Note on a Property of the Cycloid

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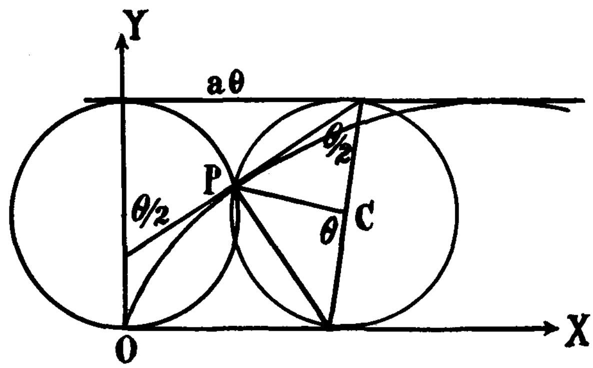

The area of the triangle whose sides are a variable tangent to the cycloid, the tangent at the cusp and the tangent at the vertex is maximum when the point of contact of the variable tangent is the point of intersection of the cycloid and the generating circle at the cusp.

With the usual notation, the cycloid is given by

$$x = a(\theta - \sin\theta), \quad y = a(1 - \cos\theta) \tag{1}$$

and A, the area of the triangle, by $A = \frac{1}{2}a^2\theta^2 \cot\frac{\theta}{2}$.

Whence $$\frac{dA}{d\theta} = \tfrac{1}{2}a^2\left(2\theta\cot\frac{\theta}{2} - \tfrac{1}{2}\theta^2\operatorname{cosec}^2\frac{\theta}{2}\right).$$

When $\theta = 0$, P is at 0, and we have a zero value of A. Also $\frac{dA}{d\theta} = 0$ when $4\cot\frac{\theta}{2} = \theta\operatorname{cosec}^2\frac{\theta}{2}$, or when

$$2\sin\theta = \theta, \tag{2}$$

and this obviously gives a maximum value of A.

For values of θ which satisfy (1) and (2), we have

$$x = a\sin\theta, \quad y - a = -a\cos\theta,$$

whence $$x^2 + (y-a)^2 = a^2,$$

which proves the theorem.

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