POLAR MOTION FROM ILS OBSERVATIONS WITH LEAST-SQUARES COLLOCATION

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ABSTRACT. Least-squares collocation is a powerful method for combining interpolation, filtering and parameter determination in one single computational step. We show that the method is applicable to the computation of polar motion values from a very large set of basic observational data. In this study, we use the ILS observations from 1900 to 1978.

1. INTRODUCTION

Least squares collocation has been extensively studied [1] and used [6] for the determination of the gravitational potential of the Earth. The method has a rather general statistical [2] and mathematical [3] foundation and can be applied to quite a number of geophysical problems (see [1], Chapter 18). In an attempt to apply the method to the problem of the computation of the polar motion coordinates (x,y) and other accompanying non-polar effects (instrumental constants, corrections to astronomical and other constants), we are faced with two major problems: a fundamental one, which is the determination of the covariance matrix of the observations and the corresponding covariance function of the polar motion, and a numerical one, which is the treatment of a huge number of data, represented as large sparse matrices.

In a first application [4], we presented a limited application to the observations of the main MERIT campaign. The limitation was expressed by the fact that the method was not applied to the original observations (latitudes, distances, etc) in the individual stations, but to the (x,y) coordinates, derived for each observation technique. The number of parameters was also limited to some mode parameters and origin biases between the different observation techniques. The numerical problems were therefore also restricted to the inversion of a moderately extended sparse matrix, which could be performed by standard techniques. In this application we want to go one step further and apply the method to the original ILS latitude observations [5].

2. COLLOCATION FORMULAS APPLIED TO ILS OBSERVATIONS

The observations we consider in this case are instantaneous latitudes observed with the

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VZT between 1900 and 1978. They are related to the coordinates of the pole of rotation, declination and proper motion corrections of the star pairs with the following relation [5]:

$$\phi_i^k - \phi_o = \Delta\phi_i^k = x(T_i)\cos\lambda_k + y(T_i)\sin\lambda_k + \Delta\delta_m + \Delta\mu_m(T_i - T_o)$$

for observation i of star pair m in station k. Other corrections (instrumental erros, astronomical constants) will not be considered in this application. Identifying this equation with the general form of the observation equation (see [1], Chapter 16)

$$z = A \cdot X + t + n$$

in least squares collocation, we must first define the signal s we want to determine. As in [4] we define s to be the continuously varying part of the observed quantity, in this case the set of polar coordinates (x,y). We can regard them as a vector or complex quantity. This signal is as such not fully present in a latitude observation, but only through its projection t on the direction of the observing station. But as t may be a linear function or functional of s (see [1], Chapter 16), we can apply the method on this example. We can express the transformation of the signal s to the observed signal t by the operator L:

$$t = L \cdot s = x\cos\lambda + y\sin\lambda$$

It is then possible to prove the following relations for the covariance matrices, needed to solve the problem:

$$C_{tt'} = C_{tt}\cos(\lambda - \lambda')$$

$$C_{tt} = C_{xx} = C_{yy}$$

$$C_{xt} = c_{tt}\cos\lambda$$

$$C_{yt} = C_{tt}\sin\lambda$$

$$x = C_{xt}C_{zz}^{-1}(z - AX)$$

$$y = C_{yt}C_{zz}^{-1}(z - AX)$$

The declination and proper motion corrections are included in the parameter vector X. The most important point remains the empirical determination of the covariance matrix C_{tt} of the observed signal t. This signal is composed of an oscillating part (Chandler and annual wobble) and some randomly distributed deviation, which we call the anomalous polar motion as in [4]. The covariance function of this signal will be dominated by the periodic part, so that the covariance matrices will be completely filled in. If we could remove this part from the signal and include it in the parameters of the observation equation, by modelling the periodic part with a number of suitable parameters, then the remaining signal will only be composed of the randomly generated deviations. Its covariance function will, as in [4], have a limited influence and will be representable by an analytical covariance function, characterized by so-called "essential parameters" (C_o = signal variance, ξ = correlation length, ξ = curvature parameter: see [1], chapter 22). The covariance matrices will be band-limited and this will reduce the numerical work considerably.

Modelling the periodic part with a drift and two periodic oscillations for this long period of 80 years has not been adequate due to changes in amplitude and phase of the Chandler motion. For this reason we decided to subtract a numerical model (a set of values x_o, y_o corresponding to a first approximation) from the observation equation. This is the usual procedure in linear least squares to obtain better numerical stability. Putting

$$(x, y) = (x_o, y_o) + (x_a, y_a)$$

the observation equation becomes then

$$\Delta\phi_i^k - x_o \cos\lambda - y_o \sin\lambda = x_a \cos\lambda + y_a \sin\lambda + \Delta\delta_m + \Delta\mu_m(T_i - T_o)$$

3. NUMERICAL MODELLING OF THE COVARIANCE FUNCTION

An empirical covariance function of the signal t can be determined by subtracting the model (x_o, y_o) and a first approximation of the corrections $\Delta\delta$ and $\Delta\mu$ (p.e. those published in [5]). A Gauss function is the simplest analytic covariance function for approximating the empirical one. We computed plots for these functions for each two year period and for each station. An example is shown in Figure 1 for the station Mizusawa. These plots include also the noise variance, so the we can separate signal covariance and noise variance. From these plots we could derive nearly constant values for the essential parameters of the gaussian covariance function. On the other hand the noise variance shows some slight increase during the 80 years observation. These values are (Figure 2.):

$$C_o = .004 \text{ arcsec}^2, \quad \xi = 30\text{days}$$

The correlation length is in good agreement with the one determined for the MERIT campaign [4], while the signal variance is much higher. This is probably due to the different modelling of the polar motion, which gives rise to larger deviations because of the longer time span used.

4. NUMERICAL SOLUTION OF THE EQUATION

The number of observations amounts to 750 000 while there are 281 star pair declination and proper motion corrections. The inversion of the covariance matrix is impossible however and a solution has been sought in the approximation as has already been suggested by Sunkel [7], but only for easier computing of very elaborate covariance functions. We have shown [8] that in this case it is also possible to invert the matrices formally and that the actual inversion reduces with a factor proportional to the length of the step. We used step sizes of 30, 20 and 10 days respectively. Using the step of 30 days, the size of the matrix reduces to 4945 with a maximum bandlength of 60. We do find then interpolated values for the anomalous polar motion (x_a, y_a), which must be added to the model (x_o, y_o) to obtain the final pole coordinates. In figures 3a–3c we show a plot of the pole coordinates in these three cases for a period of two years chosen out of the 80 years.

We obtain values for the corrections of star pair declinations and proper motions. A limited number of these are plotted in Figures 4a–4b against the values published by Yumi and Yokoyama [5]. Except for some star pairs which were not included in their results, there is a good agreement between both sets.

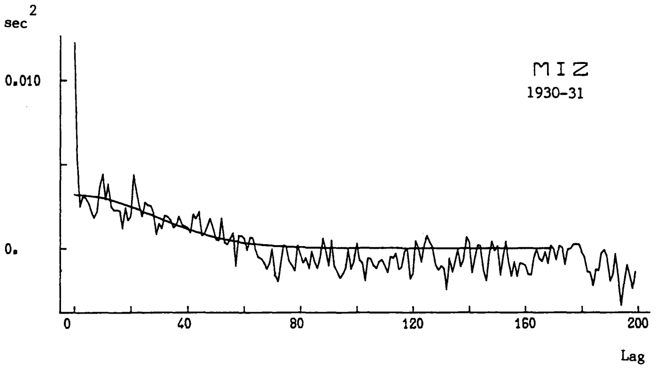

Fig. 1: Empirical and analytic covariance function

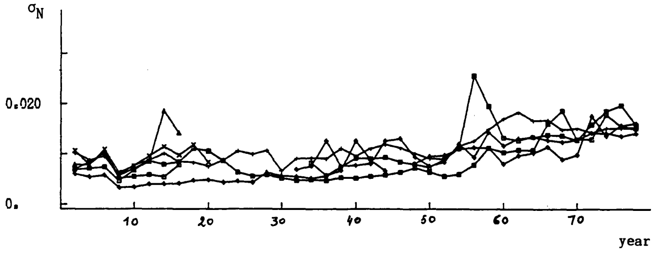

Fig. 2: Evolution of the noise variance

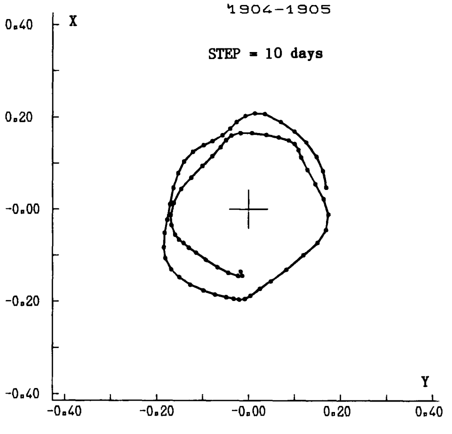

Fig. 3: Polar coordinates

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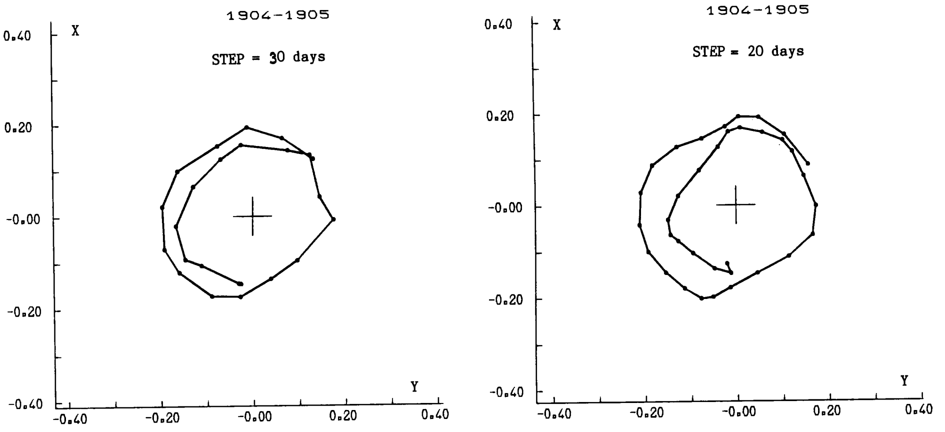

Fig. 3: Polar coordinates

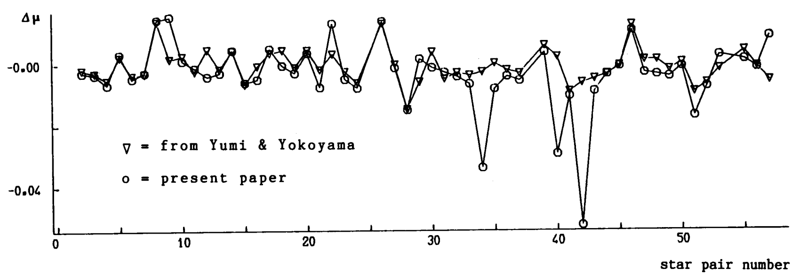

Fig. 4b: Proper motion corrections

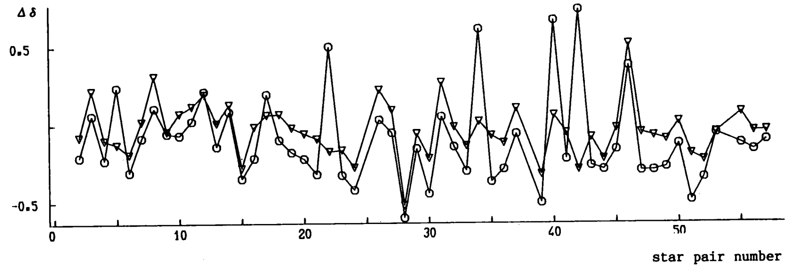

Fig. 4a: Declination corrections

5. CONCLUSIONS

We have been able to compute, in one single step, polar motion values at predefined epochs together with a number of corrections to observational parameters and to include a very large number of observations. In this way the interpolation problem is solved on a correct statistical basis, the true noise has been removed, and corrections to the observational constants are computed in one step, using all available data, instead of a chaining procedure where part of the corrections are lost or absorbed in other variables. A full account of the results will be given in a more elaborate paper.

The numerical difficulties for this size of problem have mostly been solved and we will direct our future attention to the determination of a better polar motion model (x_o, y_o) and a thorough investigation of the covariance function of the anomalous polar motion (x_a, y_a). We will also try to apply the method to a larger set of stations and more recent data, such as the IPMS and BIH latitude observations.

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