Now $a_1 + a_2 + \dots + a_n > b_1 + b_2 + \dots + b_{n-1} + G$

if $a_1 + a_2 + \dots + a_n > b + a_2 + a_3 + \dots + a_{n-1} + G$ (by (3)).

i.e. if $a_1 + a_n > b + G$,

i.e. if $G(a_1 + a_n) > a_1 a_n + G^2$ by (2),

i.e. if $0 > (G - a_1)(G - a_n)$. (14)

But
$$G \equiv (a_1 a_2 \dots a_n)^{\frac{1}{n}}$$
$$< (a_n a_n \dots a_n)^{\frac{1}{n}} \quad (i.e.\ a_n)$$
($\because$ a_n is the greatest),

and
$$> (a_1 a_1 \dots a_1)^{\frac{1}{n}} \quad i.e.\ a_1$$
($\because$ a_1 is the least).

$\therefore$ $G - a_1$ is positive, and $G - a_n$ is negative, *i.e.* $(G - a_1)(G - a_n)$ is negative, which proves the theorem.

Thus
$$a_1 + a_2 + \dots + a_n$$
$$> b_1 + b_2 + \dots + b_{n-1} + G$$
$$>> c_1 + c_2 + \dots + c_{n-2} + G + G$$
$$>>> d_1 + d_2 + \dots + d_{n-3} + G + G + G$$
....

i.e. $> nG$.

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Additional "Note on Right-Angled Triangles."— On pp. 95 and 96 of No. 8 (October 1911) of *Mathematical Notes* is given a numerical method of finding rational right-angled triangles.

It has been known for centuries that
$$p^2 - q^2,\ 2pq,\ p^2 + q^2 \qquad \text{(A)}$$
are the sides of a rational right-angled triangle whatever be the values of p and q; for
$$(p^2 - q^2)^2 + (2pq)^2 = p^4 - 2p^2q^2 + q^4 + 4p^2q^2 = p^4 + 2p^2q^2 + q^4 = (p^2 + q^2)^2.$$

Take $p = 2$, $q = 1$, and we have the "hackneyed" triangle whose sides are 3, 4, 5.

Take $p = 3$, $q = 2$; then the triangle is 5, 12, 13.

Take $p = 4$, $q = 1$, and we have 8, 15, and 17.

When $p = 4$, $q = 3$, the sides are 7, 24, 25.

The sides will have no common divisor when p and q are prime to each other, one odd and the other even, in which case the above expressions in (A) will give all possible prime rational right-angled triangles. (See Table of Prime Rational Right-Angled Triangles, *Mathematical Magazine*, Vol. II., No. 12, pp. 301–308.

By imposing certain restrictions upon p and q, we can obtain general expressions for particular classes of triangles.

(1). Let $p = q + 1$, and the expressions in (A) become

$$(q+1)^2 - q^2, \quad 2q(q+1), \quad (q+1)^2 + q^2;$$

or

$$2q+1, \quad 2q^2+2q, \quad 2q^2+2q+1. \qquad \text{(B)}$$

Whatever be the value of q in (B), the hypotenuse will exceed the longer leg by unity.

Examples.—1. Take $q = 1$, and we have 3, 4, 5, the sides of the "hackneyed one."

2. If $q = 2$, the sides are 5, 12, 13.
3. Take $q = 3$, and we have 7, 24, 25.
4. Let $q = 4$, then we have 9, 40, 41.

(B) will give all possible rational integral-sided right-angled triangles having consecutive numbers for hypotenuse and longer leg.

(2). Let $q = 1$ in (A), and the expressions for the sides become

$$2p, \ p^2 - 1, \ p^2 + 1. \qquad \text{(C)}$$

In this case the difference between the hypotenuse and one leg is always 2 if p is an even number.

Examples.—5. Let $p = 2$, and again we get the "hackneyed" triangle whose sides are 4, 3, 5.

6. Take $p = 4$, then the sides are 8, 15, 17.
7. If $p = 6$, the sides are 12, 35, 37.
8. Let $p = 8$, and we have 16, 63, 65 for the sides.

(3). To find right-angled triangles whose legs differ by unity or are consecutive numbers.

Take $p_1 = 2$, $q_1 = 1$ and we have once more the "hackneyed" triangle 3, 4, 5.

Take $p_2 = 5$, $q_2 = 2$, and we have the sides 20, 21, 29.

Take $p_3 = 12$, $q_3 = 5$, and we have the sides 119, 120, 169.

In *general*, if

$$p_{n+1} = 2p_n + p_{n-1} \text{ and } q_{n+1} = p_n, \qquad \text{(D)}$$

the legs of the triangle will be consecutive numbers. (See *Mathematical Magazine*, Vol. II., No. 12, pp. 315–19, and pp. 322–324).

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