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A NOTE ON PRIME RADICALS OF CERTAIN GROUP RINGS

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Let R be a ring (with identity) and $P(R)$ denote its prime radical. R is called semi-prime when $P(R) = (0)$. If G is a group, the group ring of G over R will be denoted by RG.

In Tan (1974), we ask the following question: if R is left Goldie and G torsion-free abelian, is it true that $P(RG) = P(R)G$?

In this note, we will prove that the answer is affirmative. In fact, we will establish the following more general result.

THEOREM. *If R is left Goldie and G is torsion-free, then* $P(RG) = P(R)G$.

To prove this, we need the following

LEMMA 1. *If K is an ideal of R such that R/K is semi-prime, then* $K \supseteq P(R)$.

PROOF. See Lambek (1966), page 56.

LEMMA 2 (Connell-Passman). *RG is semi-prime if and only if R is semi-prime and the order of no finite normal subgroup of G is a zero divisor in R.*

PROOF. See Lambek (1966), Proposition 8, page 162.

We now prove the theorem.

Since R is left Goldie, it follows that $P(R)$ is nilpotent, then so is $P(R)G$. Consequently, $P(R)G \subseteq P(RG)$.

On the other hand, we have

$$RG/P(R)G \cong (R/P(R))G = T,$$

say. However T is semi-prime by Lemma 2. It then follows by Lemma 1 that

$$P(R)G \supseteq P(RG).$$

This proves the theorem.

References

J. Lambek (1968), *Lectures on Rings and Modules* (Blaisdell, Waltham, 1966).

D. S. Passman (1962), 'Nil ideals in group rings', *Mich. Math. J.* **9**, 375–384.

K. T. Tan (1974), 'Some torsion-free group rings with nilpotent prime radicals', *J. Austral. Math. Soc.* **18**, 372–375.

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