FORUM

An Abacus for Windward Performance

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THE prudence of determining the best windward and running courses for sailing yachts can be seen from Fig. 1 in which the heart shaped curve represents a polar diagram of yacht performance with respect to the true wind direction. It is a diagram of yacht velocity V and the angle α between the yacht's track and the true wind direction. The gap in the curve dead to windward is an area over which no performance is possible, whereas any destination lying outside this zone can be reached directly; there is only one sailing angle at which a destination lying between X and Y can be reached the quickest. A similar situation occurs on the
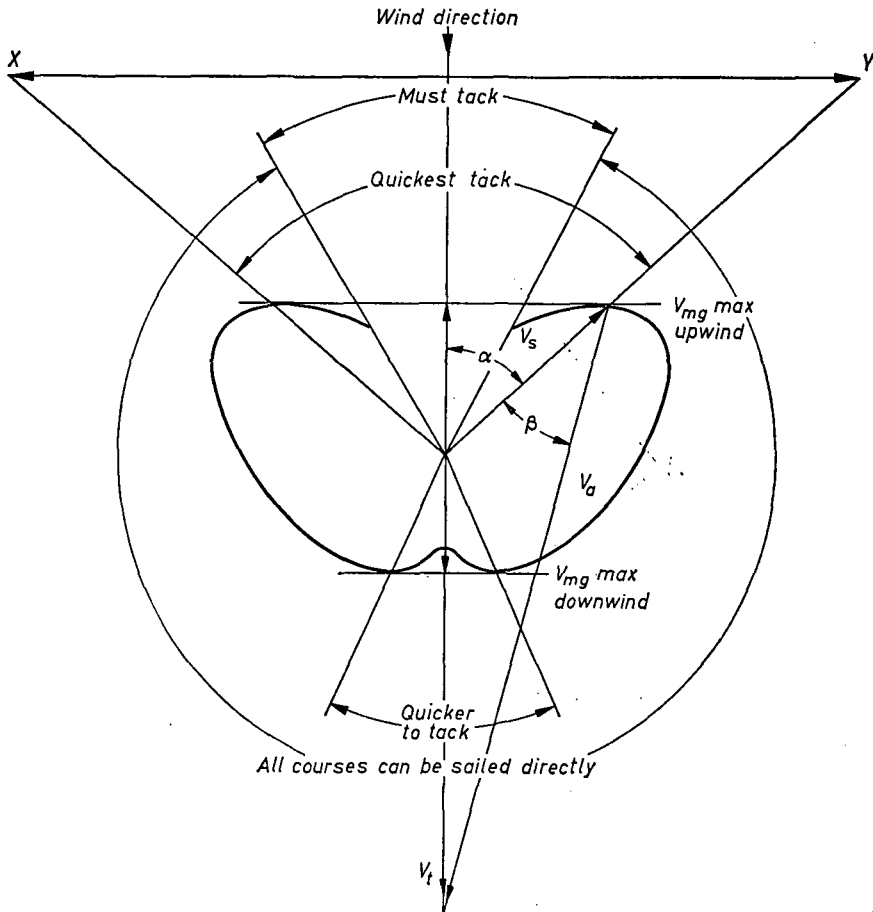

FIG. 1. Polar diagram of sailing performance

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running point of sailing where the curve may have an inflexion, i.e. there is profit in tacking a course downwind.

The rate of progress to windward for any sailing angle and velocity may be assessed by resolving the yacht velocity along the true wind vector. The angle at which this resolved velocity (V_{mg}) reaches a maximum yields the quickest course. From the trigonometry $V_{mg} = V_s \cos \alpha$. Although the angle α cannot be measured on board, due to the forward motion of the yacht, it can be deduced from other measurable parameters, the apparent wind angle β and the apparent wind speed V_a. These parameters may be measured by masthead equipment, but some kind of computation must be made to deduce α, and hence V_{mg}, from V_s, V_a and β.

FIG. 2. Resolution of velocity vectors

In Fig. 2 the measured parameters are shown in solid lines and the developed parameters in dashed lines; the pecked lines complete the right-angled triangles. In the triangle BAD:

$$\cos \alpha = (V_a \cos \beta - V_s)/V_t$$

and from the original equation:

$$V_{mg} = (V_a V_s \cos \beta - V_s^2)/V_t$$

This however still contains the unmeasurable parameter V_t, which can be found by using the cosine rule $c^2 = a^2 + b^2 - 2ab \cos C$, so that:

$$V_t^2 = V_a^2 + V_s^2 - 2 V_a V_s \cos \beta$$

The solution using measured parameters is therefore:

$$V_{mg} = V_a V_s \cos \beta - V_s^2/(V_a^2 + V_s^2 - 2 V_a V_s \cos \beta)^{1/2}$$

This equation is obviously too unwieldy for on-board measurements to be taken and solutions calculated. To realize a graphical solution the four terms of the equation must be reduced to three by pairing off the velocities V_{mg}/V_t and V_s/V_a. To produce the appropriate graphs the boundaries of interest must be declared, for which purpose the following points have been taken into account.
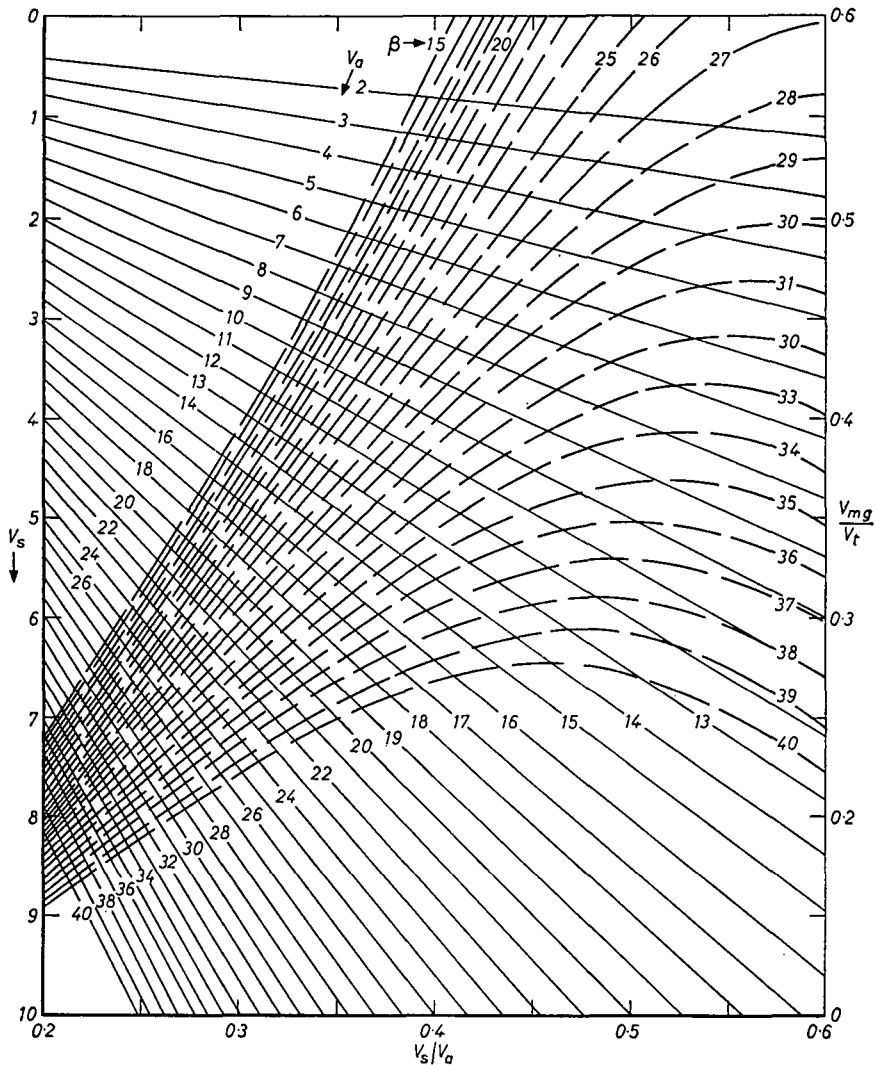
FIG. 3. The Abacus

A yacht is unlikely to sail at speeds in excess of 0·6 V_a and the maximum values of V_{mg} will occur within the apparent wind angles of 15° to 40°, the corresponding cosines of which are 0·966 and 0·766. V_{mg}/V_t can never reach unity (by sailing directly into the eye of the wind) and even 0·6 is unlikely to be attained. On this basis the curves for β from 15° to 40° are shown in Fig. 3.

To apply the curves a known practical polar diagram, Fig. 4, for a true wind velocity V_t of 11·5 kt., can be considered. The relevant values are shown in Table I and the curves for each parameter in Fig. 5, from which it will be seen that the V_{mg} curve has a maximum almost coincidental with the maximum of V_a. This feature will be dealt with later but, in general, the apparent wind velocity will maximize slightly further off the wind than does the curve of V_{mg}. This provides a ready guide to the vicinity of the V_{mg} maximum and a course steered to maximize V_a is a logical starting point.

In the example this will be in the region where β is between 24° and 26·5°, and a course steered at the former angle under steady wind conditions would indicate $V_a = 18·4$ and $V_s = 8$. The entry point on the abacus will be at $V_s = 8$
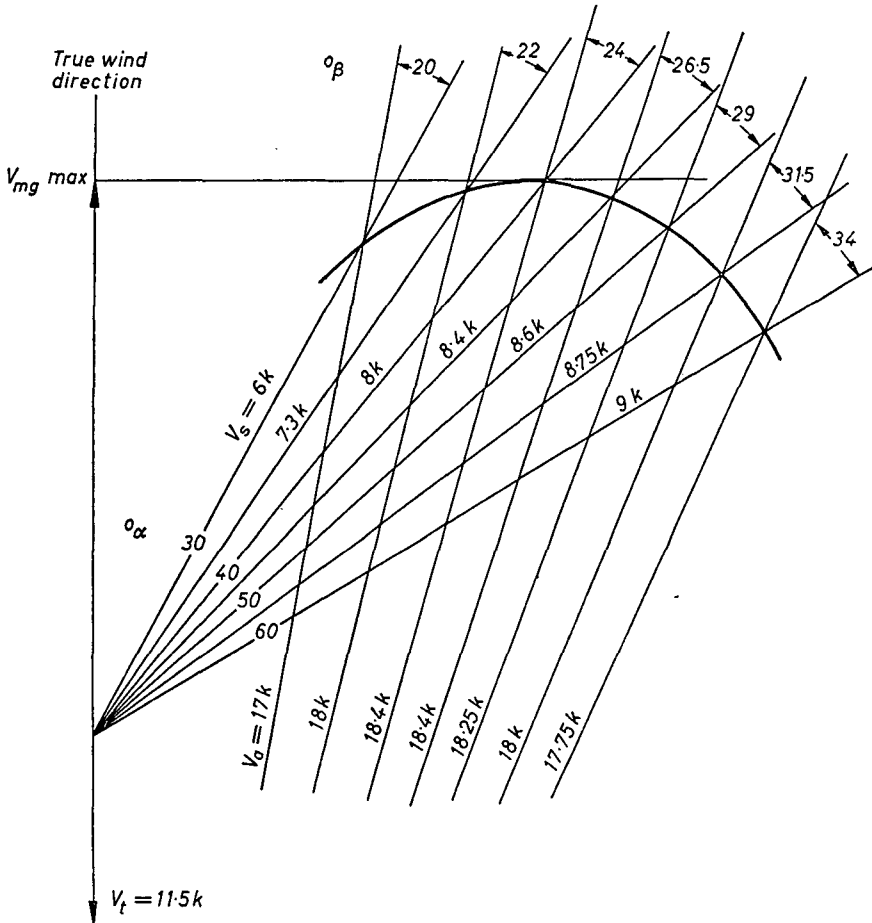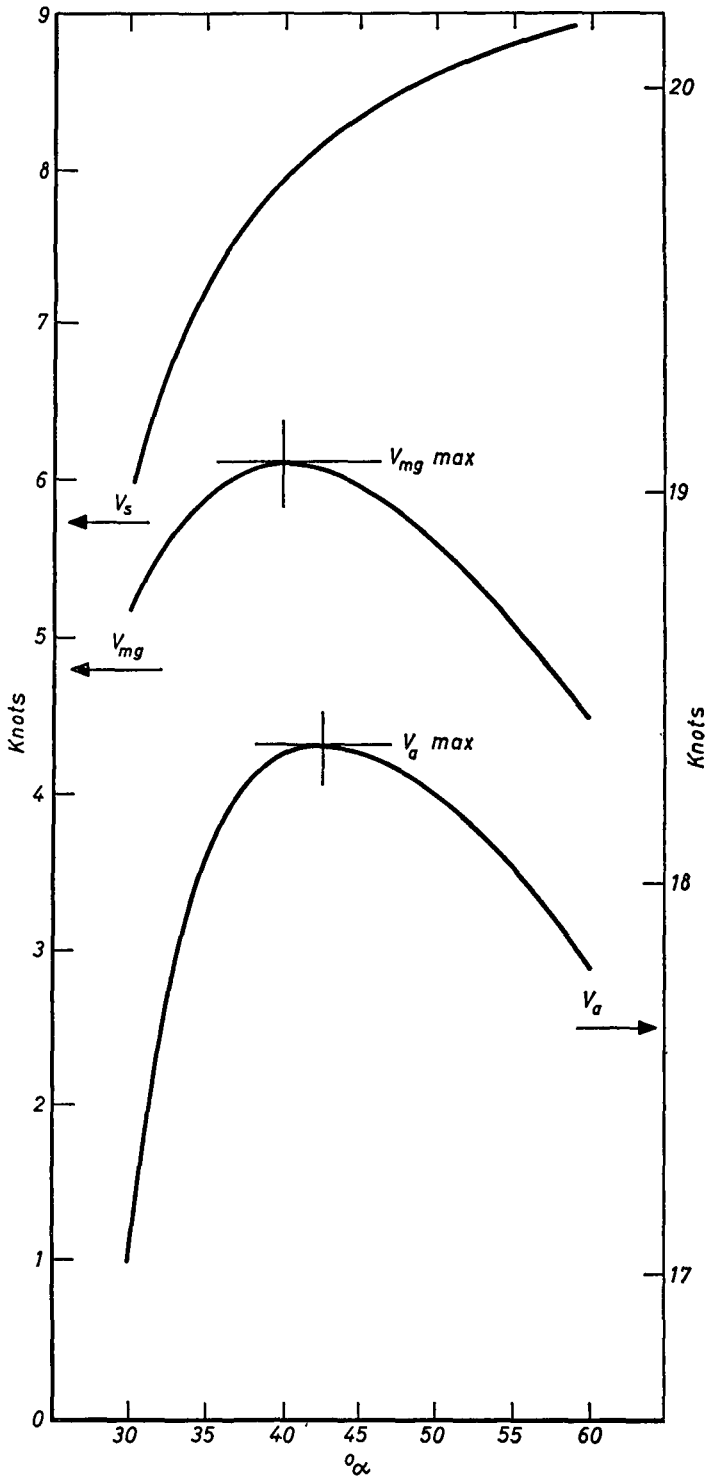

FIG. 4. Typical polar diagram (upwind)

FIG. 5. Upwind polar plots

TABLE I

α	β	V_a	V_s	V_s/V_a	V_{mg}	V_{mg}/V_t
30	20	17	6	0·354	5·196	0·450
35	22	18	7·3	0·405	5·986	0·512
40	24	18·4	8	0·435	6·128	0·530
45	26·5	18·4	8·4	0·456	5·880	0·512
50	29·5	18·25	8·6	0·470	5·500	0·476
55	31·5	18	8·75	0·485	4·990	0·434
60	34	17·75	9	0·505	4·50	0·390

on the left-hand vertical axis. This value is now traced horizontally until the relevant V_a curve (straight lines running diagonally downward from left to right) is met, and from this point tracing either up or down vertically until the corresponding $\beta(=24°)$ curve is encountered, to find V_{mg}/V_t (i.e. at 0·53). Trial courses at $\beta=22°$ and 26° will result in lower values of V_{mg}/V_t, indicating that the best course has been found first time.

As with any practical method of tackling the problem there is the difficulty of reading the three dials simultaneously, and the method invites instrumental

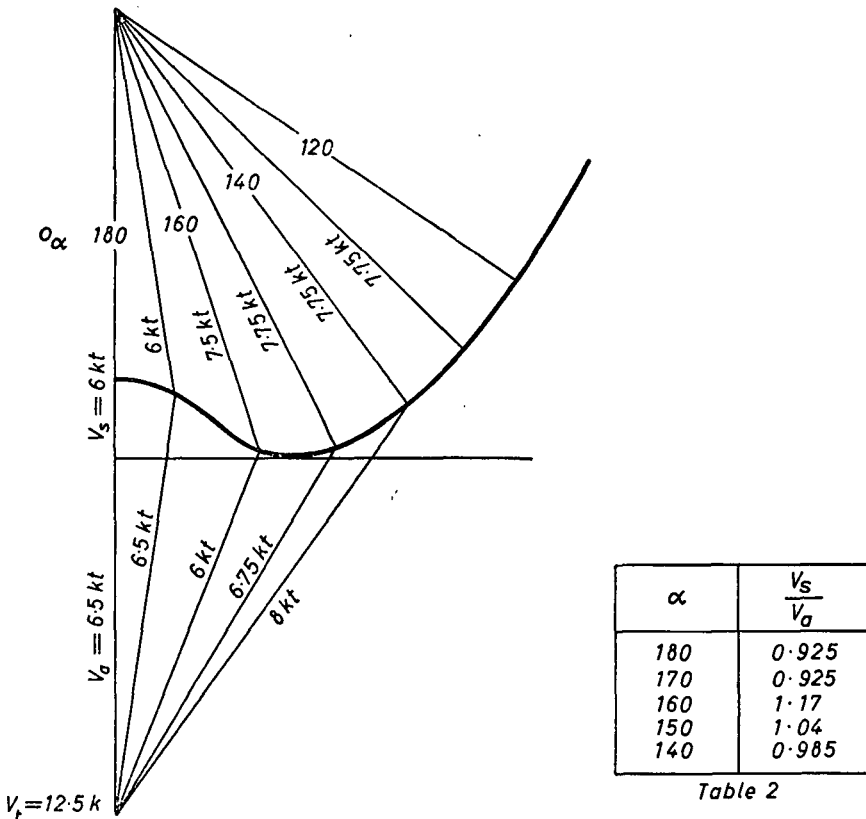| α	$\dfrac{V_s}{V_a}$
180	0·925
170	0·925
160	1·17
150	1·04
140	0·985

Table 2

FIG. 6. Typical polar diagram (downwind)

refinement by providing sample and hold facilities. By this means the angle meter may be watched for coincidence with the trial value and, when steady, the V_a and V_s values may be held for say 20 sec. while the readings are noted.

Mention was made earlier of the proximity of the V_a and V_{mg} peaks and it follows from the geometry that the higher the wind velocity, the closer the peaks become. At what point this occurs in the wind scale depends upon the shape of the polar curve and the size of the yacht. It has been observed that up to the point where the maximum velocity for the yacht has been realized the V_{mg}/V_t curve is linear and that beyond this point performance apparently tends to fall off. The point where the V_a meter alone may be used to indicate the best course will have to be a matter of individual subjective determination. That the effect does occur can be seen from a manipulation of the V_{mg}/V_t equation, which may be re-written:

$$(2\,V_{mg}/V_t) + 1 = (V_a{}^2 - V_s{}^2)/V_t{}^2$$

from which it will be seen that, as V_a becomes larger and V_s attains its maximum value, V_s cannot substantially alter the angle α at which V_{mg} becomes a maximum, i.e. when V_a also becomes a maximum. In the downwind case, where the polar curve may have an inflexion as shown in Fig. 1, the maximum value of V_{mg} occurs when V_s/V_a is at a maximum. How this happens will be readily seen from Fig. 6 which is an enlarged portion of the downwind area.

In conclusion it must be said that the β meter should have a scale capable of indicating 1° increments. The other two velocity instruments could profitably be repeating instruments with sample and hold facilities. The V_a repeater would also benefit the user if an expansion (i.e. non-linear) amplifier, with offset and a qualitative scale, could be brought into commission once the sailing conditions warrant its use as a V_{mg} meter.

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The Use of ABC Tables in Preparing for Star Observations

Captain P. A. Thompson

As part of their navigational outfit many of the ships in service today are equipped with *Sight Reduction Tables* (A.P. 3270 = H.O. 249), and many navigating officers include amongst their personal kit a *Rude Star Identifier* (H.O. 2102), or some similar device according to individual preference, for the purpose of obtaining the approximate altitude and azimuth of a selected body as a preliminary to making observations, particularly for star sights. However, for the benefit of those without such means at their disposal and as a matter of interest for others, the use of ABC tables for this purpose is suggested. The origin and author of this method are not known and it does not appear to be included in any of the navigational manuals which have been in circulation in recent years, but there