40. THE DETERMINATION OF JUPITER'S MASS FROM LARGE PERTURBATIONS ON COMETARY ORBITS IN JUPITER'S SPHERE OF ACTION

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Abstract. Necessary and sufficient conditions are formulated for determining the mass of Jupiter from large perturbations induced in cometary orbits in the sphere of action of Jupiter. A procedure for the investigation has been developed and programmed for an electronic computer. Comparison of heliocentric and jovicentric computations shows that the perturbations on P/Wolf could be determined with great accuracy when this comet passed through Jupiter's sphere of action in 1922. The first attempt has been made to determine the mass of Jupiter using this passage and the observations of the comet in 1925. The resulting value for the reciprocal mass is 1047.345.

1. Statement of the Problem

There is increasing interest in determining the masses of major planets from the motions of minor bodies of the solar system.

The study of the passage of a comet through Jupiter's sphere of action is advantageous for determining the mass of Jupiter, because the slightest variation in Jupiter's mass produces an appreciable change in the large perturbations on the cometary orbit and significantly affects the representation of the observations of the comet after the encounter (Kazimirchak-Polonskaya, 1961, p. 19).

We select for our study P/Wolf, because in its revolution 1918–1925 it passed within 0.125 AU of Jupiter. For our initial elements we adopt the most accurate set obtained by Kamieński (1959, Table 3) for the last apparition of the comet before the encounter with Jupiter.

We make use of two values for Jupiter's mass, within the possible range of uncertainty, namely: $1/m_1 = 1047.325$, $1/m_2 = 1047.400$ (de Sitter's value). For these values we have performed two identical integrations of the comet's equations of motion, with allowance for the perturbations by the planets Venus to Uranus, and the nongravitational secular deceleration. Accordingly, we have obtained two representations of the comet's normal places in 1925. These are given in Table I, and the fourth and seventh columns show that there exist significant differences in $\Delta\alpha \cos \delta$ and $\Delta\delta$ derived from the two values.

Thus, the formulation of the following problem is justified: to perform a series of integrations of the comet's equations of motion based on the same initial set of elements but varying the mass of Jupiter within definite limits, to obtain from each computation a representation of the comet's normal places after the encounter with Jupiter, and to choose the one that gives the least value for the mean error of one normal place; this representation will correspond to the most probable value for the mass of Jupiter.

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TABLE I

Representation of normal places of P/Wolf in 1925 for the two values for the mass of Jupiter

1925 UT	$(\Delta\alpha\cos\delta)_1$	$(\Delta\alpha\cos\delta)_2$	$(\Delta\alpha\cos\delta)_1-(\Delta\alpha\cos\delta)_2$	$(\Delta\delta)_1$	$(\Delta\delta)_2$	$(\Delta\delta)_1-(\Delta\delta)_2$
July 18.0	$-7''.3$	$-14''.9$	$+7''.6$	$+5''.5$	$+3''.2$	$+2''.3$
Aug. 19.0	-11.7	-21.8	$+10.1$	$+6.7$	$+3.6$	$+3.1$
Sept. 14.0	-11.1	-22.3	$+11.2$	$+4.6$	$+1.6$	$+3.0$
Oct. 12.0	-10.4	-21.5	$+11.1$	$+0.7$	-0.7	$+1.4$
Nov. 11.0	-4.3	-15.0	$+10.7$	$+0.5$	$+1.2$	-0.7
Dec. 19.0	$+5.4$	-5.2	$+10.6$	-2.2	-0.4	-1.8

2. Conditions for Determining the Mass of Jupiter

Encke (1831a, 1831b) was the first to determine a correction to the mass of Jupiter while improving the orbit of a minor body of the solar system. To do this he introduced into the equations of condition both a correction to the mass of Jupiter and corrections to the orbital elements of the body. This procedure was used afterwards by Möller (1872), Asten (1878), and Haerdtl (1889) in their studies of P/Faye, P/Encke, and P/Pons-Winnecke, and it is nowadays widely used when determining corrections to the values of the masses of the major planets.

We have decided to use a substantially different procedure, the basic principle of which is to formulate necessary and sufficient conditions whereby a single unknown, the mass of Jupiter (or some other planet), is left in the problem. The solution will then be unambiguous and the most probable one.

These conditions may be formulated as follows:

(1) The numerical theory of the comet's motion before the approach to Jupiter should be constructed over a large interval of time and to a high degree of accuracy, with allowance made for the perturbations by all the planets and nongravitational effects.

(2) Sufficiently extensive and accurate observational material should be collected both at the comet's last return before the approach and at its first return afterwards.

(3) At least three apparitions of the comet should be linked after the encounter with Jupiter to the same accuracy as before, because otherwise it will be impossible to allow for the nongravitational effects after the encounter.

(4) The variations in the elements due to nongravitational forces should be as small as possible; thus comets with very small perihelion distances should not be chosen because their nuclei are exposed to strong solar radiation.

(5) The approach to Jupiter should be close enough that the effect of a change in Jupiter's mass can be detected in subsequent observations.

Nevertheless, since Jupiter's oblateness is not known to the necessary accuracy, it is not practicable to make use of extremely close encounters, such as that of P/Brooks 2 to Jupiter in 1886, because a second unknown, the oblateness of Jupiter, is introduced, and this would also affect the motion of the comet and the representation of its observations. This complicates the problem and might lead to fictitious results. It is most advantageous to utilize those encounters where the least separation is between 0.01 and 0.20 AU.

P/Wolf is a very suitable object for solving the problem because its theory of motion and its passage through Jupiter's sphere of action in 1922 satisfy all the stated conditions.

3. Method of Investigation

In addition to the method of integration in special coordinates, programmed in double precision on the BESM-4 electronic computer (Kazimirchak-Polonskaya, 1972), we make use of the following:

(1) A highly accurate procedure for computing large perturbations on cometary orbits in Jupiter's sphere of action in both heliocentric and jovicentric form (Kazimirchak-Polonskaya, 1961, p. 191). Table II compares the application of the two forms to P/Wolf in a five-month interval during the comet's passage through Jupiter's sphere of action. Altogether, the gravitational attractions of the Sun and the planets Venus to Uranus have been taken into account.*

TABLE II

Perturbations in the orbital elements of P/Wolf in the sphere of action of Jupiter 1922 July 8.0 to December 15.0

	Jovicentric	Heliocentric	Jovicentric – Heliocentric
δM	$+20°05'52''.50$	$+20°05'52''.48$	$+0''.02$
$\delta\pi$	$-13\ 25\ 34.87$	$-13\ 25\ 34.89$	$+0.02$
$\delta\Omega$	$-\ 0\ 31\ 36.05$	$-\ 0\ 31\ 36.05$	0.00
δi	$+\ 1\ 23\ 46.18$	$+\ 1\ 23\ 46.17$	$+0.01$
$\delta\varphi$	$-\ 9\ 55\ 16.84$	$-\ 9\ 55\ 16.87$	$+0.03$
$\delta\mu$	$-96''.36763$	$-96''.36769$	$+0''.00006$

(2) Extremely precise coordinates of Jupiter during the comet's passage through the sphere of action. These are obtained by integrating Jupiter's orbit (using a step-size of 2 days, 1 day, or even less, nine decimal places, and considering perturbations by all the planets), rather than by interpolating from the magnetic tape that gives coordinates at 20-day intervals. This is exceedingly important when a comet goes deeply into the sphere of action, say, to within 0.10 AU.

(3) Additional procedures for increasing the accuracy of the determination of Jupiter's mass which we do not have the opportunity to describe here.

* The figures given are from manual calculations because only the heliocentric method has so far been programmed. Computations in double precision will undoubtedly yield still higher accuracy.

4. The First Attempt to Determine the Mass of Jupiter from the Passage of P/Wolf through Jupiter's Sphere of Action in 1922

Since Kamieński allowed for the perturbations only by Venus to Uranus (those by the other planets being almost undetectable to the accuracy he used), we also find it practicable to carry out the calculations over the critical revolution 1918–1925 using the same planets.

We have made several accurate integrations from 1918 to 1925 starting from the same set of elements and changing only the value for the mass of Jupiter within the limits

$$1047.325 \leq 1/m_J \leq 1047.400.$$

Kamieński (1933) took into account secular decelerations $\Delta\mu$ and ΔM along with the planetary perturbations. In addition, we have allowed for nongravitational effects in the orientational elements of the orbit (Kazimirchak-Polonskaya, 1972). The nongravitational effects were considered from the epoch of osculation $T_0 = 1918$ December 16.0 Berlin Mean Time to the time of the comet's approach to Jupiter. Precise positions of the comet, in relation to the Sun and the six planets, were thus obtained while the comet was in Jupiter's sphere of action. We also took the nongravitational effects into account until after the last observation in 1925.

For each integration we made a comparison with all the observations and normal places in 1925. It made little difference whether we used observations or normal places, so we adopted the latter. We have given (Kazimirchak-Polonskaya, 1972, Table I) residuals for the normal places in 1925 as found by Kamieński and Bielicki (1936) and as found by us for Hill's value of the mass of Jupiter. We find that the normal places are best represented by the value

$$1/m_J = 1047.345,$$

and the corresponding residuals are shown in Table III.

TABLE III

Representation of normal places for P/Wolf in 1925 for the most probable value for the mass of Jupiter

1925 UT	$\Delta\alpha \cos\delta$	$\Delta\delta$
July 18.0	$+0^{s}.03$	$+1''.0$
Aug. 19.0	-0.21	$+1.6$
Sept. 14.0	-0.20	$+0.7$
Oct. 12.0	-0.20	-0.4
Nov. 11.0	-0.05	$+2.1$
Dec. 19.0	$+0.21$	$+0.6$
m_J^{-1}	1047.345	
Nongravitational effects included	$\Delta\mu$, ΔM, $\Delta\Omega$, $\Delta\pi$	
ϵ	$\pm 3''.3$	

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When we compared our elements for the epoch of osculation 1925 July 12.5 UT with those by Kamieński and Bielicki from linking the three apparitions 1925–1942, we found differences of the order of those obtained earlier (Kazimirchak-Polonskaya, 1972).

Our results are of threefold importance:

(1) The elimination of the discontinuity in the theory of motion of P/Wolf has been more accurately shown.

(2) The new value 1047.345 for the reciprocal mass of Jupiter has a fair degree of confidence.

(3) The method is basically correct.

We intend to continue our studies on the problem in the future using a still more accurate procedure. To this end the theory of the motion of P/Wolf should be constructed to a very high degree of accuracy, with allowance for the perturbations from the nine planets (Mercury-Pluto) and the nongravitational effects in all the elements, for two time intervals 1884–1918 and 1925–1967. Other features that have already been developed, such as allowance for the perturbations by Jupiter's Galilean satellites, should be introduced into the study of the motion of the comet in the vicinity of and inside the sphere of action of Jupiter. To our regret, we have no opportunity to concern ourselves with such matters in this paper.

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References

Asten, E.: 1878, *Mem. Acad. Imp. Sci. St. Petersb.* **26**, No. 2.
Encke, J.: 1831a, *Math. Abh. Akad. Wiss. Berlin* **1**, 93; **2**, 35.
Encke, J.: 1831b, *Astron. Nachr.* **9**, 317.
Haerdtl, E.: 1889, *Denk. Akad. Wiss. Wien Math.-Nat. Kl.* **55**, 215.
Kamieński, M.: 1933, *Acta Astron. Ser. A* **3**, 1.
Kamieński, M.: 1959, *Acta Astron.* **9**, 53.
Kamieński, M. and Bielicki, M.: 1936, *Repr. Astron. Obs. Warsaw Univ.* **32**, 1.
Kazimirchak-Polonskaya, E. I.: 1961, *Trudy Inst. Teor. Astron.* **7**.
Kazimirchak-Polonskaya, E. I.: 1972, this Symposium, p. 95.
Möller, A.: 1872, *Astron. Nachr.* **79**, 119.

Discussion

V. A. Shor: One of the reasons for your being able to link successfully the apparitions of P/Wolf in 1918 and 1925 was that the coordinates of Jupiter were consistent with its adopted mass. You destroyed this agreement because you did not vary the coordinates when varying the mass.

E. I. Kazimirchak-Polonskaya: The correction I introduced to the mass of Jupiter is negligible, and hence the representation of the observations and correction to the mass would be unchanged if I varied Jupiter's coordinates. When I am able to include the perturbations by all the planets and full nongravitational effects, however, I shall carry out a supplementary investigation to preserve the agreement between the coordinates and mass of Jupiter. Eventually I intend to establish the limits by which the mass may be varied without destroying the theories of the motions of the major planets published in the *Astronomical Papers*, and a procedure will be developed for applying appropriate corrections to Jupiter's coordinates.

M. Bielicki: I can confirm that your new value for the mass of Jupiter is so negligibly different from Hill's value that adoption of the new value will not appreciably affect Jupiter's coordinates and hence your results. When Kamieński and I adopted de Sitter's value there was no effect on the coordinates.