By repeating indefinitely this process we arrive at the fixed diameter through P as the limiting chord. Hence the theorem is true generally.

The proof is applicable unchanged to the case of secants if the sign convention is admitted, and with simple modification if it is not.

G. D. C. STOKES.

On Recurring Decimals.

§ 1. The present note gives two methods by which vulgar fractions may often be converted into decimals with great rapidity. It is not pretended, however, that they are more than mere curiosities or that they are of practical use. The first method is not new, but I have never seen the second mentioned anywhere. The proofs are almost self-evident.

§ 2. If the vulgar fraction have its numerator and denominator multiplied, if necessary, by the same factor so that the significant digits of the denominator become $m \cdot 10^n - 1$, where m is an integer so small that we can perform short division by it mentally, then the complete recurring period can be written straight down with ease. The best description will be provided by an example.

Ex. To convert into a recurring decimal $\frac{53}{87}$.

We have $87 \times 45977 = 3999999 = 4 \times 10^6 - 1$,

$53 \times 45977 = 2436781.$

(It will be shown later how we find the multiplier 45977.)

$$\therefore \quad \frac{53}{87} = \frac{2436781}{3999999}.$$

Write down the numerator, 2436781, and divide it by 4. When six digits have been obtained (six, because of the 10^6), transfer the quotient to a new row just below, continuing division of the row just quitted, "carrying" of course any remainder from the first row. When this new row of the quotient has six digits begin another below it again, and continue this process until we obtain the digits 436781, the last six digits of the original numerator. The complete period has then been obtained.

$$\begin{array}{r|l} 4 & \underline{2436781} \\ & \cdot 609195 \\ & 402298 \\ & 850574 \qquad \therefore \tfrac{53}{87} = \cdot\dot{6}09195402298850574712 \\ & 712643 \qquad\qquad\qquad\qquad\quad 643678\dot{1} \\ & 6781 \end{array}$$

Ex. II. Write down the recurring decimal for $\frac{1}{97}$, given that $97 \times 6185567 = 6 \times 10^8 - 1$.

§ 3. The question at once arises: how is a suitable multiplier obtained in any particular case? The process is very simple, and will be shown for the above example, where the denominator is 87.

Since the product is to end in 9, the units digit of the multiplier is 7. We have then

$$\begin{array}{r} 87 \\ 7 \\ \hline \end{array}$$

$609 = 610 - 1$, but since 61 is not a convenient divisior we must try again, and make the tens digit of the product also 9 by adding 9 to the 0. Thus.

$$\begin{array}{r} 87 \\ 77 \\ \hline 609 \\ 609 \\ \hline \end{array}$$

$6699 = 6700 - 1$, but 67 is not convenient. Hence we make the hundreds digit 9 by adding 3 to the 6. Thus

$$\begin{array}{r} 87 \\ 977 \\ \hline 609 \\ 609 \\ 783 \\ \hline \end{array}$$

$84999 = 85000 - 1$, but 85 is not convenient.

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At length we find

$$\begin{array}{r}
87\\
45977\\
\hline
609\\
609\\
783\\
435\\
348\\
\hline
3999999
\end{array} = 4000000 - 1.$$

In practice the method is much more rapid than appears from the description, for we do not need to evaluate the whole product at each trial.

§ 4. As might be expected, there is a somewhat similar method for converting a vulgar fraction into a decimal by making the denominator $m \cdot 10^n + 1$.

Ex. To convert into a recurring decimal

We have $87 \times 23 = 2001 = 2 \times 10^3 + 1$

$53 \times 23 = 1219.$

$$\therefore \quad \frac{53}{87} = \frac{1219}{2001}$$

Write down the numerator *minus unity*, 1218, and divide it by 2. When three digits have been obtained (three, because of the 10^3), write below each of them the digit that with it adds up to 9; bracket this new row, and continue division by 2 of this *bracketed* row. Below the quotient obtained write a similar bracketed row, and continue division of it, and so on. The process is continued until we obtain the quotient 781, the digits which with the three last digits of 1218 make 999. The decimal is then read off from the unbracketed rows.

1218			
·609	(597)	574	(356)
(390)	298	(425)	678
195	(701)	712	(321)
(804)	850	(287)	1
402	(149)	643	

$\therefore \; \frac{53}{87} = \cdot\dot{6}09195402298850574712643678\dot{1}$, agreeing with the previous result.

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