ADDRESS ON RELATIVITY.

By A. S. Eddington.

It is a familiar principle in elementary mechanics that a uniform motion of the whole system under consideration makes no difference to the phenomena, and may be ignored. For example, in calculating the motions of the planets round the sun, we do not need to pay any attention to the fact that the whole solar system is travelling towards the constellation Lyra. It does not seem to us that there is anything surprising or needing explanation in this principle; in fact, when we try to examine the idea of motion through an empty space without fixed landmarks, we are conscious of something illusory in the conception.

Einstein's principle of relativity of 1905 reiterates and emphasises this principle—that uniform motion of the whole system can be entirely ignored. But it arouses opposition because, since Newton's time, we have filled space with an aether which is the seat of all electrical and optical phenomena. The "whole system" is now the system of material bodies plus the aether; and it would seem that we can no longer give an arbitrary uniform motion to the whole system, because, whatever we do to the material bodies, the aether will slip through the interstices and remain as it was. In short, it is claimed that the aether provides the fixed landmarks which were lacking before. It is no longer a question of travelling through void—a conception to which it is difficult to attach any meaning—but of travelling through an ocean of aether.

But now comes the bombshell of experiment, to shatter the position gained by this nineteenth century offensive. Simply, as a matter of hard empirical fact, the aether does *not* provide the fixed landmarks that we imagined. The experimental test (the Michelson-Morley experiment and others) is one of the most interesting parts of this subject; but I have not time to enter into a discussion of it. It is sufficient to say that these experiments quite unexpectedly refused to give an answer to the question, what is our motion through the aether? If aether-landmarks exist, there is no known way of finding them out.

Perhaps we ought not to build too much on the experiments alone. You cannot very well prove a universal negative by experiment, or feel certain that your successors will always fail to find this aether-motion. But the experiments suggested a re-examination of the theory. Fortunately there is no disagreement about the theoretical laws of the aether; they were laid down many years ago by Maxwell in his famous set of equations. From time to time, in solving a mathematical equation, one meets with a rather annoying experience. The reduction and simplification goes on excellently; various complicated terms cancel out; and at length, just as you think you are going to arrive at the desired value of x, behold! everything disappears, and you are left with the profound but irritating truth that $0=0$. It is always like that with velocity through the aether. You form the equations relating to some actual or hypothetical experiment, and v the velocity of the apparatus through the aether duly appears in them. Then you eliminate the various unknowns, trying to isolate v. But it is a fiasco; v disappears as well, and steadily refuses to be equal to anything in particular. It is capable of strict mathematical proof that Maxwell's equations are of such a form that this must always be the result.

You will see then that both theory and experiment agree that motion through the aether is undetectable. Velocity through the aether is like that elusive personage Mrs. Harris; and Einstein has inspired us with a daring scepticism—"I don't believe there's no sich a person." It is this disbelief *

* Note that we do not reject the aether, but only velocity with respect to the aether; that is to say, the nature of the aether is such that it does not provide a standard with respect to which a velocity can be measured.

which starts us on the way to revolutionise the physical ideas of space and time.

Now, if I may pursue the simile, Mrs. Harris's adherents are rather indignant with us; it is too bad to say the lady is a myth merely because they are unable to produce her. They are determined to bring her in somehow, and they point with justifiable pride to the scheme suggested by FitzGerald and elaborated by Larmor and Lorentz, which supposes that there is a definite velocity of the earth through the aether, but compensations have been arranged by nature so that under no circumstances can this velocity ever make any difference to anything. You probably know the kind of compensations required—contraction of all standards of length, change of rate of a moving clock, increase of mass with velocity, etc. Now let us frankly admit that this scheme of nature is a tenable one. But to the relativist, its fault is that it is like the schemes of the White Knight; you remember the verse:

But I was thinking of a plan
To dye one's whiskers green,
And always use so large a fan
That they should not be seen.

Why go out of your way to introduce this hypothetical thing, velocity through the aether, and then make elaborate arrangements to keep it out of sight? Of course, in the old days when the aether was regarded as a rarefied kind of matter, it was necessary to suppose that it must have some definite velocity. But few people nowadays believe that the aether is material in the ordinary sense; and all reason for imagining a velocity relative to it as a fixed framework has disappeared.

Suppose that far away in space we had an exact replica of the solar system, travelling uniformly at a thousand miles a second relative to the actual solar system. Newtonian dynamics tells us that the motion makes no difference—that the replica would remain perfect, and the same sequence of eclipses, occultations, conjunctions, etc., would take place in it as with us. In fact, the same Nautical Almanac would serve for both. The experimental and theoretical results that we have been discussing, confirm this; and they tell us that this is true not only for the mechanical part of the phenomena, but also for electrical and optical phenomena. In the behaviour of the system there is nothing to give away that thousand miles a second velocity; it could only be detected by observing some material landmark, *e.g.* another star outside the system.

Now any point x, y, z, in the original sytem at an instant t, has an exact counterpart x', y', z' in the replica at an instant t'. You know that in ordinary mechanics there is a relation between these corresponding points and instants, viz.,

$$x' = x + ut, \quad y' = y, \quad z' = z, \quad t' = t,$$

where u is the velocity of the replica, supposed along Ox.

We now find that these are only approximate formulae, and the true relations are

$$x' = \beta(x + ut), \quad y' = y, \quad z' = z, \quad t' = \beta(t + ux/c^2),$$

where c is the velocity of light, and $\beta = (1 - u^2/c^2)^{-\frac{1}{2}}$.

These relations mix up space and time in a remarkable way. Instead of $t = t'$, *i.e.* one uniform absolute time respected by all systems, we have

$$t' = \beta(t + ux/c^2),$$

i.e. a different time for the moving system, related not only to time but also to space in the fixed system. Let us be quite clear what we mean by time in the two systems. We set an astronomer in each system to determine by means of his telescope, Nautical Almanac, etc., the time of an event visible to both. Our own astronomer finds the time to be t, the astronomer in the other system finds it to be t'; t is not equal to t', nor do they differ merely in the zero from which time is reckoned, but are connected by the complicated relation given above.

We are now ready for the next step. We see that, instead of our time-reckoning being unique and universal, the inhabitant of another star would disregard it and employ an altogether different reckoning. The present instant "NOW!" seems to us to run through space, like a clean section through the changing world, marking an instantaneous state. The changing world is thus analysed into a succession of instantaneous states. I want to persuade you that this is nothing more than a mathematical analysis—sometimes useful, sometimes mischievous—it is not the natural taking to pieces of a puzzle, but a rude hacking through with the carving knife. The instant "Now," so far as I am aware of it, exists just where I am; there is no reason to picture it as extending into other parts of space where I am not.* It is true that if I am excited watching a football match, I may imagine myself among the players, carrying my "Now" with me; but that is poetry, not science. Cool science tells me that what I see happen now *did not happen now*; the impression has travelled to me with a finite velocity. Science—even of the most old-fashioned type—will have none of my instinctive extension of the instant "Now"; in place of it it has constructed an extension defined according to mathematical rules, which gives us the world-wide instant used in astronomy, physics, and to some extent in ordinary affairs of life. But this agreed continuation is a mathematical construction, fixed up by a caucus of astronomers, who agreed chiefly because they had not sufficient imagination to differ. When I say that it is agreed, I can answer only for terrestrial astronomers; the astronomers on that replica of the solar system we have been speaking of will profoundly disagree. They (if they are replicas of astronomers here) have adopted a different continuation of the instant "Now"—because of their different motion. Two events, which we call simultaneous, may be 100 years apart in their reckoning.

Our world-wide instant "Now" is in no way better than theirs; it is a despicable mathematical fiction convenient for us, as theirs is for them. To say that the world consists of a succession of instantaneous states means just as much and as little as to say that a piece of bacon consists of rashers. The rashers are not predetermined, and until the knife has been used we can only contemplate the solid block of bacon. The rashers can be cut in different directions, and so too the different observers cut the changing world in different directions to obtain the slices which they call instantaneous states. Until we fix the observer who is to make the dissection, we can only contemplate the four-dimensional block of undivided space-time.

I can only touch on one or two outstanding points, and will now pass on to what is called General Relativity. We progress by considering how nature would appear regarded from the point of view of some one differently, and perhaps better, situated than ourselves. Thus Copernicus revolutionised astronomy by considering how the solar system would appear seen from the sun. We have just now reached the four-dimensional world of space-time, by considering the point of view of observers in very rapid motion relative to us. For the next step, I must ask you to adopt the point of view of a man who has fallen out of an aeroplane.

Now, of course, it is easy to say that that is a ridiculous and unnatural situation to contemplate the world from, and to treat it as a joke. What a humorous old fellow Copernicus was when he told us to take the point of

* Some complication is introduced by the fact that the word "Now" may refer not to an exact instant of time, but to a period of time which, though short, is not infinitesimal. The instant in New York which is simultaneous with the instant here is indeterminate (except by arbitrary mathematical convention); but there are limits to the ambiguity, and we can fix on a period of about $\frac{1}{30}$ sec. in New York, which certainly overlaps the instant "Now" here. As we go further away the limits widen. Thus the lover who says to himself "She is thinking of me *now*" need not be troubled by the ambiguity, for her thought would certainly endure $\frac{1}{30}$ of a second. But if the absent one were on the planet Neptune (where the ambiguity owing to the increased distance amounts to about four hours), he would be deprived of this consolation.

view of a man sitting frizzling in the sun, instead of remaining on this solid immovable earth ! Need we take him seriously ? Let me explain why I think the falling aviator is better situated than we are. My body is a delicate piece of apparatus for recording observations of nature ; but unfortunately the soles of my boots are being battered by the molecules of the ground all the time with a force equal to some ten stone weight. (Action and reaction, you remember, are equal and opposite.) Now, if you were using some delicate galvanometer, would you willingly allow someone to hammer on it like that ? To put it mildly it would complicate things a great deal. So I propose that we should get rid of this molecular bombardment by jumping out of an aeroplane, so that for a little while we may be quite undisturbed in our investigations.

Newton contemplated the falling apple and produced his famous theory of gravitation. But had it fallen to the lot of the apple to produce the theory of gravitation, I think the result would have been something like Einstein's. At least the story goes that Einstein was inspired with the principles of his theory by talking to a man who had fallen from a third-floor window.

Let our falling aviator perform the experiment of dropping an apple which he was holding in his hand. The apple cannot fall any more than it was doing already, and remains in contact with his hand—at rest, as the aviator would say. He has no knowledge of any field of force tending to accelerate the motion of unsupported objects ; he reports that things at rest remain at rest, and things in uniform motion remain in uniform motion. No doubt he notices a tree coming faster and faster to meet him ; but the reason is obvious —it is not moving freely, but is being shoved up from below by molecular bombardment. For the falling aviator the field of gravitational force, which has so puzzled us, has vanished.

A new point of view is not in itself a new theory, though it may lead to one ; but, when we are seeking to understand the nature of gravitation, it is something to have shown that two people may disagree as to whether in any region there exists a gravitational field to be explained. I am not so much suggesting that the gravitational field is an illusion, as that it is slippery ; and in any picture we try to form, we must somehow recognise this slippery character. Suppose that, instead of watching things close at hand, the aviator looks further afield. If he looks through the earth to the antipodes, he finds free bodies there are falling towards him, not merely at 32 feet/sec^2., but at 64 feet/sec^2. By his own fall he has abolished gravitation near him, only to pile it up elsewhere—as though he had smoothed out a pucker at one point and it had run off somewhere else, as puckers do. May we not picture the field as a pucker in the world ? You may say that the pucker is only an analogy ; but, after all, what is any description we can give of the things around us but an analogy of those inscrutable processes of nature, which necessarily must transcend the pictures of our senses ? Our old picture of a force on the falling body—something tugging at it—is only an anthropomorphic analogy. If on examination the pucker is a good analogy, if we can discover no point where the analogy breaks down, if we can press it even further than our old analogy of a field of force, so that it explains things which the force does not—then it seems to me we are bound to remodel our theory of nature according to the new outlook which it gives.

The theory of non-Euclidean or puckered space-time has all these advantages. Needless to say I cannot here give any account of that wonderful mathematical analysis by which Einstein followed up the initial idea. I venture to say that there has never been anything to equal it in the history of mathematical physics.

In Euclidean geometry the fundamental locus is the straight line. In non-Euclidean geometry the corresponding fundamental locus is called the geodesic —it is the analogue of the straight line, but you must not call it a straight line,

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because it has not the familiar properties of one. We can construct a non-Euclidean geometry associated with the solar system which has the following property—that the tracks of the planets round the sun (both the planets which exist, and planets which might, but do not, exist) are the geodesics of the new geometry. This geometry is four-dimensional, so that it gives correctly not only the tracks of the planets through space, but their times in these tracks. This non-Euclidean geometry then is a way of describing the sun's gravitational field; I do not at the moment commit myself to saying that it is the geometry of space in the solar system in the ordinary sense. I will rather speak of it as a correlated geometry—correlated so that the geodesics in this geometry coincide with the tracks of particles in the gravitational field. Now the phrase non-Euclidean is, of course, merely a negative epithet; the geometry is not Euclidean, it is of some other kind, and we must find out what kind. We can make a few broad classifications of non-Euclidean geometries, and it turns out that the geometry correlated with a gravitational field is always of one of these fundamental classes, namely, that of a space curved "no higher than the first degree." Never mind what "the first degree" means. The point is that the mathematician can recognise the geometry alluded to, and if we tell him that the geometry correlated to the region round the sun is that of a space curved no higher than the first degree, he can immediately calculate the orbit of any planet or comet just as well as from Newton's statement about the inverse square. The statement is not quite so concise as Newton's, but I think the underlying idea is almost more concise. And as regards accuracy, the two statements have as you know been put to the test, and Einstein's comes off best, with Newton's *proxime accessit.*

There are some who would like to stop at this point. I think, for example, Prof. Whitehead would say—"Einstein's method of summarising mechanical laws by describing a correlated geometry is ingenious; it may seem a little outlandish, but if you examine any advanced text-book of mechanics (Prof. Whittaker's for instance) you will find the same kind of process. It is a fashion due mainly to Hertz. But do not go mixing up this *correlated* geometry with the *actual* geometry of space in the solar system." But that is just what Einstein insists on doing, and if you stop short of this identification of the correlated and the actual geometry—the geometry of mechanics and the geometry of extension—you miss the most beautiful part of the idea. The so-called mechanical problem of observing the course of a moving particle, and the so-called geometrical problem of observing the indications of a measuring scale are not to be separated into water-tight compartments. One law can be formulated which covers all the mechanics of motion and the geometry of measurement. It is this unification which constitutes the great advance of Einstein's theory.

I have rashly undertaken to say something about the application of these new ideas to elementary mathematical teaching. Let me say at once that I have no pressing reforms to advocate nor any definite policy to urge. I am convinced that the relativity outlook will in time lose its strangeness and become a commonplace of educated thought, just as the Copernican system has done. It is clear too that the relativity outlook presents less difficulty to the younger minds than to the older, so that there is good reason for introducing it at an early stage. But its introduction into the school must be a matter of atmosphere rather than of specific teaching. It would be unwise for one who has no experience of your problems of teaching mathematics, to attempt to suggest the precise opportunities for spreading this "Bolshevist atmosphere" among youthful minds. I can only speak in general terms.

The emphasis on the practical and experimental side of geometry, which is, I suppose, now generally admitted in schools, is a help in the right direction. I sometimes think how surprised and gratified our old mathematical masters must be nowadays; how many stalwarts do we not find now defending

Euclidean geometry as a necessity of thought, who certainly did not seem to feel any overwhelming necessity of that kind in their schooldays! But I think the belief that we can prove by internal cogitation that things must necessarily be as they are, is obsolescent; and now, when we discover that cardboard triangles on the same base and between the same parallels do actually turn out to be equal in area, our attitude is that described by Einstein as of "thankful surprise"—that nature should conform to so simple a rule when things might have been much more complicated. By theoretical geometry we bind together these experimental results, just as by mathematical physics we bind together the results of experimental optics. But we seek to learn humbly from nature what actually are the laws she has chosen, whether they be of space or of light; we do not invent a code out of our heads, and call on nature to conform to it, and blame the materials she supplies when it appears to go wrong. Also we can now perceive clearly what is the subject-matter of geometry. What is the subject of the experiments on which your schoolboys are instructed to base it? Cardboard, compasses, divided scales—in short, matter of some kind. Laws of geometry are laws relating to matter,—in particular to that special property of matter known as *extension.* It is a harmless abstraction to imagine all the other qualities of matter removed, leaving only its extension; but we realise that school-geometry deals with an extension which presupposes matter. Dismiss the absurd notion that your experiments are teaching you the properties of emptiness.

Mechanics, I fear, presents difficulties. For example, we now know that the law of the parallelogram of velocities is untrue; and any argument which seeks to establish it begs the question. But I am inclined to think that that may perhaps have to be regarded as a professional secret, scrupulously guarded from those of tender years. With regard to force, mass and inertia, I am inclined to think that general relativity may even relieve difficulty. If any one is contemplating writing a text-book on elementary mechanics, I would strongly recommend him to examine the relativity idea of these things; and, if practicable, to strike out rather boldly. But the detailed application requires much careful thought. The golden rule is—when in difficulty, appeal to experiment. Do not always think it necessary to give a reason why nature conforms to certain fundamental laws. Nature no doubt has her reasons; but they will appear (if at all) in the last chapters of some profound treatise on electromagnetic theory, not in the initial pages of a school text-book.

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In the discussion that followed, Professor E. T. Whittaker, Dr. Sheppard, Mr. C. Godfrey, Col. Alan Cunningham, Mr. C. V. Durell, and Dr. Brodetsky took part.

The next paper—by Dr. Brodetsky—is printed as March *Gazette*, No. 152.

GLEANINGS FAR AND NEAR.

70. The Educational Value of Experiment.

"It may be said that the fact makes a stronger impression on the boy through the medium of his sight, that he believes it the more confidently. I say that this ought not to be the case. If he does not believe the statements of his tutor, probably a clergyman of mature knowledge, recognised ability, and blameless character—his suspicion is irrational, and manifests a want of the power of appreciating evidence, a want fatal to his success in that branch of science which he is supposed to be cultivating."—Todhunter's *The Conflict of Studies*, 1873, p. 17.

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