SECTION VI

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THEORY OF WOLF-RAYET SPECTRA*

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1. Introduction

From the time of their discovery in 1867, Wolf-Rayet stars have been objects of great interest for theoretical astrophysicists. Their spectra, dominated by extremely wide emission lines, set them apart from the run-of-the-mill stars. Moreover, the classic assumptions that have enabled accurate model atmospheres to be constructed for hot main sequence stars: hydrostatic equilibrium, radiative equilibrium, local thermodynamic equilibrium, and negligible curvature effects, are probably all violated in Wolf-Rayet stars. As early as 1894, Scheiner proposed that

an enormous gaseous envelope (of unknown composition) surrounds the absorbing atmosphere, and produces bright lines in the spectrum by supplying to the slit of the spectroscope a greater quantity of light than the star's photosphere, in spite of the higher temperature of the photosphere.

In 1929 Beals presented evidence to support the hypothesis that the great width of the emission lines is due to the Doppler effect in a rapidly expanding envelope. Based on earlier work by Milne (1926), he proposed that radiation pressure in the spectral lines provided the propulsive force to drive the outflow. This work stimulated several papers on line and continuum formation in extended atmospheres. However, as it became realized that the theories were too rudimentary to apply to real stars, remarkably little theoretical work was done over the next thirty years. Only recently, with the availability of electronic computers, have quantitative studies of the expanding envelope model been made. These form the bulk of the present review.

Three alternatives to this model have been proposed. Thomas (1949) envisioned a type of super chromosphere supported by nonisotropic macroscopic motions in which the electron temperature T_e exceeds the radiation temperature T_r. Code and Bless (1964) advocated prominence-like activity in which streams of material are ejected into a thermalized shell. Finally, Limber (1964) discussed the possibility that forced rotational ejection of matter forms the circumstellar envelope. Whatever the strengths or weaknesses of these three hypotheses, no theoretical work exists to review beyond the original suggestions. We therefore limit discussion in the following sections to the expanding envelope hypothesis. As the results in Part IV show, this model has some success in describing the emission observed in Wolf-Rayet stars.

In attempting to present a coherent view of the theoretical interpretation of Wolf-Rayet lines and continua, there is a problem in that the number of investigations is quite small. This is especially true of theories of continuum formation. I have therefore endeavored to supplement the material available with calculations of an illustrative nature made for this review. It is hoped that the reader will not be too impatient with

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this material, in particular the comparison of the gray and non-gray model atmospheres in Part II. Finally, those empirical arguments that have been raised over the years for various aspects of Wolf-Rayet spectra are not discussed. If it cannot be expressed as an equation, it is not here.

2. The Continuous Energy Distribution

Although it is the spectacular emission line spectrum which is the outstanding characteristic of a Wolf-Rayet star, the continuous energy distribution is anomalous in a more modest fashion. Compared to hot main sequence stars, there is

(1) a slight ultraviolet excess for WN stars

(2) a large infrared excess, especially pronounced for the WN stars.

The result of this behaviour is that the color temperature T_c depends on wavelength: the longer the wavelength, the cooler the star appears (Kuhi, 1966; Kuhi, 1968). This accounts for the extremely low color temperatures for Wolf-Rayet stars quoted throughout the literature, when all other indications pointed to very high temperatures.

The construction of a model atmosphere to account for the continuous distribution in Wolf-Rayet stars is fraught with difficulty. The usual assumptions of radiative and hydrostatic equilibrium, local thermodynamic equilibrium (LTE) and plane-parallel geometry, which have enabled very satisfactory model atmospheres to be computed for hot main sequence stars (e.g. Mihalas, 1964), may all be invalid for Wolf-Rayet stars. It is not unexpected, therefore, that true model atmospheres for Wolf-Rayet stars do not now exist. There are, however, studies which relax one or another of the classical assumptions and whose results may be relevant to the Wolf-Rayet phenomenon. In this category there are but two papers, separated in time by nearly forty years. The major achievement of both is that plane-parallel geometry is not assumed, rather the atmosphere is taken to be spherically symmetric. Both investigations assume radiative equilibrium and LTE. First in time is the paper by N. A. Kosirev (1934) on the radiative equilibrium of extended photospheres. Since Kosirev meant his theory to apply to Wolf-Rayet and P Cygni stars, he assumed steady outflow of matter with constant velocity. He also assumed a gray opacity law of the Kramer's type; $\kappa \sim \varrho T^{-4}$. With these assumptions, an opacity variation $\kappa\varrho \sim r^{-1.5}$ resulted. Independently, and exactly at the same time, Chandrasekhar (1934) published his investigations of spherical atmospheres governed by the opacity law $\kappa\varrho \sim r^{-n}$, although without application to specific stars.

Since the assumptions of radiative equilibrium, LTE and gray opacity seem hardly applicable to Wolf-Rayet atmospheres, there would appear to be no virtue in following this approach further. Nevertheless, the Kosirev and Chandrasekhar method does lead to some interesting, if expected, results in a rather straightforward manner, and sets the stage for the modern computer models. Moreover, the techniques they introduced are finding use in recent investigations of radiative transfer in extended atmospheres. We will therefore give some space to the solution of the transfer problems in a gray spherical atmosphere.

To start, an opacity law of the form

$$\kappa\varrho = c_n/r^n \tag{1}$$

is assumed, so that the optical depth is

$$\tau = \int_r^\infty \kappa\varrho \mathrm{d}r = \left(\frac{1}{n-1}\right)\frac{c_n}{r^{n-1}}. \tag{2}$$

Rather than use the Eddington approximation to solve for the mean intensity of radiation, an expression derived by Larson (1969),

$$J = \frac{3H_0}{r^2}\left(\frac{n-1}{n+1}\right)\left[\tau + \frac{1}{3}\left(\frac{n+1}{n-1}\right)\right], \tag{3}$$

is employed. In a spherical atmosphere $H(r)/r^2$ is a constant (denoted H_0), where $H(r)$ is the first moment of the intensity, i.e

$$H(r) = \tfrac{1}{2}\int_{-1}^{1} I(r, \mu)\,\mu \mathrm{d}\mu$$

and μ is the cosine of the angle between the radius vector and the direction of propagation. Hummer and Rybicki (1971) have given accurate numerical solutions for atmospheres satisfying Equation (1), and find Equation (3) to be a rather good approximation. From the assumption of radiative equilibrium it follows that

$$J = B(T) = \sigma T^4/\pi. \tag{4}$$

If T_1 is the temperature at $\tau = 1$, it is easily shown from Equations (2), (3), and (4) that

$$T(\tau) = T_1 \tau^{1/2(n-1)}\left[\frac{\tau + \frac{1}{3}\left(\frac{n-1}{n+1}\right)}{1 + \frac{1}{3}\left(\frac{n-1}{n+1}\right)}\right]^{1/4} \tag{5}$$

The intensity of radiation is found most simply using a Cartesian coordinate system we will call the (p, z) representation. Figure 1 shows a ray which passes a distance p from the center of the sphere. The emergent intensity is $I_\nu(p, \infty)$ and the energy emitted at frequency ν in all directions per unit time is

$$F_\nu = 4\pi \int_0^\infty I_\nu(p, \infty)\, 2\pi p \mathrm{d}p. \tag{6}$$

It is convenient to introduce as a variable the angle ϕ between the ray at any point

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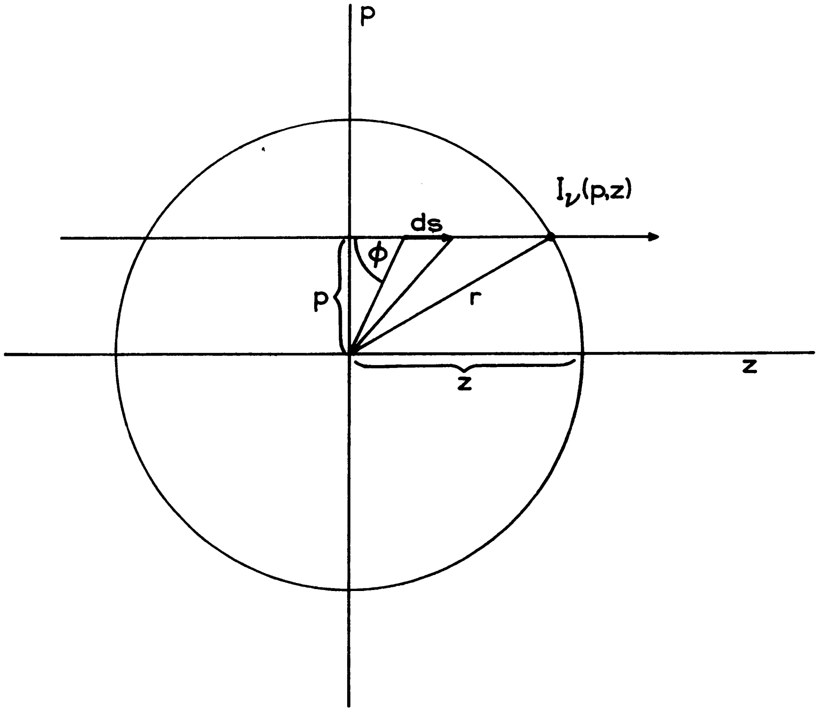

Fig. 1. The transfer equation is solved for radiation propagating along the ray $p = \text{const}$ in the (p, z) representation.

along the line of flight and the radius vector to that point. Then Kosirev's analysis results in the following expression for F_ν:

$$F_\nu = 4\pi^2 R_1^2 \int_0^\infty B_\nu(T)\, \tau^{-2/(n-1)} \Phi_n(\tau)\, \mathrm{d}\tau \tag{7}$$

where R_1 is the radius at which $\tau = 1$ and the function $\Phi_n(\tau)$ is defined by

$$\Phi_n(\tau) = 2 \int_0^\pi \exp\{-(n-1)\csc^{n-1}\phi\,\psi_n(\phi)\,\tau\}\sin\phi\, \mathrm{d}\phi \tag{8}$$

and

$$\psi_n(\phi) = \int_0^\phi \sin^{n-2}\phi'\, \mathrm{d}\phi' \tag{9}$$

Equations (5), (7), (8) and (9) give the solution of the spherical gray atmosphere in radiative equilibrium and in LTE. We apply these results to a group of stars which are distinguished by different values of n in the opacity. All the atmospheres extend to infinity; however if n is high the optical thickness of most of the atmosphere is negligible. Thus different values of n determine the extension of the atmosphere, with lower values of n corresponding to more extended atmospheres. In the limit $n \to \infty$, the plane-parallel gray atmosphere should be obtained.

By demanding LTE in an extended atmosphere, a large fraction of the atmosphere is forced to a temperature lower than the boundary temperature in a plane parallel-model. Compared to a plane parallel atmosphere, fluxes from extended atmospheres with the same values of T_1, should exhibit increased emission in the infrared and decreased emission in the violet. This does not imply that observations of an extended atmosphere would show an infrared excess and ultraviolet deficiency. In usual practise, the observed flux is compared to that of a black body (or model atmosphere if available) which best matches its behavior in the visible, at say 5000 Å. It is relative to this that excesses or deficiencies are said to exist.

We therefore compute the gradient

$$\phi_c = 3\lambda - \frac{\mathrm{d}}{\mathrm{d}\,(1/\lambda)}\,(\ln F_\nu)$$

from Equation (7) at $\lambda = 0.5\ \mu$, and compute the temperature T_c which a black body would need in order to give the same gradient. This is found from the expression

$$\phi_c = (c_2/T_c)/[1 - \exp(-c_2/\lambda T_c)]$$

where $c_2 = 1.43879$ cm deg (Allen, 1960). Table I gives ϕ_c and T_c for spherical models in which $T_1 = 50{,}000$ K and the extension parameter n is varied.

TABLE I

Color temperatures and gradients for spherical model atmospheres with different extensions

n	ϕ_c	$T_c(10^4\ \mathrm{K})$
2	1.32	1.20
3	0.90	2.18
4	0.77	3.05
5	0.73	3.54
7	0.70	3.99
10	0.69	4.26
∞	0.66	5.00

It should be noted that the effect of increasing the extent of the atmosphere (decreasing n) is to make the continuous spectrum appear cooler in the visible. It is possible, therefore, that a hot extended model and a more compact cool model would look alike. For example, $\phi_c = 0.90$ for $T_1 = 30000$ K and $n = 5$ matches the gradient for $T_1 = 50000$ K and $n = 3$. A comparison of the fluxes from these models shows that they are nearly identical over all frequencies of interest.

Figure 2 shows the behavior of F_ν/F_{ν_1}, where ν_1 corresponds to a wavelength of 5000 Å, for a star characterized by $T_1 = 5 \times 10^4$ K and $n = 2$ (great extension). This is compared to $B_\nu(T(\frac{2}{3}))/B_{\nu_1}(T(\frac{2}{3}))$, which approximates the relative flux from a gray

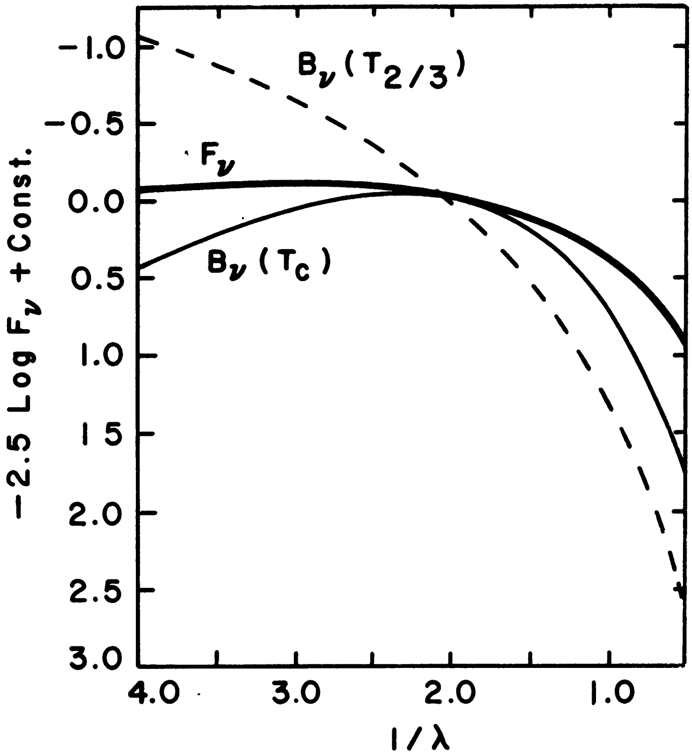

Fig. 2. The emergent flux from a gray extended atmosphere with $T_1 = 5 \times 10^4$ K and $n = 2$ compared with black body curves at $T(\frac{2}{3})$ (dashed line) and T_c. The black body curve at T_c matches the slope of the computed flux distribution at 5000 Å. In the infrared and ultraviolet the computed flux exceeds the black body value.

plane parallel atmosphere. The infrared excess and ultraviolet deficiency are striking. When compared to a black body distribution $B_\nu(T_c)/B_{\nu_1}(T_c)$, where $T_c = 1.2 \times 10^4$ K, there is now an ultraviolet, as well as an infrared, excess, reminiscent of the WN stars.

Let us now consider the very recent work of Cassinelli (1971a; 1971b). Cassinelli considered the Kosirev problem but without the gray opacity assumption. However, since he was concerned with extended, but stable, atmospheres, he assumed hydrostatic equilibrium. Thus, for Wolf-Rayet atmospheres, this represents one step forward and one step back. The solution of the non-gray problem is not amenable to analytic methods and involves numerical techniques for a computer. We therefore present only the results of this investigation. Cassinelli finds that (1) models which have the same temperature, $T(\frac{2}{3})$, but different geometrical extensions, can produce very different flux distributions and (2) a hot star with a very extended atmosphere has an optical continuum similar to that of a star with a cooler less extended atmosphere. Cassinelli measures the geometrical extension by the ratio $R(\tau = 0.001)/R(\tau = \frac{2}{3})$, where τ is a mean optical depth scale. His results are shown in Figure 3 and illustrate the above conclusions.

Since Cassinelli has assumed hydrostatic equilibrium, his atmosphere models are

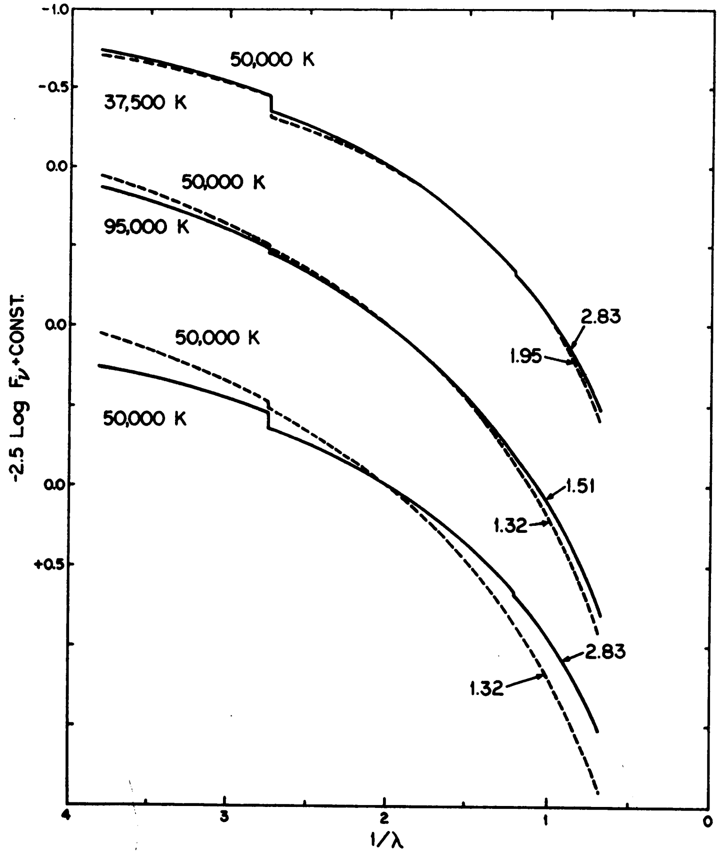

Fig. 3. Emergent fluxes in non-gray extended atmospheres after Cassinelli (*Astrophys. Letters* **8**, 108). The parameter labeling each curve is a measure of the extension, $R(0.001)/R(\frac{2}{3})$. The resemblance of hot extended to cooler, more compact models is illustrated.

compact compared to the gray calculations illustrated. Nevertheless, both the gray and non-gray calculations lead to the same qualitative results. As can be seen from Figure 4, the agreement is also quantitative, for apart from an emission jump at the Balmer limit, the gray and non-gray results are nearly indistinguishable. The non-gray calculation illustrated here has $T(\frac{2}{3})=48865$ K, $T_1=54450$ K and $R(0.001)/R(\frac{2}{3})=1.89$ Both gray models have the same value of T_1 as the non-gray model.

Cassinelli compared the results of this non-gray model with the observed continuum flux (Kuhi, 1968) from the WN 6 star, HD 191765 and found excellent agreement. In view of the similarity of the gray and non-gray models, the gray atmosphere also fits the observed points. However, Smith and Kuhi (1971) have made substantial corrections to the original observational data, and consequently the theoretical curves are not nearly as convincing. Because of the rapidity of the gray atmosphere calculations, it is possible to generate a large number of models. In Figure 5, the uncorrected observations (open circles) are seen to be well described by the curve corresponding to

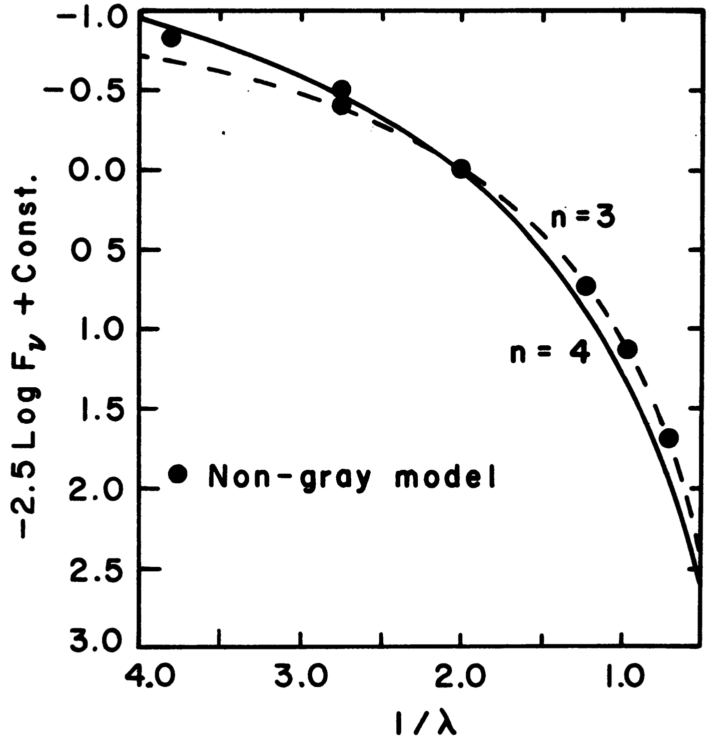

Fig. 4. Comparison of two gray models at $T_1 = 54450$ K with the non-gray model at the same value of T_1 and $R(0.001)/R(\frac{2}{3}) = 1.89$ (dots). The agreement is quite remarkable.

$T_1 = 5 \times 10^4$ K and $n = 5$. The corrected data (filled circles) follow the curve for $n = 2.5$ and for the same value of T_1. It is likely, therfore, that a non-gray model, of greater extension than the one used initially, can be constructed which will adequately describe the observations.

Although the relative flux distributions of WR stars can be simulated by the model atmospheres discussed, there is real doubt as to whether the physical parameters of the models are representative of those actually found in WR stars. What is greatly suspect here is the assumption of LTE, since in an extended atmosphere matter density becomes low and photon mean free paths large. Both effects inhibit an approach to LTE. Thus, a non-LTE model for the continuous energy distribution in stars with spherically symmetric atmospheres is certainly required before a satisfactory explanation of WR spectra will be obtained. Nevertheless, the calculations presented here show sufficient agreement with the observations to be at least suggestive of the processes operative in a real atmosphere. The infrared continuum *may* be formed deep enough in the envelope that the geometrical extension of the atmosphere is not great, and radiative equilibrium and LTE may not be as egregious approximations as might at first be surmised. The line forming region is believed to lie outside that in which the continuum is produced. Since the temperature in that region must be about 5×10^4 K

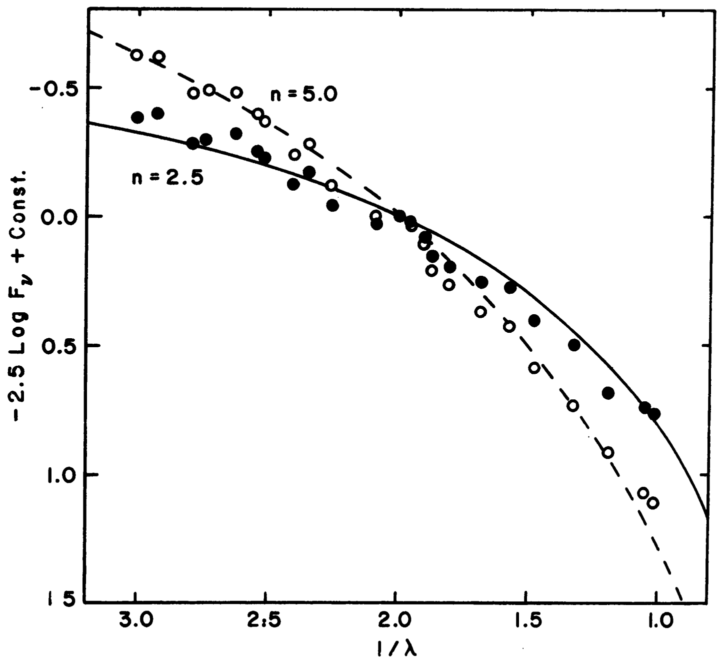

Fig. 5. Emergent flux from two gray models with $T_1 = 5 \times 10^4$ K and $n = 5$ and 2.5. These are compared to observations of HD 191765 both before (open circles) and after (closed circles) additional reddening corrections made by Smith and Kuhi.

(in a WN 6 star) to produce the degree of excitation observed, the possibliity exists that a temperature inversion occurs within the envelope. The continuum would be formed mainly in the inner region of declining temperatures, the lines further out, with perhaps additional ultraviolet continuum also arising from these outer regions. This is, of course, entirely speculative, but with so little in the way of theoretical work to fall back on, one can do little more than speculate.

3. Line Formation in an Expanding Atmosphere

3.1. Introduction

Although by no means extensive, the literature on line formation in moving atmospheres is far more substantial than that on the continuous energy distribution. The early papers by Gerasimovic (1934), Chandrasekhar (1934), Wilson (1934) and Bappu and Menzel (1954) and Chapter XX in Rosseland's textbook (1936) incorporate many of the techniques used at present. However, the assumption implicit in these investigations, of complete transparency of the atmosphere, makes them unsuitable for Wolf-Rayet models. The theory developed by Sobolev in his monograph *The Moving Envelopes of Stars* (1960) and in Chapter 28 of Ambartsumian's text *Theoretical Astrophysics* (1958) has been employed in several studies of Wolf-Rayet and other stars believed to be losing mass. One should especially note the papers by Rublev

(1961; 1963) on Wolf-Rayet atmospheres and another in the same spirit by Van Lyong (1967). Sobolev's method has recently been extended by Castor (1970) and subsequently applied to the excitation of helium lines in Wolf-Rayet stars by Castor and Van Blerkom (1970). It seems worthwhile therefore to review this theory in a somewhat leisurely manner.

The star is assumed to eject mass from a well defined surface at a radius r_c. That part of the star with $r \leqslant r_c$ we will call the core. Since the actual physical process which causes the mass ejection is not known with certainty, the velocity distribution in the atmosphere must be regarded as somewhat *ad hoc*. The comments of Chandrasekhar (1934) are still relevant today:

If one postulates that the parent star is continually ejecting atoms then from a dynamical point of view there are not many possibilities of the ways in which this could happen. The ejection process could, in fact, take place in one of two ways:

(A) At the boundary of the star the atoms (presumably only those with a relatively small but finite outward velocities) are 'repelled' by some kind of force which is, say, f times the gravitational attraction. Unless f is very nearly unity we could reasonably assume that f is a constant, i.e. the repulsive force – whatever its nature – falls off like gravity inversely as the square of the distance.

... this hypothesis includes, as a special case, the emission of particles arising from unbalanced radiation pressure, ...

(B) The atom at the boundary of a star might receive a large initial outward velocity (either in a single process or in stages), and in escaping from the star be continually de-accelerated in the gravitational field of the star, the atom either escaping from the star with a finite outward velocity, or after ascending a certain distance begin to fall back towards the parent star. We could have an atmosphere of high-speed particles set up in this way.

The dependence of line width on excitation potential originally discovered by Beals (1929) and recently reinforced by Smith and Aller (1971) indicates that process A occurs in Wolf-Rayet atmospheres. Then, if g is the acceleration of gravity at r_c

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = (f-1)\, g r_c^2/r^2 .$$

It follows easily that

$$v(r) = [2(f-1)\, r_c g\, (1 - r_c/r)]^{1/2}$$

if $v(r_c) = 0$. Since the velocity at infinity is

$$v_\infty = [2(f-1)\, r_c g]^{1/2}$$

the velocity distribution takes the form

$$v(r) = v_\infty (1 - r_c/r)^{1/2} . \tag{10}$$

This law was used e.g. by Castor (1970) in his study of line formation and, in a slightly modified form, by Lucy (1971). We will therefore use Equation (10) for $v(r)$ in all subsequent calculations.

Consider a line emitted by an expanding atmosphere which has a central frequency, measured in the laboratory, of ν_0. Since different parts of the atmosphere approach and recede from a stationary observer, the radiation received will be spread in frequency due to the Doppler effect. If $\nu(v)$ is the photon frequency as seen by an observer moving with the material at a velocity v and $\nu(0)$ is the frequency seen by a stationary observer,

$$\nu(0) = \nu(v) + \frac{\nu_0}{c}\mathbf{n}\cdot\mathbf{v} \tag{11}$$

where $\mathbf{n}$ is the direction of propagation of the photon. The maximum frequency displacements occur for radiation emitted by material with the limiting velocity v_∞. Because $\nu(v) \approx \nu_0$, i.e. emission occurs locally close to line center, the spectral line has a total half width of

$$\Delta_s = \nu_{max}(0) - \nu_0 = \frac{\nu_0}{c} v_\infty . \tag{12}$$

It is convenient to measure frequency displacement in units of Δ_s; so a dimensionless frequency parameter is defined by

$$x = [\nu(0) - \nu_0]/\Delta_s . \tag{13}$$

The observed line extends from $x = -1$ (red) to $x = +1$ (violet).
If thermal broadening is assumed, then from Equation (11), the normalized absorption coefficient of an element of gas moving with velocity v, for a photon having frequency $\nu(0)$ in the stationary observer's frame is

$$\phi[\nu(0), v] = \frac{1}{\sqrt{\pi}\Delta} \exp\left\{-\left[\nu(0) - \frac{\nu_0}{c}\mathbf{n}\cdot\mathbf{v} - \nu_0\right]^2 / \Delta^2\right\} \tag{14}$$

where Δ is the local value of the Doppler width,

$$\Delta = \frac{\nu_0}{c} v_{th},\ v_{th} = (2kT/m_A)^{1/2} . \tag{15}$$

Let

$$\begin{aligned} \delta &= \Delta/\Delta_S \\ u &= v/v_\infty \\ \mu &= \mathbf{v}\cdot\mathbf{n}/|v| . \end{aligned} \tag{16}$$

Then the normalized absorption coefficient can be written in terms of the dimensionless variable x as

$$\phi(x - u\mu) = \frac{1}{\sqrt{\pi}\delta} \exp[-(x - u\mu)^2/\delta^2] . \tag{17}$$

The notation here is that of Hummer and Rybicki (1968).

An important difference exists between lines formed in atmospheres which are stationary and those which are rapidly expanding. In the former, a strong line is formed very near the geometrical boundary of the star. In an expanding atmosphere, however, contributions can come from matter distributed across the entire envelope. One can see this in the following simple example. Consider a ray passing a distance

p from the center of the star, as shown in Figure 1, where the radiation is seen by a stationary observer to be displaced to the violet of line center by an amount x. Let T be constant, so Δ does not vary through the envelope. The optical depth along the ray p=constant is

$$\tau(x, p, z) = \int_z^{\infty} k(r)\,\phi(x - u\mu)\,\mathrm{d}z \tag{18}$$

where $\mu = z/r$ and $r = (p^2 + z^2)^{1/2}$. The absorption coefficient $k(r)$ contains only atomic constants and the population density of absorbing atoms $N_A(r)$. The continuity equation states that the total number density of atoms $N_{\text{tot}}(r) \sim r^{-2}v^{-1}$. For simplicity, we assume that $N_A(r)/N_{\text{tot}}(r)$ is constant throughout, so $k(r) \sim r^{-2}v^{-1}$. Finally

$$\tau(x, p, z) \sim \int_z^{\infty} \frac{\phi(x - u\mu)}{vr^2}\,\mathrm{d}z \tag{19}$$

where the constant of proportionality is determined by the normalization $\tau(x, p, -\infty) = 1$.

Equation (19) is integrated numerically for a model in which $v_\infty = 2000$ km s^{-1}, $T = 5 \times 10^4$ K, $x = 0.5$ and $p = 2r_c$. The composition is taken to be pure helium, so that with these parameters, $v_{\text{th}}/v_\infty = 7.2 \times 10^{-3}$. The results are shown in Figure 6. The optical depth makes a rather abrupt jump from a value near zero to its final value $\tau(x, p, -\infty) = 1$, the transition occurring within a geometrical distance of only 0.1 r_c. Outside of the transition zone, the atmosphere is transparent to radiation of the given frequency. The sharpness of transition is due to the narrowness of the line profile, which falls off on a scale of atomic Doppler widths Δ. As seen from Equation (17), the profile function peaks when

$$x = u_z$$

where $u_z = u\mu$ is the z component of the velocity distribution. Expressing μ in terms of p and z, this becomes

$$x - u\left[(p^2 + z^2)^{1/2}\right] z/(p^2 + z^2)^{1/2} = 0\,. \tag{20}$$

For a given value of p, that value of z which satisfies Equation (20) is denoted z_0. In the specific example considered, $z_0 = 1.655\ r_c$, and as shown in Figure 6, z_0 lies in the middle of the transition zone. When Equation (20) is solved, for different values of p but a given frequency x, the roots $z_0(p)$ define a surface in the expanding envelope, such that the atmosphere is transparent except within a small region about this surface. The surface may be termed a surface of constant (line of sight) velocity, since u_z is constant there. In the remainder of this paper, the subscript zero refers to a variable evaluated on a surface of constant velocity: thus, $r_0 = ({z_0}^2 + p^2)^{1/2}$. Different surfaces correspond to different frequency displacements x, and a number of such surfaces is shown in cross section in Figure 7 for the velocity distribution (10). These surfaces, distributed over all r, contribute to the observed spectral line.

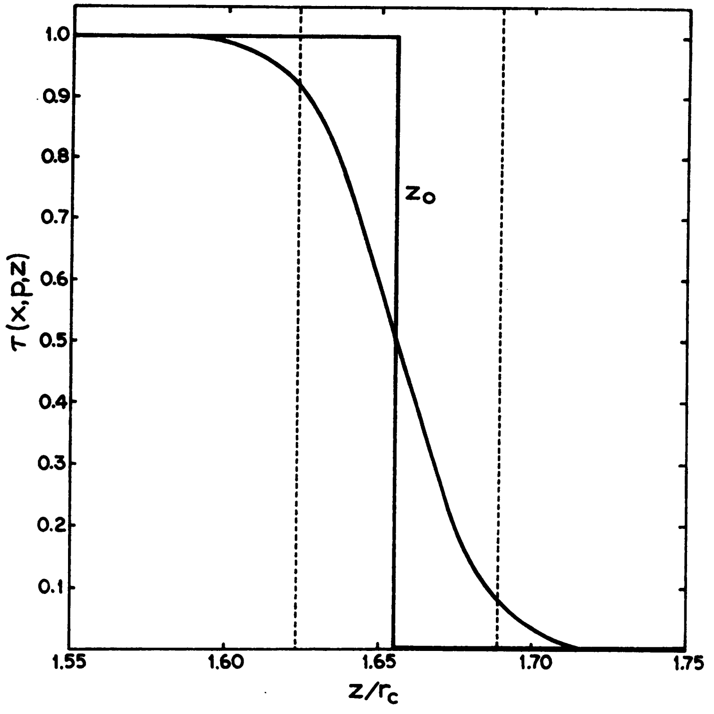

Fig. 6. Optical depth along a ray $p = \text{const}$ at a fixed frequency x and normalized such that the total optical thickness across the ray is unity. The step function approximation, with a jump at z_0 on the constant velocity surface is also shown. The dashed lines correspond to the locations of the constant velocity surfaces at $x \pm \delta$.

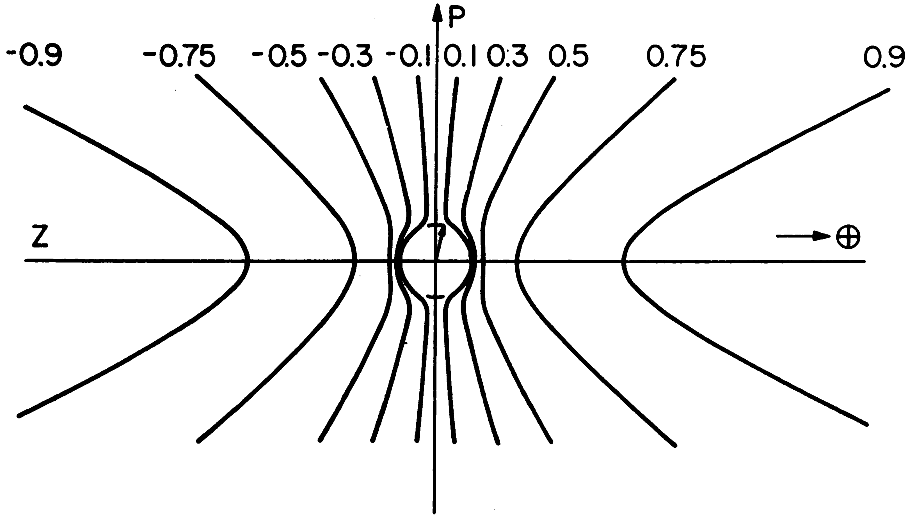

Fig. 7. Cross section of the constant velocity surfaces in a star with a velocity distribution as given by Equation (10). From Castor (*Monthly Notices Roy. Astron. Soc.* **149**, 112).

3.2. Escape probability method

In a rapidly expanding atmosphere only a geometrically narrow zone is non-transparent to radiation of a given frequency, so that a transfer problem exists only in this region. The abruptness of the jump in opacity along a ray at a constant velocity surface may be exploited to yield an approximate solution of the transfer equation. In the limit of vanishing profile width, $\delta \to 0$, the optical depth approaches a step function

$$\tau(x, p, z) = \tau(x, p)\, y(z) \tag{21}$$

where $\tau(x, p)$ is an abbreviation for $\tau(x, p, -\infty)$, the total optical depth along the ray $p=$constant, and

$$y(z) = \begin{cases} 1 & z < z_0 \\ 0 & z > z_0 \end{cases} \tag{22}$$

We now assume this limiting case applies for profiles of finite width, and evaluate $\tau(x, p)$. Because the approximation involves infinitesimally narrow lines the actual profile does not enter. Sobolev (1958) assumes $\phi(x)$ to be rectangular, while Castor (1970) does not specify a line shape; both find easily that

$$\begin{aligned} \tau(x, p) &= k(r_0)\left[\left(\frac{\partial}{\partial z}\right)_{x,p} u_z\right]^{-1} z_0 \\ &= k(r_0)\left[1 + \frac{z_0^2}{r_0^2}\left(\frac{\mathrm{d}\ln u}{\mathrm{d}\ln r} - 1\right)\right]^{-1} r_0/u(r_0). \end{aligned} \tag{23}$$

The absorption coefficient for a line transition between levels $n=1$ and $n=2$ is

$$k = \frac{\pi e^2}{mc}(gf)_{12}\left(\frac{N_1}{g_1} - \frac{N_2}{g_2}\right)\left(\frac{c}{\nu_0 v_\infty}\right) \tag{24}$$

where f_{12} is the oscillator strength, g_1 and g_2 are the statistical weights and N_1 and N_2 are the population densities of levels $n=1$ and $n=2$ respectively. The last term in parentheses converts k to the dimensionless frequency scale x. It should be remarked that $\tau(x, p)$ can be very large in spite of the geometrical narrowness of the line forming region.

The equation of transfer along a ray $p=$constant takes the familiar form

$$-\frac{\partial I(x, p, z)}{\partial z} = k(r)\,\phi(x - \mu u)\,[I(x, p, z) - S(r)] \tag{25}$$

where the source function is assumed to be independent of frequency and isotropic in all frames. The calculation of the intensity must take into account the two possible lines of sight in the atmosphere: (i) rays with $p < r_c$ which encounter the core and (ii) rays with $p > r_c$ in which case the core is bypassed. We shall assume that the core radiates only a continuous spectrum I_c, which for the sake of simplicity, is assumed to be independent of ν and p, i.e. there is no limb darkening.

Expressions for $I(x, p, z)$ are given by Castor (1970) and as would be expected reflect the step function behavior of the opacity. In particular, the emergent intensity $I(x, p, \infty)$ is

$$I(x, p, \infty) = \begin{cases} S(r_0)(1 - \exp[-\tau(x, p)]) & (p > r_c) \\ S(r_0)(1 - \exp[-\tau(x, p)\, y(z_c)]) + \\ \quad + I_c \exp[-\tau(x, p)\, y(z_c)] & (p < r_c) \end{cases} \tag{26}$$

where

$$z_c = (r_c^2 - p^2)^{1/2}. \tag{27}$$

The terms involving the step function $y(z_c)$ are simply interpreted. Radiation of frequency $x > 0$ (shortward of line center) originates in the hemisphere nearest the observer, so that $z_0 > z_c$ and $y(z_c) = 1$. Radiation longward of line center, $x < 0$, originates in the far hemisphere where $z_0 < z_c$ and $y(z_c) = 0$. Thus, for $p < r_c$

$$\begin{aligned} I(x > 0, p, \infty) &= S(r_0)(1 - \exp[-\tau(x, p)]) + I_c \exp[-\tau(x, p)] \\ I(x < 0, p, \infty) &= I_c. \end{aligned} \tag{28}$$

The intensity shortward of line center is composed of a part emitted by the envelope proportional to $S(r)$, and a part emitted by the core, proportional to I_c. The continuous radiation is absorbed in the region of the atmosphere directly between the observer and the core. This accounts for the absorption component which often appears in the violet wing of Wolf-Rayet emission lines. In the expression for the intensity of radiation longward of line center, the envelope emission term does not appear. This is due to the occultation by the core of that region of the far hemisphere which lies directly behind it. Equations (26) therefore describe the basic physical processes which effect the emission line profiles in expanding atmospheres: envelope emission, absorption by material in front of the core, and occultation of material behind it. The power emitted by the star, per unit frequency interval, is then

$$F_x = 4\pi \int_0^\infty 2\pi p \, \mathrm{d}p I(x, p, \infty). \tag{29}$$

Let us consider a particular atomic transition which takes place between levels 1 (lower) and 2 (upper). The source function is (e.g. Avrett and Hummer, 1964)

$$S = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} \tag{30}$$

where N_1 and N_2 are population densities and the other terms are the Einstein rate coefficients. The source function is assumed to be isotropic and independent of frequency. Magnon (1968) has indicated that these assumptions which appear to give accurate results in stationary atmospheres, may be less satisfactory in moving atmospheres. However, there is little evidence to corroborate this assertion, and we will continue to use Equation (30) for the line source function.

The rate of downward transitions $2 \rightarrow 1$ is given by

$$R_{21} = N_2 A_{21}$$

while the rate of upward transitions (diminished by stimulated emission) is

$$R_{12} = (N_1 B_{12} - N_2 B_{21}) \bar{J}.$$

The term $\bar{J}$ represents the intensity of radiation integrated over the line profile and averaged over angle:

$$\bar{J} = \tfrac{1}{2} \int_{-1}^{1} \mathrm{d}\mu \int_{-\infty}^{\infty} \phi(x - u\mu) \, I(x, p, z) \, \mathrm{d}x . \tag{31}$$

Because $\phi(x-u\mu)$ is zero except for a narrow range of frequencies about $x=u\mu$, a photon of frequency x can be absorbed only when the equality is satisfied. This, however, defines the constant velocity surface for frequency x, from which all the envelope emission at that frequency arises. Therefore, (apart from the core radiation) a photon which is absorbed at a point (p, z) must have been emitted at that point. Expressed differently, a photon emitted in the envelope is either reabsorbed at the same spot or escapes from the region entirely. Let β be the escape probability, i.e. the fraction of photons emitted in the transition $2 \rightarrow 1$ which escape. A fraction $(1-\beta)$ are therefore reabsorbed, so

$$R_{12} = (1 - \beta) \, R_{21}$$

or

$$(N_1 B_{12} - N_2 B_{21}) \, \bar{J} = (1 - \beta) \, N_2 A_{21}$$

With Equation (30) for the source function, this becomes

$$\bar{J} = (1 - \beta) \, S . \tag{32}$$

The mean intensity $\bar{J}$ may be formally evaluated by substituting the intensities found previously with the narrow line approximation into Equation (31). This has been done by both Sobolev (1958) and Castor (1970) with the result that

$$\beta = \int_{0}^{1} \mathrm{d}\mu \, [1 - \exp(-\tau(x, p))]/\tau(x, p) . \tag{33}$$

We have ignored, however, the upward transitions caused by absorption of the continuous core radiation. When this is included, Equation (32) is replaced by

$$\bar{J} = (1 - \beta) \, S + \beta_c I_c \tag{34}$$

where, as Castor (1970) shows, if $\mu_c = (1 - r_c^2/r^2)^{1/2}$,

$$\beta_c = \tfrac{1}{2} \int_{\mu_c}^{1} \mathrm{d}\mu \, [1 - \exp(-\tau(x, p))]/\tau(x, p) . \tag{35}$$

To a fair approximation,

$$\beta_c = W\beta \tag{36}$$

where

$$W = \tfrac{1}{2}\left(1 - \left(1 - r_c^2/r^2\right)\right)^{1/2} \tag{37}$$

is the usual dilution factor.

3.3. The two level atom

In order to calculate the emergent intensity of radiation in a spectral line from Equation (26), it is necessary to know the run of the line source function. Because of the difficulty in determining $S(r)$, workers attempting to describe real atmospheres relied on dubious assumptions. Rublev (1963) for example took $S(r)=\text{const}$, while Van Lyong (1967) assumed $S(r)=S_0(r_c/r)^t$. There is one case, however, in which the source function can be obtained in a more reliable manner; that is for an atmosphere composed of atoms with only two bound levels. It is worth studying the two level atom for this reason, although as is often the case, application to Wolf-Rayet stars is not immediate.

The source function for the two level atom, in the case of complete frequency redistribution, is given by Equation (30). This is combined with the equation of statistical equilibrium,

$$\left(B_{12}\bar{J} + C_{12}\right) N_1 = \left(A_{21} + B_{21}\bar{J} + C_{21}\right) N_2 \tag{38}$$

to yield

$$S = (1 - \varepsilon)\,\bar{J} + \varepsilon B(T) \tag{39}$$

where

$$\varepsilon = C_{21}/\left[C_{21} + A_{21}\left(1 - \exp\left(-h\nu_0/kT\right)\right)^{-1}\right] \tag{40}$$

is the probability per scattering that a photon is lost from the line by a collisional de-excitation of the excited state, and $B(T)$ is the Planck function at line center frequency ν_0. Equations (34) and (39) yield

$$S = \frac{(1 - \varepsilon)\,\beta_c I_c + \varepsilon B(T)}{(1 - \varepsilon)\,\beta + \varepsilon} \tag{41}$$

which gives the source function in terms of known variables.

With a view towards applying this result to actual model atmospheres there is a disconcertingly large number of variables which must be supplied. Runs of temperature and population densities are required to fix ε, $B(T)$ and the absorption coefficient $k(r)$. The model which calls for fewest external parameters is one in which pure line scattering ($\varepsilon=0$) occurs. Nevertheless, this simple case is one of considerable current interest in the interpretation of emission lines from quasi-stellar objects.

Scargle, Caroff, and Noerdlinger (1970) suggest that the profile of the C IV resonance line λ 1548.2 in PHL 5200 ($z=1.98$), which is observed to have an abrubt and deep absorption trough shortward of line center, is formed by scattering in a very rapidly expanding envelope ($v_\infty = 10000$ km s^{-1}) surrounding a continuum emitting core. The model is thus precisely the same as ours for Wolf-Rayet stars. Let us therefore employ the escape probability method to this case, and use the physical model given by Lucy (1971).

The velocity distribution (10) is assumed along with the equation of continuity:

$$-\frac{\mathrm{d}M}{\mathrm{d}t} = 4\pi r^2 \varrho v \tag{42}$$

where $-\mathrm{d}M/\mathrm{d}t = 2M_\odot$ yr^{-1} is the rate of mass loss. Carbon is postulated to have the cosmic abundance and 10% of the atoms are C IV throughout the flow. The core radius $r_c = 5$ pc and $v_\infty = 10000$ km s^{-1}. In terms of atomic constants

$$k(r) = \frac{\pi e^2}{mc} fN$$

where $f = 0.2$ is the oscillator strength (Allen, 1955) and N is the number density of C IV ions. From Equations (23) and (24) the total optical depth along a ray p = constant is

$$\tau(v, p) = \tau_0(r_0) \Big/ \left[1 + \frac{z_0^2}{r_0^2}\left(\frac{\mathrm{d}\ln v}{\mathrm{d}\ln r} - 1\right)\right]_{r_0}. \tag{43}$$

where

$$\tau_0(r) = \frac{\pi e^2}{mc} fNcr/v_0 v(r). \tag{44}$$

The quantity $\tau_0(r)$ is a convenient measure of the distribution of scatterers.

With the above assumptions, it is found that $\tau_0(1.1r_c) = 8.0$ and decreases monotonically, so that, for example $\tau_0(5r_c) = 0.2$. Figure 8 shows the flux F_ν/F_c computed by the escape probability method. The computed profile does not resemble the observed profile in that (i) the central intensity is only 1.5 the continuum value while it is observed to be nearly 4 times the continuum intensity and (ii) the absorption is far too gradual compared to the observed absorption which is nearly complete just shortward of line center. In this, we agree exacly with Lucy (1971). Other choices of physical parameters do not change this result significantly, and apparently the CIV resonance line is not formed by scattering in a spherically symmetric, expanding envelope.*

Perhaps the most informative calculation that can be made for Wolf-Rayet stars is one that demonstrates the wide range of profiles possible from a moving atmosphere. As Rublev (1963) and Castor (1970) have shown, these encompass all the line shapes observed on Wolf-Rayet spectra. In Castor's computations, ε and $B(T)$ are assumed to be constant throughout the envelope and different opacity distributions are chosen.

* Recently, Caroff, Noerdlinger, and Scargle (1971, preprint) argue that the observed central intensity can be obtained with a very different model than that considered above. In their model, the parameter $Q_0 = \tau_0(r)/\mathrm{d}\ln v/\mathrm{d}\ln r)$ goes suddenly to zero as the core is approached, while in the above model Q_0 remains finite.

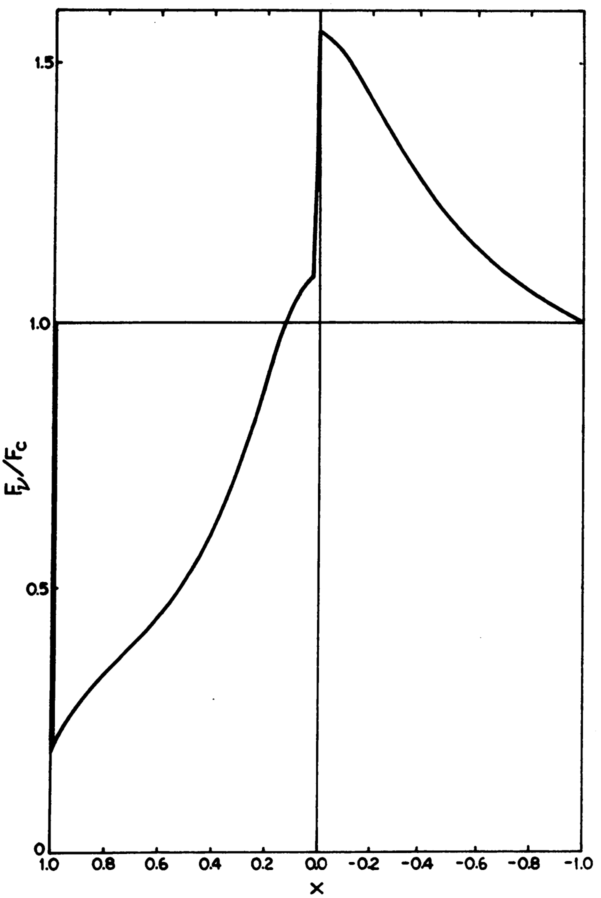

Fig. 8. Spectral line produced by a pure scattering expanding envelope with the parameters appropriate to the C IV resonance line λ1548.2 in PHL 5200. The theoretically generated profile does not resemble the observed profile and casts doubt on this interpretation of the line.

Figure 9 given one example of a resulting line profile. It demonstrates that the rounded appearance of most Wolf-Rayet emission lines does not perforce rule out their formation in an expanding atmosphere. In particular, we do not require a turbulent region with random velocities in excess of 1000 km s^{-1} as suggested by Underhill (1968).

Nevertheless, one possible flaw in the theory should be pointed out. The optical depth parameter $\tau_0(r)$ used in Figure 9 has been chosen to be small very near the core, when $v(r) \leqslant 0.2v_\infty$. If, however, $\tau_0(r)$ is a monotonically increasing function of r as $r \to r_c$, e.g. in the quasi-stellar model treated above, a profile quite asymmetric at line center results. This is evident in Figure 8 for the quasi-stellar line. No line in a Wolf-Rayet star has this appearance, and one must conclude either that (i) the two level atom and escape probability method fail to present a physically accurate picture of line formation near the core or (ii) the density of absorbing atoms or ions vanishes near the core. If the temperature increases monotonically as $r \to r_c$ the atoms in the highest

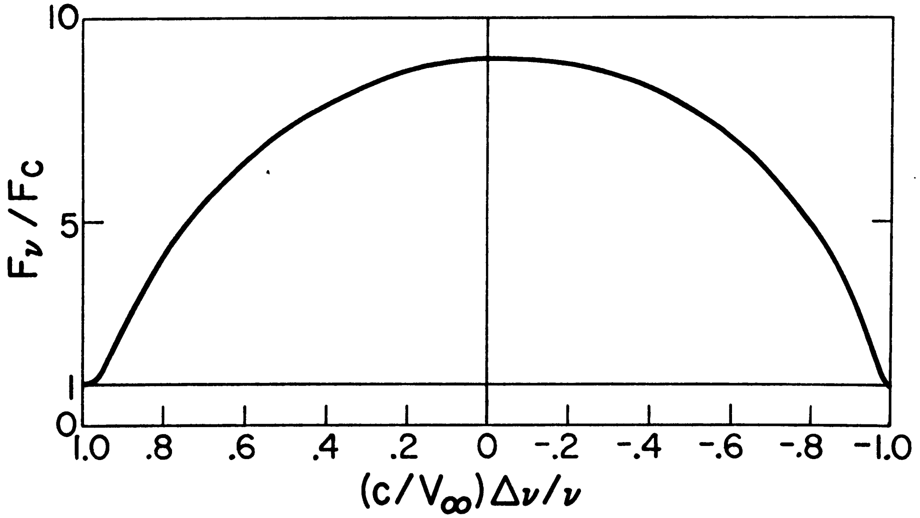

Fig. 9. A representative line shape of the rounded type. The parameters are $\varepsilon = 0.021$ and $B(T)/I_c = 0.105$. From Castor (*Monthly Notices Roy. Astron. Soc.* **149**, 124).

stages of ionization should exist in the vicinity of the core. The lack of strongly asymmetric lines from these ions suggests, if the theory is not itself at fault, that the temperature rise inward is not monotonic. One may recall the speculation at the end of Section I that a temperature inversion occurs between the continuum and line forming regions. It is then conceivable that the ionization equilibrium in a narrow zone about the core shifts to low stages of ionization. The asymmetries in the lines produced in this part of the envelope are masked by the emission from the more extensive sections of similar ionization further out.

4. Application to Wolf-Rayet Stars

4.1. Laser action in a C III line

The theory of line formation in a moving atmosphere developed in Part III has so far been merely descriptive. We have shown that the commonly observed Wolf-Rayet line shapes can be reproduced by an appropriate choice of parameters. A more useful diagnostic approach is the inverse, i.e., to deduce the physical parameters from the observed spectrum. Rublev (1963) and Van Lyong (1966) have attempted to do this, and derive a velocity distribution and a limb darkening law for the continuous radiation. The analysis is too detailed to be reproduced here, however the results should be regarded with some caution due to the unsatisfactory choice of source function mentioned previously. Nevertheless, their investigations show the type of information which may be obtained from a detailed comparison of line profiles with theoretical predictions.

A somewhat less ambitious project is to attempt to account for the total energy in a line rather than its shape. West (1968) has reported some preliminary calculations of this nature for the C III intercombination line λ1909. The line is observed to be rather strong in γ Vel, i.e. if

$$R = \int_{-1}^{1} (F_\nu/F_c)\,\mathrm{d}x$$

is the ratio of the total line to continuum flux, $R \approx 4$. Because the oscillator strength is very low, $gf = 3.1 \times 10^{-7}$ (Garstang and Shamey, 1968), there are intimations of laser action. West's results appear to require population inversions with amplification ($\tau_0 < 0$) if the star has a normal carbon abundance and if the total density at the base of the envelope is not to exceed 10^{12} cm^{-3}. However, for reasons not clear to this reviewer, West limited the line emission to only that part of the envelope directly between the observer and the core, thus removing the greater part of the emitting volume. Only shortward displaced radiation is produced in this region and the total flux in the line cannot therefore be found. West could compare only the line center intensity ratio F_{ν_0}/F_c with R, which is of questionable value.

Let us therefore consider the problem in the context of the theory developed here. The core radius is taken to be $r_c = 5R_\odot$, and the C III line emitting region in which carbon is all C^{+2}, is assumed to lie within $5r_c$. The density decreases outward from the core as r^{-2}, and at $r = 5r_c$, $v(r) = 1500$ km s^{-1}. The continuous radiation field is that appropriate to a 40000 K main sequence model, or $I_c = 4.78 \times 10^{-3}$ ergs cm^2/s/Hz. This is West's model, except the velocity distribution (10) is used in place of his linear law.

The population ratio N_2/N_1 of the upper to lower levels of the line is postulated, as is the total density $N_1 + N_2$ at the base of the envelope. The latter is chosen such that the corresponding hydrogen density at r_c would be in the neighbourhood of 10^{12} cm^{-3} if the cosmic abundances of the elements were present. With N_1 and N_2 specified, the source function follows from Equation (30) and the optical depth, including stimulated emission is

$$\tau_0(r) = \frac{\pi e^2}{mc} (gf)_{12} \left(\frac{N_1}{g_1} - \frac{N_2}{g_2}\right) \frac{r\lambda_0}{v(r)}. \tag{45}$$

Note that S is independent of r since the ratio N_2/N_1 is assumed constant within the emitting region. Equations (26) yield the emergent intensity $I(\nu, p, \infty)/I_c$ and quadratures over p and ν give the desired ratio R.

Figure 10 shows R as a function of N_2/N_1 for several values of $N_1 + N_2$. The existence of laser action is neither confirmed nor denied by these results. Apparently, all that can be stated is a lower limit on the density, since even with amplification $R = 4$ is not attainable for $N_1 + N_2 = 1 \times 10^8$ cm^{-3}.

The problem is certainly that too much information is being wrested from one line. The C III spectrum of γ Vel is very fully developed, however, and a more reliable analysis would incorporate many lines. This has only just now been done by Castor and Nussbaumer (1971). We discuss this work in more detail later, and simply note

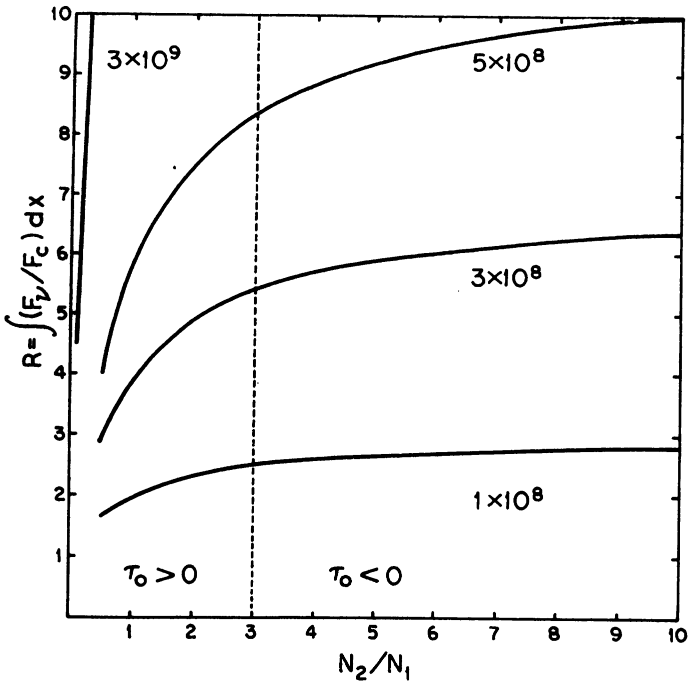

Fig. 10. Ratio of total line to continuous flux in the C III intercombination line λ1909. The curves are generated using the escape probability method and West's model of γ Vel. Laser action is suggested only in the low density models.

here that the lowest five levels of C^{+2} are found to be nearly in LTE, at an electron temperature $T_e = 2.2 \times 10^4$ K. Thus, the population ratio, $N_2/N_1 \approx 0.1$, is thirty times less than that required for laser action. If this ratio is used in West's model, a total density $N_1 + N_2 \approx 3 \times 10^9$ cm^{-3} is necessary in order that $R \approx 4$. The population density of C^{+2} ions is approximately the sum of the ground state densities of the singlets and triplets. Since the term statistical weight g_2^t of the state $2p^3P^0$ is 9, while the statistical weight of the upper level of λ1909 is 3, we have $N(C^{+2}) \approx N_1 + 3N_2$. The density of C^{+2} ions at the base of the envelope is therefore around 3.5×10^9 cm^{-3}. Castor and Nussbaumer find $N_e = 4 \times 10^{11}$ cm^{-3} at $r = 3.6 r_c$. This corresponds to a density $N_e = 5.2 \times 10^{12}$ cm^{-3} at the base of the envelope when an r^{-2} dependence is assumed. If the electron density is due entirely to hydrogen ionization, the ratio $N(C)/N(H)$ at $r = r_c$ is roughly 6×10^{-4}, or twice the cosmic abundance ratio.

4.2. He II LINES, A COARSE ANALYSIS

Because of the prominence of He II lines in the spectra of Wolf-Rayet stars, an analysis of these lines should be a strong test for any theory of line formation. In attempting to extract physical information from the line spectrum one encounters the usual 'inverse problem' in radiative transfer, viz. to determine the source function from the emergent flux. The following treatment is in the spirit of a coarse analysis.

We consider a given line transition for which the frequency independent parts of the emissivity and absorptivity are $j(r)$ and $k(r)$ respectively. Since the emission is assumed to be isotropic, and a fraction $\beta(r)$ of the energy released locally escapes we find

$$E = 4\pi \int j(r)\,\beta(r)\,\mathrm{d}V \tag{46}$$

for the power emitted by the whole atmosphere in the line. If the angle dependent terms in Equation (33) for β are ignored, then

$$\beta(r) = (1 - \exp[-\tau_0(r)])/\tau_0(r). \tag{47}$$

The optical depth is related to $k(r)$ through the equation

$$\tau_0(r) = \frac{cr}{v_0 v(r)}\,k(r) \tag{48}$$

and by definition, $j(r) = k(r)S(r)$.

Equation (46) thus becomes

$$E = (8\pi^2 v_0/c)\int_0^\infty S(r)\,v(r)\,(1 - \exp[-\tau_0(r)])\,\mathrm{d}r^2. \tag{49}$$

As a rough approximation it is assumed that the integrand is constant within a line emitting region of radius r_E, and zero outside of it. Then

$$E \approx (8\pi^2 v_0/c)\,r_E^2 v(r_E)\,S(r_E)\,(1 - \exp[-\tau_0(r_E)]). \tag{50}$$

It should be noted that Equation (50) ignores any occultation or absorption of continuum radiation. A quantity more likely to be tabulated by an observer is the equivalent width W_λ. This may be found by dividing Equation (50) by the continuous flux in wavelength units at the position of the line

$$F_c = 4\pi^2 r_c^2 I_c c/\lambda_0^2$$

where, as used previously, I_c is in frequency units and c/λ_0^2 effects the conversion. Then

$$W_\lambda = 2\,(r_E/r_c)^2\,(\lambda_0 v(r_E)/c)\,S(r_E)\,(1 - \exp[-\tau_0(r_E)])/I_c. \tag{51}$$

We may examine the type of errors associated with the use of Equation (50) or (51) by calculating E or W_λ exactly for a model atmosphere in which all processes are included. West's model for the C III emission in γ Vel, which was discussed in the preceding section, will serve this purpose. Accordingly we take $r_E = 5r_c$ and $v(r_E) = 1500$ km s^{-1}. The source function depends on the ratio N_2/N_1. This is assumed to have its LTE value at 22000 K, so $N_2/N_1 = 0.1$ and $S = 0.41$. All the remaining parameters are determined once $N_1 + N_2$ is specified. Table II gives $\tau_0(r_E)$, W_λ (exact), and the ratio of the approximate to exact equivalent widths.

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TABLE II

Comparison of approximate and exact equivalent widths for a simple W–R model

N_1+N_2	τ_0	W_λ(exact)	W_λ/W_λ(exact)
1×10^9	0.064	14.8	0.826
3×10^9	0.192	38.0	0.905
5×10^9	0.320	55.2	0.977
7×10^9	0.448	68.1	1.043
1×10^{10}	0.640	82.1	1.133
5×10^{10}	3.200	116.8	1.615

The agreement between the exact and approximate calculations is very good, and in fact better than one might have anticipated.

Following Castor and Van Blerkom (1970) this coarse analysis is applied to the He II lines in the spectrum of the WN 6 star HD 192163. Relative line intensities $E_{n,n'}/E_{4,3}$ are taken from the observations of Smith and Kuhi (1970). Of particular importance are the $\lambda4686\,(4\rightarrow3)$, $\lambda3203\,(5\rightarrow3)$ and $\lambda10124\,(5\rightarrow4)$ lines since they couple the levels $n=3$, 4 and 5. It is assumed, and later shown to be consistent, that these three lines are optically thick, so that $(1-e^{-\tau_0})\sim1$ and

$$E_{n,n'}/E_{4,3}=\nu_{n,n'}S_{n,n'}/\nu_{4,3}S_{4,3}\,. \tag{52}$$

The source function is given by Equation (30), which may be written as

$$S_{n,n'}=\frac{2h\nu_{n,n'}^3}{c^2}\left[\frac{N_{n'}/g_{n'}}{N_n/g_n}-1\right]^{-1}. \tag{53}$$

Equations (52) and (53) determine the ratios $N_3g_4/N_4g_3=4.65$ and $N_4g_5/N_5g_4=4.04$.

For levels $n>5$, the lines in the Pickering series are used. Then if $y_n=N_ng_4/N_4g_n$

$$E_{n,4}/E_{4,3}=\left(\frac{\nu_{n,4}}{\nu_{4,3}}\right)^4\frac{y_3-1}{1/y_n-1}\,[1-\exp(-\tau_{4,n})]\,. \tag{54}$$

The optical depths are not known a priori, and must be assumed. A scale factor A is defined such that

$$\tau_{4,n}=A\,\frac{N_4/g_4-N_n/g_n}{N_4/g_4}\,(gf\lambda)_{4,n}=A\,(1-y_n)\,(gf\lambda)_{4,n} \tag{55}$$

where

$$A=\frac{\pi e^2}{mc}\frac{N_4/g_4}{v(r_E)}\,r_E\,. \tag{56}$$

Equations (54) and (55) are solved for y_n for several values of A. The resulting population ratios are shown in Figure 11. One must now decide which of the curves in the figure are physically meaningful.

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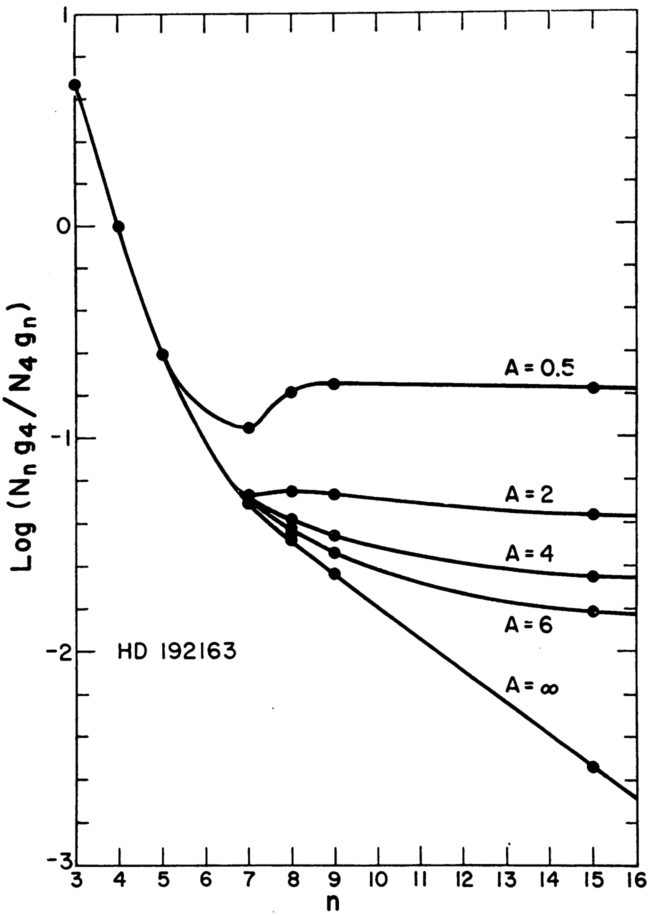

Fig. 11. Relative atomic energy level populations of He II as derived by a coarse analysis of the WN 6 star HD 192163. Physically meaningful behavior is shown by curves with $3 \leqslant A \leqslant 6$.

Electron-ion collisions act to bring the level populations to their LTE values N_n^* at the local electron temperature, where

$$N_n^*/g_n = \frac{1}{2}\left(\frac{h^2}{2\pi mkT_e}\right)^{3/2} \exp\left(h\nu_n/kT_e\right) N_e N_i \tag{57}$$

and $h\nu_n$ is the ionization potential for an atom in level n. Griem (1963) has derived a useful formula to determine those levels which are nearly in LTE by comparing the total collisional upward rate out of level with the sum of the radiative downward rates. If the collisional rate exceeds the radiative decay by a factor of 10, the population should be within 10% of its LTE value. According to Griem, this condition is met if $n \geqslant n_0$, where

$$n_0^{17/2} = \frac{7.4 \times 10^{18} z^6}{N_e}\left(\frac{kT_e}{E_H}\right)^{1/2}. \tag{58}$$

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E_H is the ionization potential of hydrogen, and $z=2$ for He II. Taking representative values of $T_e=1\times10^5$ K and $N_e=10^{12}\ \mathrm{cm}^{-3}$ for a W–R envelope, we find $n_0\approx10$. Thus, collisional processes strongly dominate over radiative processes for $n\geqslant10$. For such levels $\exp(h\nu_n/kT_e)\approx1$ for any reasonable envelope temperature. Thus, N_n/g_n will be essentially constant for $n>10$. In Figure 11 the curves which satisfy this condition, along with the requirement that no population inversions occur, lie in the range $3\lesssim A\lesssim6\ \mu^{-1}$.

If we take $A=4$, for example, the Table III gives the resulting values of y_n, $\tau_{4,n}$ and T_{ex}, where

$$y_n = \exp(-h\nu_{n,4}/kT_{ex}) \tag{59}$$

defines the excitation temperature of each line.

TABLE III

Results of a coarse analysis of the WN 6 star HD 192163

n	$N_n g_4/N_4 g_n$	$\tau_{4,n}$	T_{ex}	$\log_{10} b_n$
5	0.247	1.01×10^2	1.02×10^4	0.908
6	0.136	1.30×10^1	1.10	0.711
7	0.048	4.32×10^0	0.88	0.306
8	0.041	1.93×10^0	0.92	0.256
9	0.035	1.05×10^0	0.94	0.205
15	0.0195	1.31×10^{-1}	0.93	0.008

The excitation temperatures are about 5 to 10 times lower than the electron temperature T_e one would expect on the basis of the observed ionization of the atmosphere. If b_n is in the usual departure coefficient, $N_n=b_nN_n^*$, then one may show that

$$\log_{10}\frac{b_4}{b_n}=\frac{0.625}{\lambda_{4,n}}\left(T_{ex}^{-1}-T_e^{-1}\right). \tag{60}$$

Since we have argued that $b_{15}\approx1$, Equation (60) may be used to find b_4, and subsequently the other values of b_n. With $T_e=5\times10^4$ K as a reasonable guess of T_e, we obtain $\log_{10}b_4=1.39$. The final column of the table lists $\log_{10}b_n$ for $n>4$.

We may obtain information of the extent of the emitting region from knowledge of the line source functions. From Equations (26) and (29), the flux in a line relative to that in the underlying continuum may be obtained. If the angle factor in $\tau(\nu,p)$ is ignored and the product $S(r)\,(1-\exp[-\tau_0(r)])$ is assumed constant for $r<r_E$ and zero for $r>r_E$, as in the derivation of Equation (50), then

$$(F_\nu-F_c)/F_c=\left(\frac{r_E}{r_c}\right)^2\frac{S(r_E)}{I_c}\left(1-\exp[-\tau_0(r_E)]\right). \tag{61}$$

Read off the spectrogram, $(F_\nu-F_c)/F_c\approx8$ for $\lambda4686$, and since the line is optically thick, the exponential term vanishes. For the continuous radiation field, $I_c=B_\nu$

(40000 K) is probably adequate, and not, at any rate, very critical to the argument. With $N_3 g_4/N_4 g_3 = 4.65$ as computed previously, $S(\lambda 4686)/I_c(\lambda 4686) = 0.32$, and $r_E = 5r_c$. This is of sufficient size to account for the lack of occultation or absorption effects in the He II spectrum. It should also be pointed out that the large tabulated value of $\tau_{4,5}$ justifies the original assertion that λ10124 is optically thick.

The coarse analysis of the He II lines from HD 192163 is thus seen to lead to a self-consistent model of the Wolf-Rayet envelope. Other tests, e.g., prediction of the strengths of lines other than those in the Pickering series, do not contradict the results obtained. Of course, a detailed description of the run of physical variables with depth is not produced. Only an idea of the state of the atmosphere at a 'typical' point in the emitting region derives from this approach. Nevertheless, the simplicity of the coarse analysis is its strength since almost any attempt to go beyond it involves one in far more demanding tasks.

4.2. Solution of the Microscopic Rate Equation

The atomic level populations in a stellar atmosphere are determined by a myriad of competing microscopic processes. If we postulate that statistical equilibrium obtains, then the level populations have reached a steady state such that

$$dN_n/dt = R_n + C_n = 0. \tag{62}$$

The terms R_n and C_n are the net rates at which level n is populated by radiative and collisional processes, respectively. The solution of Equation (62) for N_n is a task which is tedious at best and hopeless at worst. Atomic parameters, such as the collisional excitation and ionization cross sections, are poorly known even for the hydrogen atom. Moreover, the transfer equation for every transition must be solved iteratively with the statistical equilibrium equations in order to evaluate the radiative rates.

Fortunately the existence of a radial outflow of matter at high velocity makes the microscopic rate approach more tractable than would be the case for a stationary atmosphere. This is because the radiation field in every line transition can be found by the escape probability method in terms of only local values of the physical parameters. The rub is that radiation in the bound-free continuum is only slightly affected by the presence of a large velocity field. Nevertheless, if one is not too scrupulous in handling the transfer problems in the continua, a start at least can be made on the microscopic determination of the level populations.

The first such calculation was done by Castor and Van Blerkom (1970) for He II in the WN 6 star HD 192163, for which a coarse analysis already provided information on the state of the atmosphere. The rate equations are solved at one 'representative point' in the emitting region, chosen to lie midway between r_c and r_E at $r \approx 40\, R_\odot$. The first 30 levels of the He II ion are considered and every collisional and radiative transition coupling these levels and the continuum are included. A crude approximation for the radiation fields in the continua is formulated so that they are treated in a manner analogous to the line escape probability method. The level populations are guessed initially, and then iterated until convergence to a desired accuracy is obtained.

The calculation is done for a grid of assumed values of helium density N(He) and electron temperature T_e at the representative point.

The suitability of the resulting models can be assayed most easily be computing the parameter A from Equation (56). Only a few choices of N(He) and T_e give values of A in the observed range. Although one unique set of parameters cannot be found, the model with $T_e = 1 \times 10^5$ K and $N(\mathrm{He}) = 2.5 \times 10^{11}$ cm^{-3} agrees rather well with the results of the coarse analysis. Since T_e exceeds the core temperature, a non-radiative energy source seems to be required to maintain the excitation in the emitting region.

A similar calculation has been attempted for C^{+2} in γ Vel by Castor and Nussbaumer (1971). Again, the statistical equilibrium equations are solved at one representative point, $r = 3.6 r_c = 55\ R_{\odot}$. The equations are solved for the 14 lowest terms of C^{+2}, with the escape probability method used for the line transitions. Because of a deficiency of atomic data only bound-bound radiative and collisional transitions could be included. Thus, the coupling of the levels to the continuum by collisions and radiative processes (including in this case dielectronic recombination) is ignored. Several model parameters are determined by fitting the theoretically derived intensities of the UV lines to observations. The range over which these parameters may vary and still yield good results is found to be fairly narrow. The best fit to the UV line equivalent widths give $T_e = 22000$ K, $N_e = 4 \times 10^{11}$ cm^{-3}, and $N(C^{+2}) = 1 \times 10^9$ cm^{-3}. If the electrons come mainly from hydrogen ionization, the abundance of carbon is about 8 times the cosmic value. Also, interestingly, T_e is significantly lower than the temperature of the core, which is assumed to radiate a continuous spectrum $I_c = B_\nu(30000\ \mathrm{K})$. This suggests that the line emitting region may be in radiative equilibrium.

5. Electron Scattering

The optical depth due to electron scattering in a Wolf-Rayet atmosphere is estimated by taking the product $\sigma_e N_e L$, where σ_e is the Thompson cross section and N_e and L are typical electron densities and lengths. An optical depth $\tau_e \sim 0.5$ is obtained in this way. It is important to recognize that although Thompson scattering is coherent in the frame of the electron, it is noncoherent to an observer who sees the electron in thermal motion. Thus, electron scattering causes frequency redistribution which the escape probability method ignores. The inclusion of this effect is a refinement which is only now being incorporated in the theory. We first resort to a very approximate treatment of noncoherent electron scattering originally developed by Münch (1950).

In Münch's procedure, a plane-parallel layer of free electrons of thickness τ_e and temperature T_e is irradiated by line photons falling on its inner boundary. Since both atomic absorption and electron scattering occur together, the results obtained must be considered as only qualitative. The redistribution function depends on the approximation used to obtain it, with somewhat different functions given by Münch (1950), Hummer and Mihalas (1967) and Weymann (1970). We use the expression given by

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Hummer and Mihalas:

$$R(x', x) = \left(\frac{1}{w}\right) \text{ierfc}\left(\frac{|x' - x|}{2w}\right) \tag{63}$$

where ierfc(z) is the integral of the complementary error function and w is the ratio of the electron Doppler width $\Delta\nu_e$ to the width of the incident line $\Delta\nu_L$. The parameter x measures the frequency displacement from line center in units of $\Delta\nu_L$. For $\tau_e < 1$, the profile of the line after it emerges from the electron layer may be described by the approximate expression

$$\psi(x) = (1 - \tau_e)\,\phi(x) + \tau_e \int_{-\infty}^{\infty} \phi(x')\,R(x', x)\,\mathrm{d}x' \tag{64}$$

where $\phi(x)$ is the profile of the line before scattering.

A study of a spectral line showing the possible influence of electron scattering has been carried out by Castor, Smith and Van Blerkom (1970) for $\lambda 3483$ of N IV in the WN 6 star HD 192163. This strong emission line shows a violet displaced absorption component, shortward of which is an extensive emission wing. Such behavior cannot be reproduced by a radial expansion model. When the incident profile $\phi(x)$ is of the normal P Cygni type, the emergent $\psi(x)$ matches the observed line very closely. In this case, the best fit is obtained for $\tau_e = 0.5$ and $w = 4$. This value of w, taken literally, would imply temperatures in excess of 5×10^5 K. However, the deficiencies of the model make any quantitative statement suspect.

Underhill (1968) has presented detailed tracings of the line spectrum of HD 191765. She shows, in particular, that the Pickering lines $\lambda 4200$ ($11 \rightarrow 4$) and $\lambda 5411$ ($7 \rightarrow 4$) are both nearly Gaussian in shape. If the results of the coarse analysis of HD 192163 can be taken to apply to this WN 6 star as well, then $\tau_{4,11} < 1$ and $\tau_{4,7} \sim 4$. One would expect different profiles for the two lines, with the thin line showing a flat-topped behavior. For the velocity distribution given by equation (10), most of the envelope is moving at nearly constant velocity. If it is assumed that $v(r) = $ constant throughout, the emergent profile can be found analytically. Sobolev (1958) shows the line to be flat-topped if the envelope is transparent and parabolic if it is opaque. These calculations ignore occultation and absorption, i.e., the core is reduced to a point. In order to see whether a parabolic line can be made to appear Gaussian by electron scattering, $\phi(x)$ is taken to be a parabola in Equation (64) and $\psi(x)$ computed for various values of τ_e and w. It is found that the core of the electron scattered line still is parabolic and changes discontinuously into a flat wing emission. No emergent line profile appears in the least Gaussian.

6. Work Following Symposium

Dr Lawrence Auer of Yale University and the author have investigated the effect of electron scattering in expanding atmospheres, in which electrons are distributed throughout the envelope and share in its expansion. Although still preliminary, it seems appropriate to comment here upon this work because of its relevance to WR stars. The model used is the usual one of a core and an expanding envelope, but the transfer of photons is treated by a Monte-Carlo technique. In this method, one photon at a time is followed from its point of emission to its final destination. Very many photons must processed in order that the result be statistically meaningful.

Let us first suppose that $w = 0$ throughout the atmosphere, i.e. no *thermal* frequency redistribution occurs at a scattering event. There is, of course, a frequency redistribution due to the macroscopic motion of the electrons. If a photon is emitted in the atmosphere at a frequency displacement x relative to a stationary external observer, it may be scattered as it traverses the envelope and emerge with a frequency displacement x'. For any model in which $v(r)$ is constant or monotonically increasing, $x' < x$, i.e. on the average the photon is shifted to the red. This is due to the fact that all points of the envelope recede from each other and the net effect of scattering is to decrease the photon energy. Thus, an extensive red wing (extending beyond $x = -1$) is formed, with no violet wing at all.

If thermal redistribution is operative ($w > 0$) energy is transferred from line center to both wings. Again, however, the red wing is far more extensive than the violet. The following simple example demonstrates these effects.

Let $v(r) = \text{const}$ for a fully ionized envelope in which the line in question is formed by recombination, i.e. the emissivity $j(r) \alpha N_e^2$. Since the product $r^2 N_e v$ is constant for steady outflow of matter, $j(r) \alpha r^{-4}$. If the envelope is *transparent* to the line radiation, the emergent profile, in the absence of electron scattering, is found by integrating the emission along surfaces of constant velocity (see, e.g., Rosseland, 1936). The profile is found to be

$$\phi(x) = \begin{cases} 0 & |x| > 1 \\ 1 & 0 < x < 1 \\ (1 - x^2)^{1/2} & -1 < x < 0 \end{cases} \tag{65}$$

in arbitrary units. To the violet of line center, the line is flat-topped, while to the red it is diminished by occultation.

We now include the effect of electron scattering by stipulating the radial optical depth in electrons from the core to infinity, τ_e, and the degree of thermal redistribution w. The Monte-Carlo technique is employed in this case. For $\tau_e = 0$, the profile given by Equation (65) should be recovered. Figure 12a shows that with 100000 photons processed, the two calculations agree almost exactly.

Figure 12b shows the profile for an electron optical depth $\tau_e = 1$ and *no* thermal redistribution. As discussed above, there is a pronounced red wing on the line, but no wing on the violet side. Thermal redistribution is 'turned on' in Figure 12c. The parameter w is the ratio of the electron Doppler width to the half width of the unscattered line profile, i.e.

$$w = \left(\frac{2kT}{m_e}\right)^{1/2} / v_\infty .$$

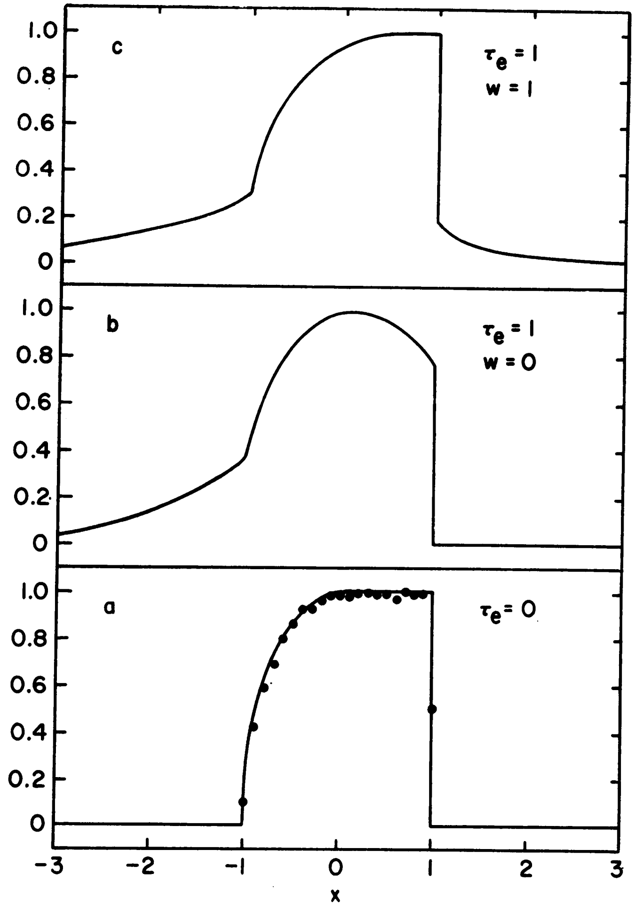

Fig. 12. Monte Carlo calculations of a line formed in an atmosphere expanding at constant velocity and having no line opacity. The exact profile is compared to the Monte Carlo result (filled circles) in Figure 12a. Electron scattering with and without thermal redistribution ($w=0$ and $w=1$) cause the profiles shown in Figures 12b and 12c respectively.

For $v_\infty \approx 1000$ km s^{-1} and $T \approx 5 \times 10^4$ K, $w \approx 1$. This case is shown in the figure. A violet wing is now apparent, and the red wing is somewhat enhanced over the $w=0$ case.

These results depend on the assumptions of constant outflow velocity, transparency of the envelope in the line and the specific line formation mechanism. A WR star would not be expected to satisfy any of these assumptions, so the line profiles shown in Figure 12 look nothing like observed WR lines. Nevertheless, the appearance of an extended red wing is a feature of electron scattering in an expanding atmosphere and is expected to show up when a more realistic model of line formation is employed. Because of the blending problem associated with lines of such widths, it is difficult to find an uncontaminated line in order to study its wing structure. A look at Dr. Underhill's line profiles (Underhill, 1968), however, does not show definite asymmetries which could be interpreted as due to electron scattering. Exactly what this signifies for WR stars is not yet certain. Perhaps estimates of electron densities in the envelopes are too high by nearly a factor of ten. Further study of this problem is under way.

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DISCUSSION

Thomas: Personally I think it futile to compare these continuum models to observations in any sense until the assumption of LTE is checked. If collisions should turn out to dominate photo-ionization, then the LTE assumption is valid. If photoionization dominates then instead of a rapid drop in temperature it begins to rise up again.

Van Blerkom: In an extended atmosphere, conditions are favourable for photoionization to dominate, since densities fall to very low values.

Thomas: Be sure to check it. Just before you put the infrared and ultraviolet excesses, ask where in terms of $\tau = 1$ in the visual, which you are talking about here, where in the atmosphere do three things happen:

(a) $\tau = 1$ in the UV,

(b) $\tau = 1$ in the infrared.

(c) the photoionization rates equal the collisional rate. Then right away you can answer the question posed.

Van Blerkom: Cassinelli originally computed his models for central stars of planetary nebulae, but realized that they described Wolf-Rayet continua rather well. It may be a fortuitous accident.

Morton: How do these models with the curved atmospheres compare with the plane-parallel non-grey models for O type stars?

Van Blerkom: Cassinelli's model would probably describe OB supergiants, since these have continua similar to Wolf-Rayet stars.

Morton: What happens to the Lyman continuum in Cassinelli's models?

Conti: It looks to me that Cassinelli's spherically extended model atmospheres might well explain the very large discrepancy between the interferometrically derived temperature and what we think is the real temperature in the case of ζ Puppis.

Morton: I wish I could be more optimistic that the limb darkening can have really that much effect, but the possibility is worth investigating.

Van Blerkom: I am very glad that Cassinelli's paper came out in 1971 because otherwise there would only have been Kosirev's. Cassinelli had to do this calculation in order to make Kosirev's results worth presenting.

Kuhi: In your integrations over the atmosphere, how do you avoid the discontinuity that results naturally in the density distribution whenever you get $r = r_c$. What do you do there?

Van Blerkom: If you let absorbing atoms exist all the way to the core of the star where $r = r_c$ you get rather severe discontinuities in the emergent line profile, because of strong occultation effects and absorption near the core. These are not observed. In order to get the line profiles that Castor gave in this paper, he had to assume that there were no absorbing or emitting atoms near the core. Either the escape probability method and the two level atom is incorrect near the core or else excitation conditions near the core are such that those atoms do not exist there. This might fit in with the view that there is a temperature inversion and it is getting too cold for atoms in the particular stage of ionization to be around.

Kuhi: These calculations always end up giving you parabolic profiles for the optically thick case. Can you suggest what you would have to do to get a Gaussian profile that seems to be the case mostly observed?

Van Blerkom: I imagine you have to play around with the velocity distribution to get a Gaussian.

Kuhi: My next question is for the P Cygni problem but applies to the WR stars as well. Most of the absorption edges that you get with this kind of calculation will give you intensities in the absorption component down to about 0.5, and it seems very hard to get the intensity down to zero when you have the intensity of the emission component say 5 or 10 times that of the continuum, which is typically what you see in Hα or Hβ. Can you suggest what you would have to do there?

Van Blerkom: That is the problem that Lucy ran across when he tried to explain the quasi-stellar case where he used the expanding atmosphere model we have used. You could not get both a deep absorption and a high emission at line center. The only explanation that one could think of then was that the part of the envelope that fills in the absorption has to be missing. In other words, you cannot have a spherically symmetric atmosphere. You must have some kind of a jet coming preferentially at you. I do not believe that.

Underhill: Why do you not believe in jets?

Van Blerkom: To have a jet preferentially directed towards you all the time, seems to me too fortuitous.

Underhill: I can see another way. The absorption can be strengthened by using scattering. In other words, you have a non-LTE like calculation. You can always fix it that lines get a strong absorption and not so strong emission. You bring your required emission intensity up by increasing the size of your shell.

Van Blerkom: These are non-LTE calculations.

Underhill: You have to work out more than for two levels.

De Groot: How certain are you that the emission line profiles of P Cygni type are really Gaussian? If the difference with the parabola is that we have a little more of a wing, may be you could do something with a little bit of electron scattering.

Van Blerkom: That was my hope when I attempted the calculation. You take a parabola, put it through an electron scattering layer and out it would come a Gaussian. It did not quite work out that way, but that might be the fault of the very crude way it was treated. It might be possible that the usual rounded parabolic type profiles with some electron scattering would look Gaussian.

De Groot: In P Cygni itself I had a lot of profiles and I do not think they are Gaussian. They had a wing on the red edge. It is also quite steep, may be not as steep as your model would indicate, but at least in between the two.

Kuhi: I did not mean to say that the profiles in P Cygni were Gaussian. I had two separate questions: Gaussian profiles in WR stars and the problem of the depth of absorption in the P Cygni stars.

Johnson: I just wanted to clarify whether your continuum comes from something you might call a photosphere, whereas when you talked about line emissions you went into an envelope. And I would like to ask what, if anything, the envelope may contribute to the continuum.

Van Blerkom: We considered that for the two WR stars that John Castor and I analyzed and we found that in the visible continuum the optical depth of the envelope is so small that there should be no observed continuum emission from the envelope.

Underhill: These calculations are for a two-level atom. In order to obtain the ionization balance between ions, you need at least a three-energy-level atom, a continuum plus two line levels. Have you any feeling for how possible it would be to extend the theory in that direction?

Van Blerkom: We did a thirty level atom for helium in which the continuum is included. You can do a multi-level atom because the line transfer is very simple, with the escape probability method. It is much simpler than if the atmosphere was standing still. The rub is that the continuum is not affected by the precence of the velocity field. Hence you have to solve the continuum problem in every single transiton. And that is a tremendous problem. We did a rough escape probability type method for the continuum which is very, very crude. We tried to include every single collisional and radiative transition.

Thomas: Why do you have to be so sophisticated on this? Certainly one or two lower levels for the ionization is sufficient, and treating the transfer problem for one continuum is quite straightforward. Anne Underhill's question is only about ionization equilibrium.

Van Blerkom: If you are just interested in ionization equilibrium, then you can get away with it.

Underhill: It seems to me you are being too elegant also. I would go along with Thomas. Every atom we have got has a continuum that we are interested in, a ground state that we do not observe lines from unless we have rocket UV-spectra, and some levels most of them clustered rather close to the continuum. The ones we observe are much closer to the continuum than they are to ground. We observe several stages of ionization, at least three. It seems to me a very plausible programme to set up the spherical moving atmosphere ionization balance, because you are taking in the major continuum of at least the three ions. Then you plot the fraction of each ion that is around. Then, depending on your density, which may be high enough, you can argue very convincingly that the levels that you want to observe from those populations are linked to the ion population of the next one by LTE with your assigned local temperature. And then you could perhaps pick this thing up. That may be the problem partly with P Cygni.

Van Blerkom: I might point out it is a non-trivial problem to treat one continuum in a spherical atmosphere even if you do not take into account the motion.

Thomas: It depends on the degree of sophistication you want. Are you talking about accuracies of a factor one per cent or a factor two?

Van Blerkom: A factor of two.

Underhill: Factor of two in the ionization balances would be quite acceptable. Then to calculate particular lines you might go back and be very fussy. If you want to match profiles you do have to be fussy.

Van Blerkom: The continuum optical depths for the very lowest levels are so high that the radiative rates are practically in detailed balance and so are the collisional rates that set things. The continuum really came into the higher levels when we wanted to compute their populations.

Thomas: It depends. Certainly in the regions where the strong resonance lines are formed, the continuum cannot possibly be in detailed balance.

Let me go now to your analysis of the He II lines. In the integration you have assumed that S times the bracket is constant. But S is going to decrease towards lower optical depth and so is the bracket. So that product is not a product of two factors, one of which increases and the other decreases. Both decrease, giving a systematic behaviour. It seems to me that the number of assumptions you have put in here are large enough that one worries about trusting the result.

Van Blerkom: This is a coarse analysis, but I think it is pretty good. Of course, I cannot prove it. What it does is to give you a feel of the average behaviour in the emitting region.

Thomas: The only reason I am worried comes from having put a lot of time on interpreting gradients, both in terms of emission decrements and of height gradients. I just find that you can really confuse things when you compare the observations and a theory in which you average things out. I remind you that we have changed the optical depth in the solar atmosphere by a factor of 50 just by being more presice. I am all for your procedure. It is just that I would be uncertain about how much I would trust my intuition on a coarse analysis as to what is most important, although I agree that a coarse analysis comes first.

Underhill: I think you are depending very strongly on the statement you made that you assumed all lines to be formed in the same region. If you consider that this is an expanding atmosphere and that the regions are defined by the constant velocity surfaces and by the on-the-spot hypothesis, then that assumption is not as difficult to accept as it is in the stationary atmosphere. I think that is what saves you. It is difficult for me to accept that all lines are formed in the same region. Now He II 4686 has a very much stronger *f*-value than one of the higher Pickering lines.

For a stationary atmosphere these lines are certainly not formed in the same region. But when you take into account the atmosphere is moving, then the only parts that count are those parts moving with just the right velocity to get you on the line center.

Van Blerkom: A line is formed in an expanding atmosphere over a large fraction of the envelope, that is, wherever the appropriate ions exist. That is why I say the lines are formed in the same region even though their *f*-values and optical depths are very different. Now, I would like to comment on the electron temperatures deduced from the theoretical models. The analysis of the WN6 star HD 192163 by Castor and myself indicated that the electron temperature in the envelope exceeded that of the core. Thus, a source of energy other than radiative is suggested. Castor and Nussbaumer have studied γ_2 Velorum. This is a very complicated system since the Wolf-Rayet is the fainter component. They found a best fit of their model to observations for a core temperature less than the electron temperature of the envelope. This might be an indication that radiative equilibirum holds.

Thomas: There is a very strong difficulty of supporting this atmosphere, and Castor says the only way he sees how to do it, is a random turbulence of some hundreds or thousand kilometers a second. So this is not exactly what you would call a self-consistent situation.

Underhill: That is right.

Morton: In regard to the analysis of the ultraviolet spectrum of γ_2 Velorum, if we accept the present point of view that the O star is brighter, one can assume that all the emission lines, except the λ1909 line, in the UV spectrum are due to the O star. Now, what C III lines were actually analyzed?

Van Blerkom: Ten lines from the fourteen lowest terms of C III; they were mainly in the ultraviolet.

Thomas: What is the highest stage of ionization observed in this star?

Smith: Probably C IV.

Van Blerkom: Let us return to the question of support of the atmosphere. Castor and Nussbaumer used a spherically expanding envelope model, so the question of support does not arise.

Thomas: I asked Castor, what he needs in the way of supporting the atmosphere? He said he needs a random turbulence. It is conceivable I misunderstood, so I may be wrong.

Morton: Now, how does his support problem differ from your support problem?

Van Blerkom: You do not have a support problem if you are not supporting anything.

Morton: You have an outflowing atmosphere.

Van Blerkom: Right, but we do not know what is causing the outflow.

Morton: I do not see that that argues against this model in particular.

Thomas: No, the problem is this. If I am going to invoke either random motion or differential expansion motion on the order of several hundreds or thousand kilometers a second, it is very difficult for me to see how you get this, without some kind of mechanical input of energy to maintain this. So when the suggestion is made that may be we can do these models in radiative equilibrium, I am afraid I just sit back and smile.

Van Blerkom: The question was raised earlier about what lines were used in the model of γ_2 Velorum The lines $\lambda\lambda$2296 and 1176 allow the core temperature and envelope electron temperature to be related. The intercombination lines $\lambda\lambda$1909 and 2846 allow the density to be determined.

Conti: Lindsey Smith and I had been writing a paper on γ_2 Velorum, and she has just sent me a print of its ultraviolet spectrum; it looks just like a B0 supergiant.

Underhill: That is what it suggests to me, too.

Conti: Which is to say, it is the O9 supergiant we observe and all the lines that you see there belong to that star. None of them can be definitely WR lines, especially none of these.

Thomas: Let us just be sure I understand Conti's remark here. Is it what you are saying that the only line here you can be sure belongs to the WR component of γ_2 Velorum, is 1909 Å? Everything else is incidental?

Conti: Everything else is the O supergiant.

Thomas: So, my question about what was the highest level of ionization you observed, is irrelevant!

Conti: I do not think you can do it from γ_2 Velorum. If you want to look at the UV you must get a rocket spectrum of a single Wolf-Rayet star.

Morton: The Princeton OAO, with its high-resolution spectrometer should be able to sort out the system by determining which UV lines shift the same way as the visible absorption lines. O VI also occurs in the UV spectra of the OB supergiants, in absorption and possibly also in emission at 1032 and 1038 Å.

Thomas: Is the spectrum good enough to see that?

Underhill: As far as I know, the available spectra of γ_2 Velorum do not have adequate resolution in that region to show you anything.

Smith: Let me strike an intermediate position. It is quite clear that between about 1000 and 2000 Å which is the region that Conti and I compared with ζ Puppis, it is very difficult to tell the difference between the spectrum of γ_2 Velorum and of ζ Puppis (See also Stecher, in the last WR Symposium). Most of the lines in this region are strong resonance lines, and apparently the O companion is chiefly responsible for these lines. But in the region from 2000 to 4000 Å, there are a large number of emission lines, and most of them are probably due to the Wolf-Rayet star. The emission lines can be conspicuous despite the disparity in the magnitudes, exactly the same way as they are in the visual region from 3000 to 6000 Å. Which lines Castor used I do not remember, but he had many C III lines to choose from.

The data used was from OAO observations, provided by Lilly. The OAO observations agree moderately well the observations of Stecher, but Stecher's have slightly better resolution. Below 2000 Å, the OAO observations are better than Stecher's because they are free of atmospheric absorption. In that region, the spectrum of γ_2 Velorum looks very much like that of ζ Puppis, so probably the very strong resonance lines are due mainly to the O9 supergiant. Below 1000 Å, we know nothing at all, but between 2000 and 4000 Å, there are many emission lines (whose equivalent width is greatly reduced because of the contribution from the O9 supergiant to the continuum) that come from the Wolf-Rayet star. I wish to say, Castor is aware of the final numbers on the relative luminosity, so that he is taken that into account.

Van Blerkom: Half his manuscript is concerned with correcting for the presence of the O star.

Underhill: That is a very difficult point. He actually uses, in the analysis two lines, between 2000 and 3000 Å. This problem really boils down to which lines are radiation dominated, and which are collision dominated. There are two different kinds of problems being solved that are of a different character. I think it is interesting, but I do not think it solves anything.

Smith: Does it need to be added here that C III λ1909, is certainly not seen in the supergiant spectra? So, that line in particular, is coming from the Wolf-Rayet star. But in the region from 2000 to 4000 Å there are many C III lines, as was already mentioned by Van Blerkom.

Underhill: That line is seen in Wolf-Rayet stars but I do not think that Lindsey Smith is sure it is not coming from the O star, or from gas in the system.

Smith: All I said was that we do not see it from other supergiants, so I assume it is not coming from the O9 supergiant in this particular case.

Sahade: Well, it should be coming from the 'outermost' envelope, is it not so?

Underhill: Yes, it is coming from the general envelope of the system.

Van Blerkom: It is a calculated risk to try to analyze a binary. I would like to point out that Bappu's results showed unblended C IV lines in some single WC stars. It might be a more profitable thing to deal with them, because there are no ambiguities at all.

Paczyński: Coming back to WN stars, as far as I remember the paper of Castor and Van Blerkom, they have calculated about six different models with different electron densities and temperatures. If you plot the results on the electron temperatures – electron density plane, you find a line on this plane along which the models give the observed line ratios. For a high electron temperature you need a high density and vice versa. My impression is that it is difficult to decide whether the electron temperature should be high or low.

Van Blerkom: Yes, it is difficult to pick a unique model. This was just one that seemed to fit, but there might be others. I would like to try putting in lower values of the electron temperature and the electron density to see if one might get reasonable agreement with observation.

Underhill: This is a WN star, a WN6. I think, using the He II spectrum by itself, you cannot get a unique answer. You may very well be able to balance it out with a lower temperature, but He II does not really care what temperature, once it is above 50000°. However, it is WN and you have N V very strong, as well as other N ions. If you consider the N V, you find 10^5 degrees is rather a nice temperature. You will not find anything seriously lower than that, adequate to give you the N V in emission.

Van Blerkom: What is the ionization potential necessary to get N V? It is substantially higher than for He II. So, it might be formed in a very different part of the envelope.

Underhill: I have some information which I intend to demonstrate on this point tomorrow, to illustrate it.

Van Blerkom: I am not saying that the entire envelope is in radiative equilibrium, and I do not think that Castor is saying that either. The results suggest that the region in which C III lines are formed, might be in radiative equilibrium. That does not apply to the entire envelope, by any means.

Underhill: And all the C III lines that you observe are not necessarily formed in the same region. In fact, you are almost certain λ1909 is formed around the system. Some of it might be formed where λ2296 is or λ1176 or λ5696. The real problem with these stars is, approximating the atmosphere, as you are forced to for an illustrative theory, with one point.

Paczyński: I would like to talk about one thing that was not mentioned in Van Blerkom's paper, and which I believe is important. If you take your and Castor's model for the envelope of a WN star, you can calculate the critical depth in different He II continua. You did this for the visual continua and you found them to be optically thin. Therefore, we should not expect any jumps in the visual part of the spectrum. It is possible to do similar computations for the three ultraviolet continua. If you take your numbers you find that Lyman and Balmer continua are very thick. Their optical depth is above 100. If you go to the third continuum, which has an edge at 2050 Å, you find that the optical depth is about 0.3. There are two important things that follow from that. The spectral region around 2050 Å is observable, and we may see an edge either in emission or in absorption at this wavelength. It would be very interesting to observe those WN stars which show a nice Pickering series and to see if there is any edge. This could help in deriving the electron densities and temperatures in the envelopes. And there is a second aspect. The temperature of the central star is assumed to be about 40000 K. The maximum of the Planck curve is just shortward of 800 Å. This means that a significant part of energy is emitted in the wavelengths in which the envelopes are optically thick. Therefore, the photospheric radius varies by a factor of 5 or 6 between visual and far ultraviolet. It will be very difficult to build a model of such an extremely non-grey and non-plane parallel atmosphere. This kind of model has never been studied from the point of view of the radiation pressure in the continuum acting as an agent for the mass outflow.

Van Blerkom: I thought Rublev studied radiation pressure in the continua beyond the principal series of He II and found that it was negligible compared to the electron scattering.

Paczyński: It depends on your model. In this case the electron scattering gives you an optical depth of about one, whereas the optical depth in the far UV is one hundred. This leaves a possibility of having radiation pressure in the continuum as the driving force for the mass outflow. There is no such model available in the literature now. Perhaps from the theoretical point of view Wolf-Rayet stars can exist, inspite of my lecture last afternoon!

Thomas: That theory depends on whether it increases or decreases the emission in the continuum. Just the fact that I have a high optical depth does not mean anything; it is whether the intensity increases or decreases relative to the black body, that is important.

Underhill: The black body is a very simple model.

Thomas: Be clear on the effect. If the intensity goes up, then I may have something; if it goes down, it just reinforces the conclusion of no effect.

Paczyński: Well, I just do not know, because I do not think that anybody has built a model atmosphere with a photospheric radius varying by a factor of 5 from one wavelength to another. I am not in a position to answer that.

Underhill: You can do that automatically in all the plane parallel atmospheres. It is the only geometry in which solutions have been obtained. To put opacity differences into a spherical atmosphere has not yet been possible, because of numerical difficulties. Although we are sounding very critical of the type of theory just presented indeed, there is a 40-year gap in which no progress was made. The new thing is a considerable step forward. I do not think at any time that the people who have offered these papers have really implied that they were more than a numerical experiment giving something that looks vaguely like a Wolf-Rayet star.

Paczyński: I really do not know. I just wanted to point out that in the particular model which fits very nicely the Pickering series observed in two WN stars you may calculate the optical depths in the three continua. And they come out to be large. One of these jumps is observable.

Thomas: It is always large if you go down deep enough. What you mean is large enough above a certain point.

Paczyński: It is large enough above the visual photosphere.

Underhill: It is larger than the model outer-atmosphere?

Paczyński: Yes.

Underhill: With the amount of gas at half the length you have there, you get vary large opacity in the ultra-violet continua. In some ways, that is why I would like to have some hydrogen there because it is not so opaque.

These are difficult problems and you have a life-time of work in front of you.