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Preface to the special issue: Computing with infinite data: topological and logical foundations

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This special issue of *Mathematical Structures in Computer Science* is composed mainly of papers submitted by participants of the Dagstuhl Seminar on *Computing with Infinite Data: Topological and Logical Foundations*. The workshop took place in the Schloss Dagstuhl - Leibniz Center for Informatics in the first half of October 2011.

A major motivation for the research presented in this seminar is the still unsatisfying situation in scientific computing where the current mainstream approach uses programming languages that do not possess a sound mathematical semantics. As a result, there is no way to provide formal correctness proofs.

The reason is that on the theoretical side, one deals with well-developed analytical theories based on the non-constructive concept of a real number. Implementations, on the other hand, use floating-point realizations of real numbers, which do not have a well-studied mathematical structure. Ways to get out of these problems have been promoted under the slogan ‘Computing with Exact Real Numbers’. Well-developed practical and theoretical bases for exact real number computations and, more generally, computable analysis are provided by Scott’s Domain Theory and Weihrauch’s Type Two Theory of Effectivity.

In both theories, real numbers and similar ideal objects are represented by infinite streams of finite objects. In contrast to the theory of computations on finite strings, the study of computations on infinite strings crucially depends on topological considerations. The study of such computations is important also for other branches of theoretical computer science including specification, verification and synthesis of reactive systems, as well as stream computability.

The seminar focused on two problem areas in the realm of computations on infinite streams and computable analysis:

— Algorithms for stream transforming functions with particular emphasis on logical and category-theoretic methods for the synthesis of provably correct programs as well as on topological investigations of particular stream representations supporting efficient stream algorithms.

— Hierarchies and reducibility relations between sets and functions of infinite data as a means of classification. Methods from topology, logic and descriptive set theory are of particular importance in this case.

Infinite streams are infinite words. So, there is a close connection to the theory of ω-languages which to study was a further aim of the seminar.

In the last years, much interest in the (logical and topological) structure of the infinite words used to represent continuous data as well as how they code the data space has emerged.

Of particular concern are approaches aiming for a constructive theory of digital computation based on coinduction, and its application to computable analysis. The intention is to create a mathematical foundation for (lazy) algorithms on analytical data such as real numbers, real functions, compact sets, etc. Since coinduction admits a particularly elegant formalization, such approaches are well suited for computer aided modelling and proving in computable analysis. A concrete goal is to use program extraction from proofs as a practical method for obtaining certified programs in computable analysis.

Besides the problem of how to represent continuous data in an 'optimal' way, it is as well an important task to distinguish between computable and non-computable functions and, in the last case, to measure the degree of non-computability. Mostly, functions are non-computable since they are not even continuous. A somewhat easier and more principal task is in fact to understand the degree of discontinuity of functions. This is mostly achieved by defining appropriate hierarchies and reducibility relations.

The aim of the seminar was to bring together researchers working in these (and related) problem areas and foster interaction between them. Such interaction was anticipated within each area, but also reaching across the two areas and to other topics.

Forty-nine top scientists and promising young researchers accepted the invitation. They came from 15 countries, mostly European countries, but also Argentina, Canada, Iran, Japan, Russia and South Africa. The atmosphere was very friendly, but the discussions were most lively. During the breaks and till late in the night, participants also gathered in smaller groups for continuing discussions, communicating new results and exchanging ideas.

As usual there are many people to be thanked. First of all we want to thank the institute and its staff, both in Saarbrücken and in Dagstuhl, for the excellent work they did to make all run smoothly in an efficient but always pleasant and friendly manner.

Moreover, thanks are due to the referees for having taken over the burden of carefully reading and commenting the submissions. Last, but not the least, we are very grateful to the Editor-in-Chief of *Mathematical Structures in Computer Science* for the opportunity to publish in this special issue of the journal.

The papers have undergone a reviewing process in accordance with the standards set for *Mathematical Structures in Computer Science*. The submissions by one of the special issue editors were handled by others so as to protect the anonymity of the reviewers and to avoid a conflict of interest.

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