GRAVOTHERMAL OSCILLATIONS

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ABSTRACT. Stellardynamical systems are modelled by gaseous spheres. The properties of an isothermal equilibrium configuration with a central singularity are discussed in the context of post-collapse evolution. For the numerical calculations heating effects are taken into account. Evolution towards the equilibrium configuration is monotonically only if much energy is delivered from hardening processes of binary stars. Since the equilibrium configuration is gravothermally unstable a minute emission of energy is sufficient to turn gravothermal contraction into gravothermal expansion. In such case equilibrium is approached by oscillations of the core.

1. THE SINGULAR ISOTHERMAL SPHERE ('SIS')

The discussion of Hénon (1975) is a cornerstone of post-collapse evolution of globular clusters. He assumed a self-regulated, central energy source, which reverses core collapse leading to a general expansion of the cluster. Post-collapse evolution assuming strict self-similarity has been discussed by Inagaki and Lynden-Bell (1983) (henceforth ILB for short). The inner part of the ILB solution is isothermal with the density decreasing as $1/r^2$. The expansion of this isothermal region is triggered by the energy input. Similar post-collapse expansion has been found for finite systems too, e.g. by Lightman and McMillan (1985) using a N-body hybrid method, by Cohn (1985) within the scheme of Fokker-Planck theory and, in the framework of gaseous self-gravitating systems, by Heggie (1984) and Bettwieser and Sugimoto (1984, henceforth BS for short).

Self-similar solutions demand systems infinite in radius and mass. However, there is an equilibrium state of a self-gravitating sphere of mass M confined within an adiabatic wall of radius R, which is related to the ILB-theory.

J. Goodman and P. Hut (eds.), Dynamics of Star Clusters, 219–230.

https://doi.org/10.1017/S0074180900147400 Published online by Cambridge University Press

It is the singular, isothermal solution (**SIS** for short), with the density ρ and temperature T given by

$$\rho = M/(4\pi R^3)\cdot(r/R)^{-2}; \quad T = GMm/(kR)\cdot 1/2 \qquad (1)$$

where k and m are Boltzmann's constant and the mass of the gas particles, respectively. The singular, isothermal equilibrium solution has the following properties: (i) The total entropy of **SIS** is rather low in comparison with intial models, hence a collapsed state cannot evolve towards **SIS** without eating negentropy. Energy has to be fed into a region with the highest temperature or must be removed from another region with a low temperature (e.g. by star evoporation from the boundary). (ii) In BS the gravothermal instability of **SIS** is demonstrated by the calculation of the inverse tensor of specific heat (cf. Fig.1) and the computation of eigenfunctions and corresponding eigenvalues, which extremize the second variation of the total entropy.

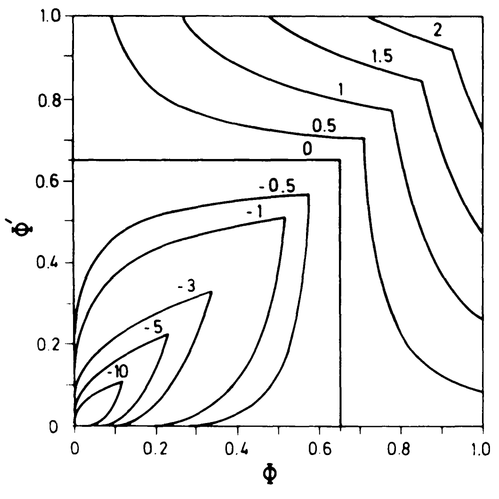

Fig.1. Tensor of the inverse specific heat $G(\phi,\phi')$ drawn as function of normalized mass coordinates. Note the large negative region and the singularity in the very centre. Functional form resembles those of regular isothermal solutions with very high density contrast.

Self-gravitating systems naturally have negative specific heats. SIS is one of the most extreme example for such case. (iii) SIS is unstable against spontaneous development of anisotropy of the velocity dispersion (Bettwieser et al., 1984). This is concluded from the functional form of the specific heats and from the numerical observation, that SIS if taken as an initial model in a pre-collapse calculation, does only marginally change its density distribution but becomes anisotropic on a secular timescale. In fact, we have constructed anisotropic and singular equilibrium configurations.

Let us turn to the general consequences of these properties of SIS. (i) Evolution triggered by heat added into the central core region will approach gradually to a structure nearby SIS. (ii) Provided effects of self-gravity dominate over the heating due to the energy fed into the core, gravothermal expansion or (and) gravothermal contraction can ensue. This will approximately be the case whenever the energy put into the core during one high density phase of evolution is small in comparison to the binding energy of the core. (iii) The post-collapse configuration becomes more and more anisotropic, and if gravothermal effects are more important than thermal effects (i.e. low energy input) even the region near the very center is expected to become anisotropic in the post-collapse phase. In the next chapter we discuss numerical results related to the topics (i) and (ii).

2. GRAVOTHERMAL OSCILLATIONS OF GASEOUS SPHERES

We investigated the evolution of gas models with a heat source designed to represent the kinetic energy added into the core by superscattering processes with hard binary stars. The model equations are given in BS for a system composed of single stars, and they are generalized to multi-mass systems in Bettwieser and Inagaki (1984). The energy generation rate per unit mass is assumed to be given by

$$\varepsilon = C/(1.95\ M_{\odot}L_{\odot}^{-1})\cdot[\rho/10^{10}\ M_{\odot}\ \mathrm{parsec}^{-3}]^{2}\cdot \cdot[10\ \mathrm{km\ s}^{-1}/\sigma]^{\ell} \qquad (2)$$

σ is the three-dimensional velocity dispersion. In most models calculated the parameter ℓ was chosen to be unity (seven) for single-mass (multi-mass) systems, respectively (cf. BS and Heggie, 1984). For the sake of definiteness the outer boundary condition was treated to be a fixed adiabatic wall. At the centre the energy flux and the velocity of mean mass motion was set to zero. Hence the code can in principle calculate a configuration such as SIS being singular at the centre. For multi-mass systems the velocity dispersions are assumed to be equal initially. As a consequence evolution is also due to the tendency to equipartition of the kinetic energies. Starting from Plummer's model we computed its time evolution for different values of the energy input rate. The value of the constant C was changed by many orders of its magnitude. For extremely large (small) values of the constant C the core becomes a perfectly thermal (gravothermal) system, respectively.

For pre-collapse models and for the first case (C large) our results are in agreement with those obtained by Heggie (1984) or by Cohn (1980, 1985). The core suffers from gravothermal collapse in the first place, until the central density reaches a threshold beyond which the energy input becomes effective. The isothermal region expands and the decreasing density contrast can even fall below the critical value of 709. Then the system evolves eventually to a completely isothermal configuration which is also uniform in density.

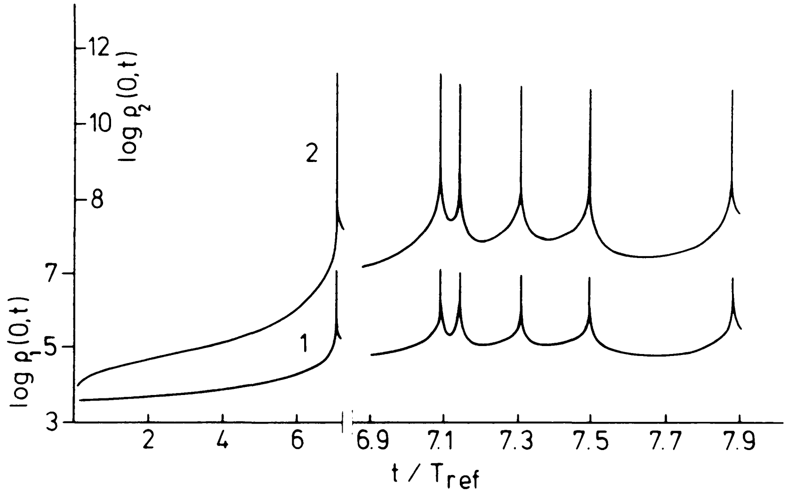

Fig.2. Time evolution of the central density ($M_{\odot}$/pc^3) for a two-component gas-system with a weak heat source ($C = 10^4$, $\ell = 7$, $M = 10^6 M_{\odot}$). Time in units of the Spitzer-Hart reference time.

For the second case (C small) the core contraction is halted and reversed at a higher value of the central density. Due to our numerical results, the energy released per mass within one local relaxation time is about constant (as a function of C). Hence the maximum of the density and the minimum of the core mass are proportional to C^{-1} and $C^{0.36}$, respectively. In fact, the collapsed state can have a very small core, with a mass fraction below 0.001. The core begins to expand and reaches a state of miminum density with a core mass of about 0.02 of the total mass for a broad range of values of C. Hence core heating is not the driving force for the phase of expansion. After this stage the core starts to contract again. As depicted in Fig.2 the system oscillates between phases of high and low central densities. In this case a two component system with a mass ratio of two was calculated for a mass fraction of the heavier component of 0.1. Energy was put into both components and Eq.(2) was used with the density replaced by the total density of the two components. The example shown is typical among our results; core oscillations have been found for one, two and three component systems; with different rates of the energy input; for extended energy sources as well as for a point source; for configurations isotropic or anisotropic in velocity space, and for different initial models and boundary conditions (Bettwieser and Fritze 1984). For these calculations three, in several aspects different numerical codes are used (cf. BS, Bettwieser and Inagaki 1984, Bettwieser et al 1984).

2.1. Physical Mechanism

The non-linear oscillations with such a great amplitude as seen in Fig. 2 is regarded to be a gravothermal flip-flop. The contracting phases are the well-known gravothermal catastrophe. Since the specific heat is negative in the core, contraction is driven by a self-sustaining heat flow directed outwards. With the accelerating increase of the central density energy input becomes suddenly appreciable. The core is supplied with a short energy impulse at a high central density. (It is worth to note here that for higher values of C energy is gradually put into the core at an earlier stage of the contraction, i.e. during a phase of slower evolution). Once the expansion commences the rate of emission of energy is quenched, since the central density decreases rapidly and the velocity dispersion changes only slightly. After the energy impulse is fed into the core there is a short phase of a thermal expansion of it. The inner core expands somewhat faster than the outer regions, since the energy input is concentrated towards the centre. As a result the slope of the temperature profile changes its sign and a heat flow directed inwards appears. Since the core is self-gravitating the heat flowing inwards enhances the bump in the temperature distribution as seen in stage 3 of Fig.3. Most of the time the expansion instability of self-gravitating systems is responsible for the evolution of the system. More and more heat flows into the central region accompanied by a decrease of its temperature. But there is a competing heat flow going outwards from the peak of the temperature profile to the halo. The total increase in entropy due to this latter process can be much larger than the entropy production invoked by perturbations, which lead to the gravothermal expansion (Hachisu and Sugimoto 1978). Therefore, the heat flowing outwards from the bump in temperature wins the game, and as a consequence the bump is smeared out. Finally, the core becomes almost isothermal and, since the density contrast is large, it enters again the phase of gravothermal contraction.

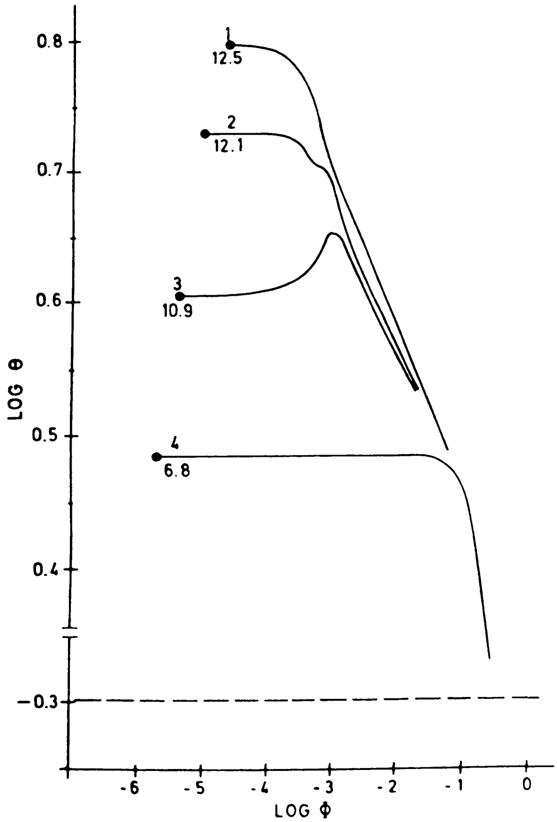

Fig.3. Temperature vs. mass (ϕ in units of the total mass of the cluster) for a one-component system, which suffers from gravothermal oscillations ($C = 10^4$, $\ell = 1$, $M = 10^5\ M_\odot$). At stage 3 the logarithm of density dropped from 12.5 to 10.9 as indicated in the figure, and the inversion in temperature is most pronounced.

It is worth to notice the following facts: (1) The energy generation is used mainly to turn the switch from unstable contraction to unstable expansion, provided the value of C is well below the threshold to excite the oscillations. Therefore the total amount of energy fed into the core during one cycle is virtually unaffected by even drastic changes of the energy generation rate (cf. table 1 in BS). But the value of C determines the maximum of the density and the typical time between successive peaks. (2) The evolution followed in the plane central temperature vs. central specific entropy looks as if the basic process works like a refrigeration cycle. The cyclic change in entropy and temperature comes from in- and out-flow of heat. The steady absorption of binding energy leads to a secular decrease of the average temperature. With each cycle the system as a whole is drifting more and more to SIS. The maximum size of the isothermal region gradually extends into the halo region. But even after many cycles the core is still regular in its density distribution. (The central singularity of SIS or of the self-similar ILB solution are mathematical artefacts). (3) The coefficient of heat conductivity is tailored to simulate the evolution of a stellar dynamical system. To a first approximation the dynamics of the core can be regarded to be independent from the outer shells, since the latter evolve on a much longer secular timescale. Therefore the appearance of the oscillations is such as if an isothermal core would be embedded in an adiabatic sphere, which follows the core's movement. (Here we disregard the secular drift during the cyclic changes of the core parameters). All the time any perturbation is renormalized to be in the linear regime. This is the reason why even a minute absorption of binding energy is sufficient to turn the contraction instability into the expansion instability. However, this remark does not mean, that the system can be divided into the weakly coupled parts of core and halo. Such system would fail to have gravothermal instabilities. (4) With increasing values of C the gravothermal effects and effects of core heating interfere. But the latter process is unimportant as long as C is below the threshold to excite oscillations (cf. table 1 in BS). When the energy supplied becomes too large the thermal perturbation is in the non-linear regime. Then an amount of energy is added into the core, which is comparable to the core's binding energy. As a result the reexpansion of an isolated cluster proceeds indefinitely. (5) In the case of multi-mass systems the kinetic energies of the core are near to equipartition during the whole post-collapse stage. The time between successive peaks in the density is shorter for a multi-mass system than for a single-mass system.

3. DISCUSSION AND COMPARISON WITH OBSERVATIONS

The role of the central binaries is often thought to reverse the collapse and serve as an efficient source of energy. If the cluster oscillates such role is played only in the long run of the evolution. The net flux of energy emitted will be reduced by inelastic two-body encounters, hence a small value of C may be more realistic. In a real star cluster the competition between heating, cooling, gravothermal and external effects, such as tidal heating, determines the net dynamical evolution and the final fate. The conditions to excite gravothermal oscillations are almost unaffected by

https://doi.org/10.1017/S0074180900147400 Published online by Cambridge University Press

the parameters of the model. In this sense the preliminary investigations are confronted with observations. If those clusters which have short relaxation times in their cores, follow an evolution prescribed by a small value of the heating efficiency C, it is comprehensible why no collapsed object is observed despite the core collapse time is shorter than the age of the cluster. The duration of the oscillation phase can be much longer than the time needed for its initial contraction, and even much longer than the age of the cluster. After a cluster passed through many cycles of core oscillations, its spatial structure is nearly that of SIS. The cluster's thermal relaxation is then well pronounced. The total amount of energy delivered by the central machine can add up to a significant fraction of the core's binding energy, either because of many energetically insignificant events (C small) or by one very efficient process (C large). As a result the observed brightness distribution of the inner halo oscillates around a 1/r profile. These oscillations around the average trend should be observable, and it seems that the theoretical prediction is met in NGC 6624 (cf. Djorgovski and King, 1983).

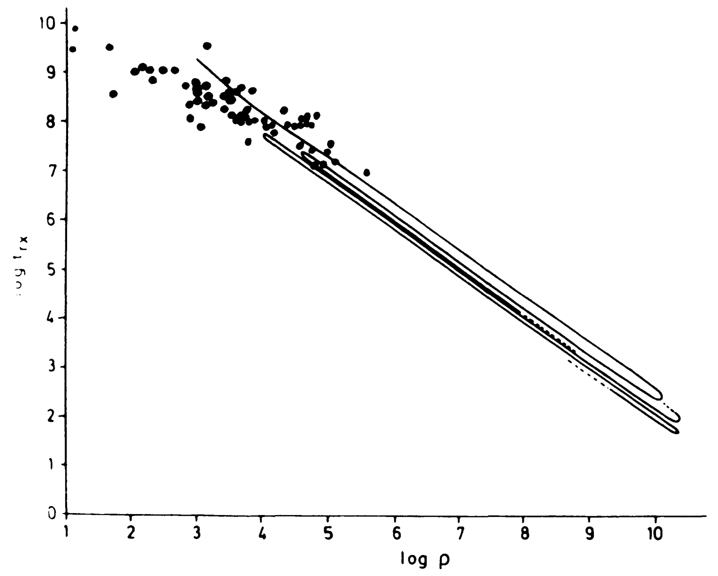

Fig.4. In the plane central relaxation time (years) vs. central density ($M_{\odot}/pc^3$) galactic globular clusters are represented by dots (data from Peterson and King, 1975). For the same model as in fig. 3 part of the evolution track is indicated.

https://doi.org/10.1017/S0074180900147400 Published online by Cambridge University Press

Recently Djorgovski and Penner (1984) found several new post-collapse clusters with a brightness distribution near to those of the singular isothermal sphere. But the total fraction of such clusters is estimated to be only a few per cent, and hence in conflict with the timing estimate based on thermally expanding models. Therefore, such models (C large) seem to be ruled out in the sense that most of the clusters with short relaxation times approach the singular isothermal sphere much more slowly, and hence evolve very different from the model's prediction.

Let us look at this problem from the following point of view. In the cases of thermal expansion (C large), either triggered by accretion onto a central black hole (cf. Bettwieser et al. 1984) or by hardening of binary stars, evolution is much slower in the post-collapse stage as compared with cases where C is small. In fact, for C large the timescale of evolution greatly exceeds the central relaxation time (cf. section 3.1 in Heggie 1984). Therefore such a post-collapse object tends to stay a long time far outside the range of parameters observed. Since tidal forces can remove the outermost stars with increasing ease we encounter the problem that at present there should be remnants of partly or completely dissolved globular clusters.

For the evolution driven by the gravothermal instabilities the ordinate of Fig.4, i.e. the central relaxation time, is the typical timescale of evolution of the central parts of the cluster. Note that the high density phases are too short to have a good chance to be observed. In low density phases the cluster's structure is not so much different from its appearance before the initial collapse. The same is true for the outer halo region. The evolution track in Fig.4 is for a case with C being small. If many clusters make a similar evolution the core densities in the observed ensemble of clusters will vary over many orders of magnitude, even if initially the densities would be confined within narrow limits. At the stage of lowest density the observation of an oscillating core would be most probable. At such stage the core mass depends but weakly on the choice of the initial model or the parameters of the energy input. The core mass as well as the energy fed during one cycle is adjusted by the system as a whole (cf. BS 1984; Bettwieser and Fritze 1984). Within rather broad limits such quantitative and integral aspects of the evolution are almost independent from the details of the model calculated. Concerning the quality of evolution, however, there is the important difference between the slow thermal expansion and the more rapid gravothermal oscillations. Since the fundamental instabilities drive the latter and since the energy source is used mainly to turn a switch, it is not surprising that we rediscover Hénon's conjecture about self-controlled post-collapse evolution.

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DISCUSSION

HUT: The Fokker-Planck post-core collapse calculations, which Haldan Cohn reported today did not show gravothermal oscillations. So you expect those to occur also in the Fokker-Planck approach?

BETTWIESER: The mechanism of gravothermal oscillations is not restricted to any specified model realization of a N-body system. Dr. Cohn discussed the class of those post-collapse solutions, which is generated by core heating rates above the upper limit to excite oscillations.

COHN: You said that with too large a normalization of the binary energy input rate the cluster simply expands rather than oscillates. How does your normalization compare with that used by Dr. Heggie?

BETTWIESER: Douglas Heggie uses normalized quantities, while our calculations are in physical terms.

HUT: Your N-body calculation using a few hundred particles in which you found oscillations must be usbject to large fluctuations in temperature, much larger than the temperature difference needed to drive gravathermal oscillations.

BETTWIESER: Concerning the 100-body calcaulation Daiichiro Sugimoto discussed briefly, I think there is a definite indication for core oscillations, on the other hand the level of fluctuations is too large to decide whether gravothermal effects play a role.

HEGGIE: (Addressed to Bettwieser & Sugimoto) I know you have studied the stability of the singular isothermal solution, but I gather from Dr. Sugimoto's question to me yesterday that you have also shown that the Inagaki & Lynden-Bell solution is unstable. Could you summarize your arguments and conclusion about the stability of the Inagaki & Lynden-Bell solution?

SUGIMOTO: I have not calculated stability of the I-L-B solution directly, because it is not an equilibrium solution to which the usual linearized stability analysis can be applied. Moreover, the I-L-B solution does not have any mechanism to control its energy generation rate. Therefore, I talked "stability" in the following senses: i) If the energy generation rate is not exactly equal in its intensity and time change to that required by the I-L-B solution, the expansion does not satisfy the I-L-B solution. ii) If the actual energy generation rate is appreciably lower than required, i.e., lower than heat conduction, the core will contract because the temperature is decreasing outwards and the state is in a stage of finite amplitude gravothermally contracting instability. These are self-evident from physics of the gravothermal catastrophe. We need not to perform any calculation for it. I-L-B's similarity solution is not a solution in equilibrium. In this sense it is difficult to apply concepts of the linearized stability theory. Discussion of stability in the frame of the expanding solution is not always impossible. However, in this particular case the expansion is as fast as the time scale of gravothermal expansion (instability). Therefore, this great amount of energy input controls the behavior of the system. Concerning this point, I have not developed any detailed calculation. We can ask in another way. Is the configuration gravothermally unstable or stable, if the energy input is turned off? This question is easily answered with certainty. Because the temperature is decreasing outward everywhere, the state corresponds to a gravothermal contraction of finite amplitude. It will therefore collapse. In I-L-B's similarity solution no physical mechanism is involved. Therefore, it is impossible to develop a stability analysis due to the associated changes in the energy generation rate.

APPLEGATE: It would be very useful if calculations presented in non-dimensional form would be related to physical units. This would help to determine if gravothermal oscillations last long enough to be observable and in comparing Dr. Bettwieser and Sugimoto's energy generation rate with Dr. Heggie's and Dr. Cohn's.

BETTWIESER: The fact, that there are several galactic globular clusters with rather short relaxation times in their cores is consistent with an interpretation, that these globulars are in post-collapse stages in between collapsed phases. The expanded phases of gravothermal oscillations can be described by standard King models.

SUGIMOTO: Concerning observational points, it is important to distinguish gravothermal expansion from thermal expansion. The gravothermal expansion takes place on the same time scale as the gravothermal contraction which is controlled by the relaxation time in the core. On the other hand the thermal expansion is controlled by the rate of energy input. The gravothermal expansion is much faster than the thermal expansion. Therefore, in the case of the gravothermal expansion and oscillation, most of the globular clusters would be observed in the expanded state without the density cusp. In the case of the thermal expansion, most of the globular clusters are in the slowly expanding state where the density cusp still exists in the central region. This difference may be used to decide observationally which expansion is the case in the real systems. The last figure shown by Bettwieser is in favor of the gravothermal oscillation.

INAGAKI: In your calculation if C is small, you have oscillations and if C is large, the core expands for ever. However, in both cases the collapsing phase is self-similar. I do not understand why the difference in energy generation rate causes the qualitative difference in evolution in self-similar stages.

SUGIMOTO: After the first beat of energy input the gravothermal expansion continues without any energy input from binaries. If C is too large, the energy input is so large that the system gets too much energy in the first expansion phase. Then the system will have so much energy that it is not a gravothermal system any more but a thermal system. We can see it clearly in the final phase of such expansion: As more energy is fed, the temperature finally begins to increase as in the case of a normal thermal system.