Using a Hand-held Calculator for Ship's Position Comparison

T. R. Meaden

THE author's personal experiences under operational conditions have been with the CBM (Commodore Business Machines) S.R. 4148R, which has proved quite adequate for position comparisons in a surface warship fitted with a Magnavox satellite navigation system primarily used as a DR recorder. It is fed with ship's course and speed, corrections to the positions found being up-dated from satellite passes. Occasionally the criteria for up-dating from a satellite pass are not met and the system may run on DR for several hours without up-dates. Since the system is unable to compensate automatically for tidal or current drift and set or for leeway, the accuracy of the DR will deteriorate with age. The possibility of component malfunction, albeit temporary, may also contribute to DR error. In such conditions a series of astronomical observations is made; the calculator is capable of finding the average of such a series if required.

The nearest DR printout from SAT NAV is noted and the time elapsed between DR and sight is used in conjunction with the speed to find the DR position at the time of the sight. The hand calculator can do this in one brisk operation, but for ease of explanation it will be treated as two distinct operations. The symbols enclosed in square brackets indicate the keying operations used on this calculator; the five normal functions (+, −, ×, ÷, and =) being common to most calculators are not boxed.

Operations 1 and 2:

1. Speed ÷ 60 × time elapsed = distance
2. Distance [x↔Y] Course [→R] D. lat
   [X↔Y] dep

Operation 2 utilizes the calculator's capability to change polar coordinates into rectangular coordinates; the inputs must always be in the order of distance followed by course. Since the calculator is capable of accepting course from 0–360° direct, the first readout will always be D. lat, positive for a northerly change and negative for a southerly change. The operation of the [X↔Y] key will now produce the departure but no sign is displayed. This information may, if required, be stored in the two available memory banks, D. lat in [Sto 1] and Departure in [Sto 2].

2(a) [Rcl 1] ÷ 60 = [Sto 1]
2(b) Lat° + (Lat′ ÷ 60) = [+ / −] (as required) [∑ 1]

Operation 2(a) cannot be performed as a continuation of operation 2 since any function operated between [→R] and [X↔Y] will destroy the departure figure. Operation 2(a) converts D. lat to decimals of a degree, whilst 2(b) converts the DR latitude into degrees and decimals and adds it algebraically (hence the [+ / −] as required) to the value in [Sto 1], the [∑ 1] key performing this operation.

2(c) [Rcl 1] [Cos] [1/x] × [Rcl 2] ÷ 60 = [Sto 2]
2(d) Long° + [(]Long′ ÷ 60[)] = [+ / −] (as required) [Xn]

Operation 2(c) changes Departure into D. long (D. long = dep sec lat). The result is decimalized and placed in [Sto. 2]. Since the amount of D. lat and Departure is usually very small, the error involved in not using middle latitude is negligible. Operation 2(d) decimalizes the DR longitude and adds it algebraically to the D. long.

At this point the contents of the memory banks must be noted, since the storage facilities are now required for further operations. This is one of the reasons why it has not been found necessary to perform the operations with suffixes. Normally it is sufficient to record the D. lat and Dep. (from Operation 2) on a pro-forma sheet and then apply D. lat to the DR latitude to find the latitude of sight, and decimalize:

3. Lat° + [(] Lat′ ÷ 60[)] =

Note this reading and then perform operation 4, which is a continuation of 3, to convert Departure into D. long.

4. = [cos] [1/x] × Dep =

.The D. long is applied to the DR longitude on the pro-forma sheet to find the longitude of the sight. This value is recorded but not decimalized.

Finding the GHA and declination of the body is done in the usual way on the pro-forma sheet, or it can be done on the calculator. The decimalization of the LHA and declination are performed as in operation 3 and the results noted. The pro-forma sheet now shows the decimalized forms of latitude, LHA and Declination.

The calculator is now used to find the computed altitude and azimuth at the DR position of the sight from the formulae:

$$\sin H_c = \cos \text{lat} \cos \text{dec} \cos \text{LHA} \pm \sin \text{lat} \sin \text{dec}$$
$$(\text{with } + \text{ for same and } - \text{ for contrary lat and dec})$$
$$\cot Z = \{\cos \text{lat} \tan \text{dec} \pm \cos \text{LHA} \sin \text{lat}\}/\sin \text{LHA}$$
$$(\text{with } - \text{ for same and } + \text{ for contrary lat and dec})$$

This is done through the following operations, which may be considered as one long chain operation:

5. lat [cos] [Sto 1]
6. dec [cos] × [Rcl 1] = [Sto 1]
7. LHA [cos] × [Rcl 1] = [Sto 1]
8. lat [sin] [Sto 2]
9. dec [sin] × [Rcl 2] = [Sto 2]
10. [Rcl 1] ± [Rcl 2] = [ARC] [sin]

which gives the computed altitude in decimalized form. This is converted to degrees and minutes:

11. $H_1 - H_c{}^\circ = \times 60 =$

The result is noted on the pro-forma sheet.

The azimuth is found from a similar chain operation:

12. lat [cos] [Sto 1]
13. dec [tan] × [Rcl 1] = [Sto 1]
14. LHA [cos] [Sto 2]
15. lat [sin] × [Rcl 2] = [Sto 2]
16. [Rcl 1] ± [Rcl 2] = [Sto 1]
17. LHA [sin] [1/x] × [Rcl 1] = [1/x] [ARC] [tan]

The value found will be in a decimalized form between 0–90° and the true azimuth must be either 360 ±Z or 180 ±Z as common sense requires.

The sextant altitude is now corrected in accordance with normal practice and compared with the computed altitude. Depending on the conditions under which the sight was taken, limits are imposed on the intercept in judging the acceptability of the DR position. Given very good all-round conditions, a limit of $< 01'$ is imposed and with poor conditions a limit of $< 03'$. These limits may sound tight but with experienced observers they have been found fairly generous on average.

There are several good reasons for using a pro-forma sheet instead of performing the whole computation by computer. The first is that seldom can the operation be performed without some interruption. Secondly the calculator has not enough storage facilities for all the required variables. Thirdly, the pro-forma sheet provides both a check on the progress of the computation and a check for eliminating mistakes.

The method described is particularly suited to Sun and Moon sights but multiple star sights have been found unduly cumbersome and are best plotted by the old and trusted method on a plotting sheet, allowing for runs.

Error Distribution in Navigation

O. D. Anderson

In an earlier paper the present writer drew attention to the distinction between mixed and aggregated distributions. Incidentally, two minor mis-statements in that paper should be corrected. The first sentence in the last paragraph of section 2 (page 72) should read 'As σ_2/σ_1 ($=k$, say) tends to either zero or infinity, X/σ increases indefinitely; whilst for k nearer unity, X/σ is smaller'. Also the last line of the Appendix should not terminate with '$X/\sigma = 3{\cdot}724$' but with '$X/\sigma \to \infty$'.

Mixed distributions arise when for example an error occurs either from one distribution with probability p, or from another with probability $1-p$, as in a set of position line errors due sometimes to one and sometimes to the other of a pair of observers of differing precision. An aggregate distribution occurs when the error consists of, say, two sub-errors, one from each of the two distributions; as when Captain Flint marks the chart and Long John subsequently reads it.

It has been suggested to the writer that the analysis for the mixed situation is all very well, but only the very special case of mixing *equal* numbers of observations from just *two* guassian distributions was considered. We here offer a rather more general theory which may be of wider interest, as for instance in quality control in a factory where a product might have been made on either a new or an older machine.

In many empirical situations it is found that observed distributions, though unimodal and symmetric, have sharper peaks and higher tails than the same-variance normal distribution. Indeed this has sometimes led to analysts playing safe and making a laplacian (two sided negative exponential) rather than a gaussian assumption. This non-normal behaviour can however be reconciled to a