

# An Ultraviolet View of the Luminous Blue Variables

Steven N. Shore

Astrophysics Research Center, Dept. of Physics  
New Mexico Institute of Mining and Technology  
and DEMIRM, Observatoire de Meudon

## 1. Introduction

It is, perhaps, tautological to say that the luminous blue variables (*LBVs*), being luminous and blue, should be ideal targets for space ultraviolet observation. In fact, many have been observed on occasion and these will be discussed in this article. Mitigating circumstances, however, conspire to make this study difficult. The *LBVs* are rare and tend to be apparently faint, and since they are massive and formed in the plane in our Galaxy and in active star-forming regions in other galaxies, they tend to be heavily reddened. The stars were not really studied at all in the ultraviolet until the launch of the *International Ultraviolet Explorer Satellite (IUE)*, with the notable exceptions of photometric observations of several of the galactic stars with *ANS* and *TD-1* and *Copernicus* observations of P Cygni (Hutchings *et al.* 1987). *IUE*, a small aperture (45 cm) spectrographic satellite, covering the spectral range from 1200 to 3300 Å, allows for an adequate characterization of the UV properties of these stars.

## 2. Why Go to the Ultraviolet?

Optical observations show that many of the *LBVs* are embedded in emission regions and that the optical emission lines from the extended envelopes render it difficult or impossible to say much about the behavior of the underlying star. For mass loss indicators, the optical depths in the absorption components of optical P Cygni lines are not sufficient to properly sample the stellar wind velocity profile; many of the stellar wind lines are formed from excited states. In the ultraviolet, we have a better view of the opacity of the envelope and a clearer probe of the ionization structure. Many of the strong UV lines are resonance transitions, which give a better handle on the radiative balance of the wind. Finally, and perhaps most important, the UV permits the direct measurement of the bulk of the stellar flux so that one can determine stellar bolometric luminosities without reliance on optical properties of normal supergiant atmospheres.

## 3. Low Dispersion Spectra and Ultraviolet Spectral Types

We do not see the naked star in the ultraviolet anymore than we do in the optical, but more information is available from the UV about the behavior of the envelope because we see a pseudo-photosphere which looks remarkably like a normal supergiant. It is better to say that the UV gives us a picture of what the star would like to believe itself to be,

and what its environment knows it as. For instance, it is possible to use the ultraviolet to connect the infrared emission to the bolometric luminosity of the star through IR line fluorescence and dust re-emission, which are driven by the absorption of UV photons (see, for example, McGregor *et al.* 1987). Because this UV continuum is also the spectrum responsible for illuminating any surrounding nebula, direct UV observation is important for chemical analysis of the ejecta.

*Ultraviolet Spectral Types (UVST):* In the absence of good quantitative stellar atmosphere models for these stars, one can form an ultraviolet spectral type using Si III  $\lambda$ 1300, Si IV  $\lambda$ 1400, C IV  $\lambda$ 1550 and Al III  $\lambda$ 1860 (Henize *et al.* 1981, Shore and Sanduleak 1984) for supergiants in the range O8 to B9; additional information about the shell structures is provided by the strong Fe II absorption systems near 1500, 1600, and 2200Å. High resolution observations of the Fe II lines between 2500 and 2600Å sometimes show P Cygni profiles, which probe the outer portions of the envelope and provide information about expansion velocities and wind strengths (*e.g.* Stahl and Wolf 1986). An advantage of the UVST method over model fitting is that it is independent of the reddening assumptions. For detailed application of this method to the LMC/SMC stars see Shore and Sanduleak (1984); recent work has extended to Galactic stars (Shore *et al.* 1988).

*General Results:* The Si IV  $\lambda$ 1400 doublet is typically resolved even at low resolution for most of these stars. R 127 is an unusual exception to this rule, suggesting that the ejection event may be initially at higher velocity than normally observed for the shells in these stars. Ultraviolet observations show that the envelopes often remain stable on long timescales, and that the high mass loss rates are perhaps present for decades. The structure of winds in these stars does not appear to greatly differ between the Galaxy and the LMC. Put more precisely, it appears that the metallicity difference between these galaxies is insufficient to produce any observable difference in the behavior of the stellar mass outflows.

Those stars which optically show the strongest Fe II and [FeII] systems, like R 50 = S 65/SMC, S 22/LMC, S Dor, and He3-407, typically show the most strongly absorbed UV systems as well. Strong optical stellar wind profiles are normally associated with the strongest resonance line absorption in the UV (S 12/LMC, S 61/LMC). Many of the stars which show S Dor-like profiles in the optical, with broad emission wings on the hydrogen lines and narrow, low velocity absorption, show B2 - B4 spectra in the UV; this may be an indication of multiple shell ejection events and could be a useful indicator of those stars which are likely to undergo shell outbursts (like R 127 = S 128/LMC).

It is generally the case that the LBVs show only an optically thick spectrum in the UV. Only a few LMC stars show emission lines: S9 (NIII], Si III]:), S 12 (N IV), S 30 (He II, N III], Si III]:), MWC 112 (N III]:), S 131 (Si IV, C IV, Si III], N III]). Only S 134 shows both narrow, nebula-like emission and P Cyg profiles on the resonance lines. The strongest emission line objects are S 18/ SMC and LMC Anon. For the Galactic stars, only He3-40 shows Mg II emission. Strong Fe II absorption systems are the rule for these stars. The stars in the LMC sample (Shore and Sanduleak 1984) had been chosen from the Henize list of H $\alpha$  emission line stars, while those observed by Stahl *et al.* (1985) have been selected optically from the same list. Not all of these stars have been known as LBVs, but several have proven to be after the fact.

*Disks vs. Cocoons:* The Hamburg group has suggested that the B[e] stars are hybrid spectra, and has explained the low excitation, narrow lines as arising from a circumstellar disk, perhaps a low velocity equatorial wind, on the basis of optical spectra. Gallagher (*this conference*) has argued that many LBVs are binaries with dissipative accretion disks. The ultraviolet spectral distributions, at low dispersion, do not match the expectations for disk systems. There are thus two different kinds of disks, and perhaps the UV observations can narrow the choices.

Turbulent disks have a characteristic spectral shape. Assuming that the disk is optically thick, but geometrically thin, and that the energy is released primarily through viscosity acting locally throughout the disk, then the temperature in the disk varies as  $T(r) \sim r^{-3/4}$ . Assuming that the disk radiates like a Wien radiator, then  $F_\nu \sim \nu^{1/3}$  with the upper portions of the disk presenting an effective chromosphere to this hot optically thick surface. The luminosity depends on the stellar mass and the mass accretion rate, and the  $M/R$  value for the accreting star. Spectra much like those of the LBVs have been observed in some of the longer period Algol systems, especially 22 Vul and HD 207739 (McCluskey and Sahade 1987). The equatorial winds (hybrid stars) will not show the same dependence of temperature on radius, but it is still to be demonstrated that they reproduce the UV spectra.

Predictions are needed of the UV and optical appearance of so luminous a disk, accreting at the Eddington limit, if this is to be the sole explanation for the UV behavior. For the LMC stars which have been well studied, and for which the reddening is low, like S 111/LMC, the continuum matches well with normal stellar atmospheres. If there are disks, they may be a factor in the emission lines but do not play much of a role in the formation of the ultraviolet continuum. If disks are involved with the LBVs, UV observations suggest that they must be excretion, rather than accretion, type.

Only a few systems, like S 18/ SMC (Shore *et al.* 1987, Zickgraf *et al.* 1988), R 99, or S 131/ LMC (Stahl *et al.* 1987) show strong UV emission lines. LMC Anon (Michalitsianos *et al.* 1988, *preprint*) looks similar to  $\eta$  Car and S 18/SMC in these characteristics and is also variable in optical He II  $\lambda 4686$ . S 18/SMC appears to require some accretion from a companion to power the far-UV radiation source (Shore *et al.* 1987; Zickgraf *et al.* 1988, *preprint*).

#### 4. Some Galactic LBVs

A good sample for Galactic stars is the Henize sample of P Cygni and B[e] stars studied optically in some detail by Carlson and Henize (1979) and in the IR by McGregor *et al.* (1987). They consist of a sample of optically related stars, which generally show strong emission lines of H and He I, sometimes N and C ions, and Fe I and [Fe II] as well. They have been observed to show IR excesses in a few cases, and at least one of them, He3-1482 = HD 316285, is a radio source, like P Cyg. Since they are all fainter than  $V = 9^m$ , they appear to be at large distance and to have very large IS reddening.

Several of these stars have been observed to show extended shells, especially He3-517, but for this star there are no UV spectra (Stahl, *this conference*). A few are hot IRAS point sources.  $\eta$  Car, has been observed on several occasions with IUE. It shows strong nitrogen enhancement in the UV spectra of the ejecta (Davidson *et al.* 1982), in agreement with the optical spectra. Another well studied star is AG Car, which also shows an extended asymmetrically structured shell. High dispersion spectra are available for this star, which show that it has a strong stellar wind (Johnson 1982). Other stars which have been studied in detail in the ultraviolet are MWC 300 (Wolf and Stahl 1985), P Cygni (Lamers *et al.* 1985), and GG Car (Brandi *et al.* 1987) (which may be a main sequence star although it has many of the LBV properties and the luminosity is poorly known).

R 126 = S 127/LMC shows the strongest resonance lines of the B[e] stars in the LMC. This star provides a useful example of the UVST method. Its properties overlap part of the range of R 127, but the absorption lines are consistently stronger; it is not notable as a photometric variable. The Galactic star, He3-395 = HD 89249 shows nearly identical line strengths (Si III, Si IV, C IV, and Al III are within about 15 percent). R 126 has been

called a rotationally unstable hybrid star (Zickgraf *et al.* 1986), but it is surprising to find that it has a *sosie* in the Galaxy. The UVST for He3-395 is  $B0.5 \pm 0.5$ , with an effective temperature of about 27000 K, and a reddening of  $E(B - V) = 0.8$ ; optical photometry and spectra give 25000 K and  $E(B - V) = 0.8$ . Assuming average reddening, this gives a distance of about 2 kpc and a bolometric luminosity of about  $L \approx 10^5 L_{\odot}$ , fainter than R 126. Few of the B[e] stars have been observed to show strong UV variability, with HR Car and R 127 being notable exceptions. Several of the shells have been quite stable, notably S 22/LMC and S 65/SMC.

Some additional results for Galactic stars are: He3-365 ( $\log T_{eff} = 4.4$ ,  $A_V \approx 2.9$ ,  $\log L/L_{\odot} = 4.9$ ), He3-1138 ( $\log T_{eff} = 4.2$ ,  $A_V = 5.6$ ,  $M_{bol} \approx -9.6$ ), He3-1300 ( $\log T_{eff} = 4.4$ ,  $A_V \approx 5.0$ ,  $M_{bol} \approx -8.3$ ), He3-1330 ( $\log T_{eff} = 4.6 : (B1.5)$ ,  $A_V \approx 4.0$ ,  $M_{bol} \approx -8.8$ ). For most of the other stars in the Carlson - Henize sample there are no good photometric observations available.

## 5. The Large Magellanic Cloud

The disadvantages introduced by the distance of the LMC, both of faintness and of crowding, are largely offset by its distance above the galactic plane and the certainty in the distances to the associated LBVs. In the LMC, we see all of the stars in their environmental context and with little foreground obscuration. This is very important, because for these stars we can address the questions of dust formation and mass expulsion that would not be possible for their Galactic counterparts.

By far the best studied LBV in the LMC subsequent to the launch of IUE is R 127 = S 128 = HDE 269858*f*. First observed as an O Iapef star by Walborn, it was discovered to be a B2 Ia by IUE observations in 1982 (Shore and Sanduleak 1984, Stahl and Wolf 1983, Stahl and Wolf 1986); it may be the first case of an LBV being found in another wavelength region than optical. Thanks to the longevity of IUE, it is possible to trace most of the history of this star from the time of its first detection as a B star through its present status as one of the most optically luminous stars in the LMC and an A2Ia<sup>+</sup> spectrum similar to S Dor. During the years 1982 - present its bolometric output has remained essentially constant. When first observed, it showed strong Si IV and Al III lines, with little C IV absorption and no emission. The Fe II lines subsequently deepened and obliterated the hotter photospheric features without the appearance of strong emission lines. With the increase in brightness in the long wavelength end of the IUE range, high dispersion spectra revealed the P Cyg profiles on the Fe II (42) multiplet.

The prototypical LBV, S Dor, is a bit more difficult to study in the UV because of the presence of a hot companion, a B0 star which is several magnitudes fainter in the optical but about equal to S Dor at 1600 Å (Wolf *et al.* 1980). This complicates the interpretation of the IUE spectra. Low resolution studies have deconvolved the spatial blend of these two stars showing that S Dor is approximately a B4 spectrum, with deep absorption bands of Fe II (Leiterer *et al.* 1985).

The best studied stars are what are now called the B[e] stars (Stahl *et al.* 1985, Zickgraf, *et al.* 1986), also called the "Zoo" sample by Shore and Sanduleak (1984). Many of these stars are known to show only microvariability, but since R 127 has revealed its LBV nature in recent years, there is every reason to suspect that many new LBV candidates are lurking in this class of stars. Several of the B[e] stars have been studied at multiple wavelengths. S 22/LMC (Besammar *et al.* 1983) shows extremely strong Fe II absorption in the UV and one of the strongest Fe II and [Fe II] optical spectra. They argue that the continuum is well fitted by a power law and could arise from an accretion disk. S 134/LMC was discussed

by Shore and Sanduleak (1983), who concluded that the star has a dusty envelope, with enhanced 2200Å absorption, and a high mass loss rate; Stahl and colleagues argue that this is a hybrid spectrum. This star's UV spectrum is unique, displaying a strong P Cyg profile on N V and on C IV with possible emission at Si IV. The probable mass loss rate for this star is about  $10^{-5} M_{\odot} \text{yr}^{-1}$ .

Wolf, *et al.* (1987) have performed studies of R 84 and S 61. R 84 = S 91 is a star with a massive red companion; Shore and Sanduleak (1984) obtained a spectral type of B1 or earlier for the blue component. The ultraviolet absorption lines are strong with no obvious P Cyg structure; the Si IV lines show some structure, the lines are separated and the wind slow, in accord with the general behavior of the absorption systems in the galactic and LMC supergiants. C IV shows no obvious P Cyg structure. Possible structure is observed in the Al III 1860 doublet, which may be stronger than the galactic comparison star HD 188209. S 61 displays strong P Cyg absorption on many of the optical profiles but shows no emission lines in the UV. The Al III lines show strong P Cyg structure, even at low S/N. There is a hint of an emission component on the Si IV doublet, which is well resolved. In general, the emission features in the UV are weaker for these two stars than in their likely galactic comparison stars. All of the UV absorption lines are blue - shifted by about 200 km s<sup>-1</sup>, suggestive of a pseudo - photosphere.

Stahl and Wolf (1987) have studied HD 37836 = S 124/ LMC = R 123. which they have also shown to be surrounded by a circumstellar envelope of high density. This paper also includes a study of S 131/ LMC and HDE 269445. High dispersion data for S 124/ LMC shows strong absorption at C IV and Si IV, showing terminal velocities of about 2500 km s<sup>-1</sup>; this is very similar to what has been seen in HD 38489 and quite high for most of the S Dor-type variables. For S 131, for which excess dust absorption is seen at 2200Å, the reddening may not be correct; this may have caused an underestimation of the temperature.

The eclipsing system R 81 has been studied by Stahl *et al.* (1987), which is a prime target for UV observations during eclipse ingress and egress. The techniques employed for the eclipsing WR systems would be well applied here.

## 6. Dust and Shells

One curious result of the LMC survey is that many of the stars show a strong absorption feature at  $\lambda 2200\text{\AA}$  which is not present in neighboring stars in their parent associations, and also not present in the field. In general, *those stars which show strong emission lines in the ultraviolet are also the stars for which the dust feature is enhanced and for which the reddening curve looks "galactic"*. For a number of LBVs, IRAS observations, and groundbased follow-up, have revealed extended nebulae. AG Car and He3-519 (Stahl 1986, Stahl and Wolf 1986, McGregor *et al.* 1988), and S 134/LMC, S 9/LMC and S 12/LMC (Stahl *et al.* 1984). None of these stars have turned up as IRAS sources, suggesting that the dust observed in the UV may not occupy a large volume or may not have sufficient mass to produce a great deal of emission. Several stars for which the dust is not a strong absorber at 2200Å show up as extended shell sources; for these stars, the dust may be more "Magellanic" in nature (Hutchings *et al.* 1987).

Near IR signatures of dust have been found in several B[e] stars (Stahl *et al.* 1984). A number of the stars show evidence for dust shells also show CO, and possibly TiO, emission (see Zickgraf *et al.* 1988, *preprint*). When the dust is formed, and how it is related to the hot wind phase we now see, are open questions.

## 7. Hubble - Sandage Variables

Humphreys, *et al.* (1984) have obtained IUE and groundbased observations of the Hubble - Sandage variables in M31 and M33. The stars look like the cooler members of the Zoo stars, especially S22/ LMC and S71/ LMC. They are especially notable for the lack of strong absorption lines, no emission lines, and flat continua in the merged spectra. The UV observations show that these stars are systematically selected for in optical studies when they are heavily blanketed in the UV and thus brightest in the visual. If the photometric variations merely reflect the changes in the bolometric correction, one would expect that the ultraviolet spectrum should be later than the stars selected on the basis of their strong optical emission lines.

## 8. Some Implications of UV Observations

The presence of LBV-type stars poses in understanding the composite spectra of star-forming galaxies. For instance, in the blue compact dwarf systems, population models have not included the effects of a few massive, bright, highly evolved supergiants on the derived initial mass function (Kunth *et al.* 1985). One B or A-type massive supergiant can strongly skew the IMF toward the middle mass range, and could be responsible for some of the anomalous mass functions that have been suggested for the blue compact dwarf galaxies. For instance, the spectrum of Haro 3 is nearly identical with the 1982 spectrum of R 127 alone, and the inclusion of several LBVs in the spectra of Haro 2 and Mk 59 produce spectra which match very well. One need only recall the experience with R136a in 30 Dor to see how difficult the problem of determining the mass function can be when LBV or B[e] stars are involved.

## 9. Conclusions

Ultraviolet observations have made it possible to better understand the placement of the LBVs and B[e] stars on the HR diagram and to detail the characteristics of their photospheres and winds. They should be even more important once Space Telescope is launched, and we are able to obtain good high resolution spectra of mass loss diagnostic features. This is a field still in its infancy. Continuing ultraviolet monitoring of the Galactic and Magellanic Cloud stars is urgently needed to provide a database for future work on these beasts. Also, model atmospheres are needed, as well as quantitative predictions of high resolution line profiles in order to take advantage of the expanding horizon of UV observational possibilities.

## ACKNOWLEDGEMENTS

I wish to thank K. Davidson, R. Humphreys, H. Lamers, E. Fitzpatrick, J. Gallagher, D. Hunter, N. Walborn, O. Stahl, M. Shara, Y. Kondo, B. Bohannan, C. Garmany, P. Conti, F. Viallefond, and D. Kunth for enjoyable discussions, and to apologize to those whose work, for want of space, has not been cited (see also Mead *et al.* 1986). I especially thank my long-time collaborators N. Sanduleak, D. N. Brown, and G. Sonneborn. Some of this work, especially on population synthesis, has been in collaboration with P. Dyer. This work has been supported over the years by NASA and the ENS (Paris).



## REFERENCES

- Bensammar, S., Friedjung, M., Muratorio, G., & Viotti, R. 1983, *Astron. Astrophys.* **126**, 427.
- Brandi, E., Gosset, E., & Swings, J.-P. 1987, *Astron. Astrophys.* **175**, 151.
- Carlson, W., & Henize, K. 1979, *Vistas in Astronomy* **23**, 213.
- Davidson, K., *et al.* 1982, *Astrophys. J. Letters* **254**, L47.
- Davidson, K. 1987, *Astrophys. J.* **317**, 760.
- Henize, K., Wray, J.D., & Parsons, S.B. 1981, *Astron. J.* **86**, 1658.
- Humphreys, R.M., *et al.* 1984, *Astrophys. J.* **278**, 124.
- Hutchings, J.B., Lequeux, J., & Wolf, B. 1987, in *Exploring the Universe with the IUE Satellite*, ed. Y. Kondo (Dordrecht: Reidel), p. 605.
- Johnson, H. 1962, *Astrophys. J. Suppl.* **50**, 551.
- Kunth, D., Thuan, T.X., Tran Thanh Van, T. (eds.) 1985, *Star-Forming Dwarf Galaxies and Related Objects* (Paris: Editions Frontieres).
- Lamers, H.J.G.L.M. *et al.* 1985, *Astron. Astrophys.* **149**, 29.
- Leitherer, C. *et al.* 1985, *Astron. Astrophys.* **153**, 16.
- McGregor, P.J., Hyland, A.R., & Hillier, D.J. 1988, *Astrophys. J.* **324**, 1071.
- McGregor, P.J., Hillier, D.J., & Hyland, A.R. 1988, *Astrophys. J.*, **334**, 639.
- McCluskey, G., & Sahade, J. 1987, in *Exploring the Universe with the IUE Satellite*, ed. Y. Kondo (Dordrecht: Reidel), p. 427.
- Mead, J.M., Brotzman, L.E., & Kondo, Y. 1986, *IUE Newsletter*, Nr. 30.
- Shore, S.N. & Sanduleak, N. 1983, *Astrophys. J.* **273**, 177.
- Shore, S.N. & Sanduleak, N. 1984, *Astrophys. J. Suppl.* **55**, 1.
- Shore, S.N., Sanduleak, N., & Allen, D.A. 1987, *Astron. Astrophys.* **176**, 59.
- Shore, S.N. *et al.* 1988, in *A Decade of UV Astronomy with IUE*, ed. E. Rohlfs (ESA SP-281), p. 417.
- Stahl, O. & Leitherer, C. 1987, *Astron. Astrophys.* **177**, 105.
- Stahl, O., Leitherer, C., Wolf, B., & Zickgraf, F.-J. 1984, *Astron. Astrophys.* **131**, 307.
- Stahl, O. *et al.* 1985, *Astron. Astrophys. Suppl.* **61**, 237.
- Stahl, O. & Wolf, B. 1986, *Astron. Astrophys.* **154**, 243.
- Stahl, O. & Wolf, B. 1986, *Astron. Astrophys.* **158**, 371.
- Stahl, O. & Wolf, B. 1987, *Astron. Astrophys.* **181**, 293.
- Stahl, O., Wolf, B., & Zickgraf, F.-J. 1987, *Astron. Astrophys.* **184**, 193.
- Walborn, N.R. 1982, *Astrophys. J.* **256**, 452.
- Wolf, B., Appenzeller, I., & Cassatella, A. 1980, *Astron. Astrophys.* **88**, 15.
- Wolf, B. 1987, in *IAU Symposium 122*, eds. I. Appenzeller & C. Jordan, p. 409.
- Wolf, B. & Stahl, O. 1985, *Astron. Astrophys.* **148**, 412.
- Wolf, B., Stahl, O., & Seifert, W. 1987, *Astron. Astrophys.* **186**, 561.
- Zickgraf, F.-J., Wolf, B., Stahl, O., Leitherer, C., & Klare, G. 1985, *Astron. Astrophys.* **143**, 421.
- Zickgraf, F.-J., Wolf, B., Stahl, O., Leitherer, C., & Appenzeller, I. 1986, *Astron. Astrophys.* **163**, 119.

## DISCUSSION

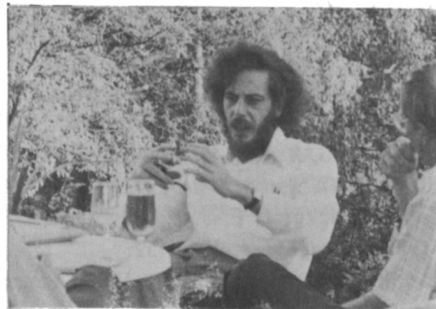
*Hillier:* A comment on HD 326823. This object appears to be different from other stars in Henize's list. It is very hydrogen-deficient and its red and infra-red spectra are very different from the other objects that you mentioned. Also, high-resolution red spectra indicate that the profiles are split and asymmetric, indicating that the envelope is not spherical.

*Zickgraf:* For R126 a high-velocity component in the wind, with maximum velocity  $\sim 1800$  km/s, is clearly indicated. Line widths in the optical region, on the other hand, show a second wind component with low velocity (several times 10 km/s).

*Shore:* When I say slow wind, I mean both the optical ( $\sim 200$  km/s) and UV ( $\sim 1000$  --  $1500$  km/s). This is slow compared to S134/LMC (= HD 38489), which is a similar star in physical parameters but which has a wind speed of about 2500 km/s.

*Schulte-Ladbeck:* Polarimetric monitoring may be able to distinguish observationally between the "pipe" and "torus" models. In the pipe model the puffs that come off asymmetrically should cause variable polarization, while in the torus model the dust ring might not show time-dependent polarization. Polarization would depend on the inclination in the latter case, providing another test for a torus model, *i.e.*, large polarization for edge-on examples and small polarization for pole-on objects.

*Shore:* Lovely point, and I agree. The torus, however, may be unstable. Models for Newtonian, radiation-dominated tori around active galactic nuclei are generally found to be unstable and tend to form rings and jets; so the polarization results may be more ambiguous than one would have hoped. It is true, however, that polarization gives the most immediate accessible information about the geometry.



Steve Shore