinions, a bookie knows that if Nayes were to learn p for sure by t', *that* he had learned p for sure would not be further, and independently of p, relevant to q (ibid., p. 80).

[4]Compare with remarks made by Jeffrey about 'dogmatic empiricism' as found in J. M Keynes and C. I. Lewis: Jeffrey 1985, pp. 114 and 116.

[5]For another example, let p be that stock in Gold Strike will 'take off' tomorrow. Let q be that I will make money on Gold Strike. It could be that for me P(q/p) is *very low*. For perhaps, I do not own Gold Strike stock, and since I think that it is *unlikely* that Gold Strike will 'take off', I have no intention of buying any. In that case I might think it is *much more unlikely* that not only will it 'take off' but that I will make money on it. Even so, I may realize that if I were to learn for sure that Gold Strike will 'take off', I would form the intention of buying some and be sure of that intention. In this case after that learning that p, P(q/p) could be very high. (Paul Weirich considers taking a position in Gold Strike in Weirich 1984.).

[6]Wlodzimierz Rabinowicz has observed that what is wanted follows given the weaker condition that, for any r such that $P(p^* \rightarrow r^*) = 1$, $P(r^* \rightarrow r) = 1$. It is sufficient that, for things Nayes is sure he would learn along with p, he be sure that he

https://doi.org/10.1086/psaprocbienmeetp.1990.1.192727 Published online by Cambridge University Press

would not be learn them unless they were true. It is not necessary that he be sure that they are things which, if true, could not escape his notice. For the entailment of what is wanted — viz., $P(q/r \& p) = P(q/p)$ — given only $P(r^* \rightarrow r) = 1$, I recall that $P(p \leftrightarrow p^*) = 1$. So, given $P(p^* \rightarrow r^*) = 1$ and $P(r^* \rightarrow r) = 1$, it follows that $P(p \rightarrow r) = 1$, and thus that $P[p \leftrightarrow (r \& p)] = 1$. And from this it follows that $P(q/r \& p) = P(q/p)$.

[7]This idealization is a part of the 'dogmatic empiricisms' of Keynes and Lewis. In Keynes' "view of knowledge and probability...our probable knowledge is founded on certainties, i.e., truths known by direct experience" (Jeffrey 1985, p. 116). Lewis bases all probable knowledge on certainties, specifically, "certainties, in the sense data initiating belief and in those passages of experience which later may confirm it" (these words of Lewis' are quoted by Jeffrey, ibid.). 'Direct experience', 'sense data', and 'passages of experience' are terms for things that could not escape a person's notice, and of which a person would be certain they were such and so if and only if they indeed were such and so.

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