GENERATION OF OSCILLATORY MOTIONS
IN THE STELLAR ATMOSPHERE

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Abstract

Thermal overstability of non-radial eigenmodes of stars is discussed as one of possible causes for generating non-thermal motions in the stellar atmosphere. The nature of oscillatory motions in stars is first considered both in the local and the global stand points. Then, the excitation of eigen-oscillations is discussed and results of numerical studies so far made are reviewed for the vibrational stability of various stellar models against non-radial oscillations. It is found that many of non-radial p-modes of high tesseral harmonics are likely excited in various stars of the HR diagram and that they possibly manifest themselves as non-thermal velocity fields in the stellar atmosphere.

1. Introduction

Two kinds of phenomena are known to indicate the existence of non-thermal motions in stellar atmospheres. One of them is the spectral line broadening, asymmetry, and shift indicating the existence of velocity fields, and the other is the mechanical source of energy that is required to heat the stellar chromosphere and corona. One of the most important questions to be addressed in this colloquium will be, what kinds of motions are involved in these phenomena? What mechanisms (or instabilities) can generate them? It is generally thought that thermal convection is one of the most likely sources for generating these motions in stellar atmospheres. However, it will be evident that all of non-thermal motions are not necessarily generated by thermal convection because various activities such as evidenced by micro- and macro-turbulences and by X-ray emissions due to the hot corona are observed also in early-type stars where any appreciable surface convection zones are not expected. Thus, some other instabilities that can generate motions in the stellar atmosphere have to be investigated as well. In this paper, I will discuss the thermal overstability of trapped waves in stars as one of possible mechanisms. This mechanism can generate wave motions of some finite amplitude from perturbations of infinitesimally small thermal fluctuation.

The thermal overstability is the basic mechanism for generating pulsation

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motions in variable stars such as Cepheid and RR Lyrae. However, modes of oscillations with which we are concerned here are different from ordinary radial pulsation, but they are those of non-radial type. The existence of non-radial oscillations as stellar eigenmodes have been known theoretically for a long while, but their importance in relation with observational phenomena in stars seems to have been recognized only recently. The five minute oscillation in the solar atmopshere is now understood as global non-radial p-modes of the sun (Deubner 1975, 1977). Although velocity fields due to thermal convection, which is manifested as the granulation in the solar atmosphere, are dominant at the photospheric level, the five minute oscillation as the velocity fields becomes increasingly important with height, and the latter dominates in the upper photosphere and the chromosphere. Non-radial oscillations have been inferred as the possible modes of observed motions in stars such as line-profile variable stars (Smith 1977) and early type supergiants (Lucy 1976a, b). Lucy (1976) has shown that the semiregular variability in radial velocity observed in the A-type supergiant star α Cygni may be due to the simultaneous excitation of many discrete pulsation modes of non-radial type and the "macroturbulence" required for large line-broadening in this star may be explained by the superposition of these oscillation modes.

One might think that two pictures of "turbulence" and "eigenmodes" are seemingly contradicting since turbulence is essentially a stochastic phenomenon with energy cascading from larger eddies to small eddies while eigenmodes are discrete in frequency and wavenumber and are coherent in time and space. However, they do not necessarily contradict each other since "turbulence" used in stellar spectroscopy is not turbulence as understood in aerodynamics and it simply means non-thermal motions responsible for spectral line broadening. On the other hand, non-radial eigenmodes are rich in physical properties and have a very dense spectrum although they are still discrete. Thus, if extremely large numbers of eigenmodes are excited in a star and are superposed, they show up spectroscopically as unresolved velocity fields. One may visualize the relation between turbulent convection and eigenmodes of the star in the power spectrum of velocity fields against the wavenumber k. The turbulent convection occupies a high wavenumber domain with $k>1/H$, while the non-radial eigenmode oscillation occupies a low wavenumber domain with $1/H \lesssim k \lesssim 1/R$, where H and R denote the scale height of the atmosphere and the star's radius, respectively. Thus, observationally, turbulent convection is more related to "microturbulence" due to its small scale and eigenmode oscillations are to "macroturbulence".

In this paper, I will argue that the non-radial p-mode oscillations of high tesseral harmonics are very likely to be excited in various stars in the HR diagram and they might possibly be responsible for the non-thermal motions in some of these stars. As far as I am aware, Christy (1962) was the first to propose in 1962 that non-spherical pulsation-type motions are present in late-type stars and they are important for the hydrodynamics in the atmospheres of these stars. However, this suggestion seems not to have been taken seriously for a long while, and it is only in recent years that numerical stability analyses for non-radial oscillations have been carried out in a number of stars. We shall discuss the nature of oscillations in stars from the local and the global points of view in sections 2 and 3. Excitation mechanisms of oscillations and results of numerical calculations will be reviewed in section 4.

2. Waves in the Stellar Atmosphere

In this section, we discuss wave motions in the stellar atmosphere from the local point of view. Wave motions in gravitationally stratified fluids have extensively been studied in geophysics (see, e.g., Eckart 1960). It is well known that two kinds of forces act on fluid elements: pressure force due to compressibility and buoyancy force due to gravitational stratification. If the medium is stably stratified (i.e., convectively stable), both of them are restoring forces and they give rise to oscillation of a fluid element if it is displaced from static state. Corresponding to this, two kinds of waves, i.e., acoustic waves and gravity waves, occur in the stellar atmosphere.

We now consider the wave propagation in a plane-parallel isothermal atmosphere under a constant gravitational field, in which the pressure p_0 and density ρ_0 of static state are known to vary with height z as

$$p_0,\ \rho_0 \propto \exp\left(-z/H\right) \tag{1}$$

where $H=p_0/(\rho_0 g)=\mathcal{R}T_0/(\mu g)=\mathrm{const}$ is the scale height, and g the gravitational acceleration, T_0 the temperature, μ the mean molecular weight, $\mathcal{R}$ the gas constant. In this case, the linearized system of equations in hydrodynamics for adiabatic perturbation allows a simple solution of plane waves for velocity $\underset{\sim}{v}$, the pressure variation p', and the density variation ρ' of the form

$$\underset{\sim}{v},\ \frac{p'}{p_0},\ \frac{\rho'}{\rho_0} \propto \exp\left(\frac{z}{2H}\right)\ \exp[i(-\omega t+k_x x+k_y y+k_z z)] \quad . \tag{2}$$

Here the exponentially growing factor with height z in equation (2)

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arises so as to conserve wave energy $\rho_0 v^2$ in the vertical direction since the density ρ_0 in the atmosphere decreases exponentially with height.

Substitution of these expressions into equations of hydrodynamics yields a dispersion relation between frequency ω and wavenumber k, which is written as

$$k_z^2 = f(\omega, k_h) = \frac{1}{c^2}(\omega^2 - \omega_{ac}^2) + k_h^2\left(\frac{N^2}{\omega^2} - 1\right) \quad , \tag{3}$$

where $k_h=\sqrt{k_x^2+k_y^2}$ is the horizontal wave number. Here quantities ω_{ac} and N, having a dimension of frequency, are called the acoustic cut-off frequency and the Brunt-Väisälä frequency, respectively, and they are given by

$$\omega_{ac} = \frac{c}{2H} = \frac{\gamma g}{2c} \quad , \tag{4}$$

and

$$N = \frac{g}{c}\sqrt{\gamma - 1} \quad , \tag{5}$$

where c is the sound velocity, and γ is the ratio of the specific heats of gas.

If ω and k_h are given, equation (3) determine k_z^2. If $k_z^2>0$, waves can propagate vertically. On the other hand, if $k_z^2<0$, no waves can propagate, and the energy density of perturbations decreases exponentially with height (evanescent waves), if perturbations are coming from below. This situation is most conveniently shown in the diagnostic (k_h, ω)-diagram, which is illustrated schematically in Figure 1.

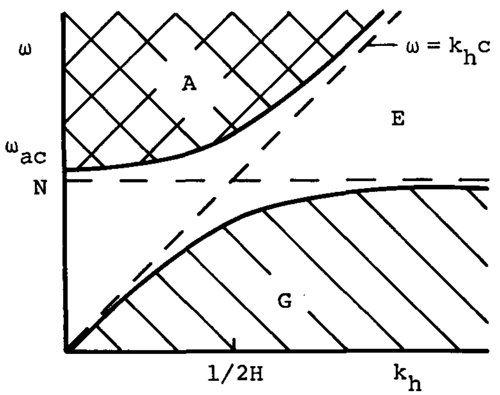

Figure 1. Diagnostic diagram

The diagnostic diagram is divided into three regions: (1) region A with $k_z^2>0$ where modified acoustic waves can propagate, which is given approximately by the conditions $\omega>\omega_{ac}$ and $\omega>k_h c$. (2) region G with $k_z^2>0$ where the modified gravity waves can propagate, which is approximately given by $\omega<N$ and $\omega<k_h c$. (3) region E with $k_z^2<0$ where waves are evanescent.

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Oscillations of our interest are trapped waves, but the isothermal atmosphere has not such resonance property. Thus, in order to have a proper waveguide character of the stellar atmosphere and envelope, we have to take into account the variation in temperature with height, and this will be considered in the next section. For global eigenmodes of the star, the atmosphere act as an outer reflecting boundary. Waves are evanescent in the atmosphere for such oscillations. It is important to note here that the velocity amplitude of evanescent waves does not necessarily decrease with height, but rather it usually increases slightly within the atmosphere. This is because the energy density of perturbations decreases with height less rapidly than the density ρ_0 itself and thus the factor of $\exp(z/2H)$ in equation (2) plays an essential role.

3. Eigenmodes of Stars

So far we have considered wave motions in the stellar atmosphere from the local stand point. We now turn to discuss stellar oscillations from the global point of view. Any persistent oscillations of a star may be considered as a superposition of normal modes of the star. There are two kinds of normal modes in stars: the radial oscillations and the non-radial oscillations. However, radial oscillations may be regarded as one of special cases of non-radial oscillations with the spherical harmonic index $\ell=0$. If we assume that the unperturbed state of the star is in spherically-symmetric time-independent equilibrium, the eigenfunction of a non-radial mode for perturbations of physical variables (e.g., density pergurbation ρ') can be expressed in the spherical polar coordinates (r, θ, ϕ) by

$$\rho' = \rho'_{k,\ell}(r)\, Y_\ell^m(\theta, \phi)\, e^{-i\omega t} \quad , \tag{6}$$

where $Y_\ell^m(\theta, \phi)$ is the spherical harmonics. The quantity ω is the eigenfrequency of the nonradial oscillation which is specified by three integrals (k, ℓ, m). For a given ℓ, the quantum number m takes integer values from $-\ell$ to ℓ, and eigenfrequencies of these $(2\ell+1)$-modes are degenerate in a spherically symmetric (non-rotating, non-magnetic) star.

The existence of two extra indices (ℓ, m), describing the horizontal dependence of eigenfunctions, makes non-radial oscillations a complicating appearance and it also gives them an extra richness in the eigenvalue spectrum. Besides that, as noted in the previous section, two different kinds of restoring forces (i.e., the pressure force and the buoyancy force) operate in non-radial oscillations, and there exist therefore two different kinds of modes: pressure (acoustic) modes and gravity modes.

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The pressure modes (p-modes) form a sequence of increasing eigenfrequency with the order of modes specified by the number of nodes, i.e.,

$$\omega_{p_1} < \omega_{p_2} < \omega_{p_3} < \cdots \rightarrow \infty \quad ,$$

while the gravity modes (g-modes) form a sequence of decreasing frequency, i.e.,

$$\omega_{g_1} > \omega_{g_2} > \omega_{g_3} > \cdots \rightarrow 0 \quad .$$

A system of equations, which describes the linear adiabatic non-radial oscillations, forms a boundary-value problem of fourth-order ordinary differential equations in the radial coordinates r, and for a given stellar equilibrium model eigenvalues and eigenfunctions are to be calculated numerically. Although these equations look complicated, they can be reduced under a certain assumption to a form analogous to the Schrödinger equation of quantum mechanics. We can then make some qualitative discussion with the help of the so-called propagation diagram (see, Unno et al. 1979). Under a given stellar model and for a fixed spherical harmonic index ℓ, we exhibit in the propagation diagram the spatial variations of the Brunt-Väisälä frequency, N, and the Lamb frequency, L_ℓ, as functions of the radial coordinates r in the stellar interior. Here two frequencies N and L_ℓ are defined by

$$N^2 = g\left(\frac{1}{\Gamma_1}\frac{d\ln p}{dr} - \frac{d\ln \rho}{dr}\right) \quad ,$$

and

$$L_\ell^{\,2} = \frac{\ell(\ell+1)c^2}{r^2} \quad ,$$

and they represent the local buoyancy frequency and the frequency (the reciprocal time) of horizontal acoustic propagation with a given horizontal wavenumber ℓ.

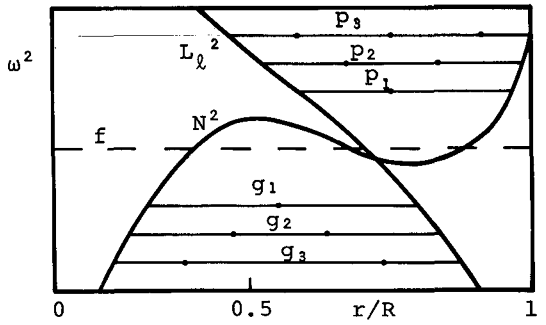

Figure 2. Propagation diagram

Figure 2 illustrates a schematical propagation diagram for a star near the main-sequence with a low order ℓ. In this diagram, the acoustic waves can propagate locally in the region with $\omega^2>N^2$ and $\omega^2>L_\ell^{\,2}$ (P-zone) and the gravity waves can propagate in the region with $\omega^2<N^2$ and

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$\omega^2 < L_\ell^2$ (G-zone) and waves cannot propagate (i.e., evanescent) in other regions with $L_\ell^2 < \omega^2 < N^2$ or $N^2 < \omega^2 < L_\ell^2$ (E-zone). This diagram shows therefore in what part of the stellar interior a wave with a given frequency has locally a propagating or non-propagating character. The propagation diagram may be comparable to the potential energy function of the one-dimensional Schrödinger equation of quantum mechanics in such a way that a propagation zone corresponds to a potential well and an evanescent zone to a potential wall. Since eigenmodes are standing waves, they occur in a region of a potential well that is enclosed by potential walls at both ends. The most important difference between the problem of non-radial oscillations and that of quantum mechanics lies in the fact that in the case of non-radial oscillations there are two different kinds of potential wells: one is that opened upward (p-wave zone) and the other is that opened downward (g-wave zone). The main characteristics of a star as an oscillator can be essentially represented by the propagation diagram. The complicated behavior of non-radial oscillations in evolved stars are mainly caused by complicated spatial variations in the Brunt-Väisälä frequency as a function of the position in the stellar interior.

Generally speaking, non-radial p-modes are oscillations trapped near the surface and g-modes are those trapped in the deep interior. Since we are interested in those oscillations that may produce observable motions in the stellar atmosphere, the most important modes are those of non-radial p-mode oscillations. Furthermore, oscillations of non-radial p-modes are more strongly concentrated near the surface as the spherical harmonic index ℓ is increased. This can be seen in the propagation diagram as the Lamb frequency L_ℓ moves upward with the increase of ℓ. The horizontal wavenumber of oscillations in the stellar surface becomes almost continuous for high value of ℓ and it is given by $k_h = \sqrt{\ell(\ell+1)}/R \sim \ell/R$. In the diagnostic (k_h, ω)-diagram, non-radial p-modes then appear as several discrete ridges corresponding to each radial-order mode (i.e., p_k-mode), which has been observed beautifully in the case of the solar five minute oscillations (Deubner 1977, Deubner et al. 1979).

4. Overstability of Eigenmodes

Since the ordinary dissipation gives damping of oscillations, some special mechanisms of excitation must operate inside the star in order for an eigenmode to be excited and to be observed. To study this, we must explicitly consider non-adiabatic effects of oscillations, i.e., the energy exchange of fluid elements with the surroundings during a cycle of oscillation. The thermodynamic excitation of oscillations have extensively been studied in connection with the driving of pulsation in Cepheid and

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RR Lyrae stars. Two kinds of excitation mechanisms are known which may operate in the outer layers of stars. The first one is the κ-mechanism (opacity-mechanism) of the hydrogen and helium ionization zones in the stellar envelope, and this is the mechanism that drives pulsations in Cepheids. The second one is the so-called Cowling-Spiegel mechanism which operates in the zone with the super-adiabatic temperature stratification. The latter mechanism operates only for non-radial oscillations and not for radial oscillations.

To get some idea how these mechanisms work, we describe briefly the operation of the κ-mechanism below. The opacity κ of partial ionization zones of the hydrogen and the helium increases at the phase of compression in a cycle of oscillation and the radiative flux flowing from the center to the surface is blocked there at that phase. The heat thus accumulated will give stronger repulsion at the next phase of expansion, and the amplitude of oscillation will tend to grow with time if the effect of excitation of this κ-mechanism is stronger than that of damping in other regions. As for the Cowling-Spiegel mechanism in a super-adiabatic temperature stratification, if some other restoring forces such as the stabilizing chemical composition gradient, magnetic fields and rotation, exist, this mechanism is known to be effective to give overstability of some non-radial modes (Moore and Spiegel 1966). However, if there exists no other restoring force, it is then not clear whether this mechanism acting as a sole agent of excitation can destabilize oscillatory modes. In this respect, Unno (1977) has pointed out that the Cowling-Spiegel mechanism associated with the variation of convective flux is more important than that of radiative flux. Then the variation of convective flux has to be taken into account in order for the problem to be settled.

Stars have been thought in the past to be less susceptible to non-radial oscillations than to radial oscillations because of the increased dissipation of the oscillations by the lateral radiative heat exchange. However, recent investigations show that much variety in the geometrical and physical properties of non-radial oscillations should give some of non-radial modes a better chance to get excited. In fact, those stars which are unstable against radial pulsations are likely to be unstable against some of non-radial modes as well. Furthermore, stars which are unstable against some of non-radial modes will probably occupy a wider range in the HR diagram than radially pulsating stars. The most important reason why some of non-radial modes are easy to get excited is that they can be "trapped" locally in the place where some excitation mechanism works effectively so that the dissipation of oscillation in other regions of the star is kept minimal. If one of non-radial modes is unstable in a star, several of other modes are very likely to be excited because their physical characters are very similar. Thus, if non-radial oscillations are ever excited in a star, it is most probable that many of non-radial modes are excited simultaneously.

The question of overstability of an eigenmode is a problem of delicate balance between excitation and damping and it can only be answered adequately by the numerical analysis of stability using a realistic stellar model. Thus, the full equations of linear non-adiabatic non-radial oscillations have to be solved numerically. The radiative heat exchange both in the sub-photospheric layers and in the atmosphere is important in the case of our interest, and, exactly speaking, the problem of time-dependent three dimensional radiative transfer has to be treated. It looks so much difficult that it is the usual practice at present to treat this by using the Eddington approximation developed by Unno and Spiegel (1966). The basic equations are then reduced to the fourth order linear differential equations with complex coefficients (under the Cowling approximation in which the Eulerian perturbation of the gravitational potential is neglected), and they form together with adequate boundary conditions an eigenvalue problem with a complex eigenvalue and complex eigenfunctions. The real part ω_R of the complex eigenfrequency ($\omega=\omega_R+i\omega_I$) gives the frequency of oscillation and its imaginary part ω_I determines the growth (or damping) rate of oscillations. This system of equations for linear non-adiabatic oscillations have been solved in a few cases of our interest and they will briefly be reviewed below.

(1) The solar non-radial p-modes.

Ando and Osaki (1975) performed extensive calculations of eigenfrequencies of non-radial p-modes of the sun in relation to the solar five minute oscillation. They solved equations of linear non-adiabatic non-radial oscillations for a realistic solar envelope model and obtained complex eigenfrequencies of non-radial p-modes covering a wide range in the spherical harmonics ℓ (ranging from $\ell=10$ to 1500) and in several overtones of the radial order. It was found that many of non-radial p-modes were overstable and that most unstable modes occupy a region in the diagnostic (k_h, ω)-diagram centered with a period of 300 sec and with a wide range of horizontal wavenumber.

It has been seen from eigenfunctions that the vertical velocity of oscillations has an increasing amplitude with height in the atmosphere, even though the kinetic energy of oscillations per unit volume $\rho|\underline{v}|^2$ is decreasing within the atmosphere. Many of non-radial p-modes are found to be unstable and they are excited by the κ-mechanism of at the hydrogen ionization zone. The Cowling-Spiegel mechanism seems to work at the convective-radiative transition zone, but quantitatively the κ-mechanism is found much more effective than the latter. The radiative dissipation in the atmosphere ($\tau<1$) contributes appreciably to the damping of oscillations and its importance increases with the increase in frequency of modes, and higher p-modes become stabilized ultimately.

The biggest uncertainty in this analysis is the problem of the interaction between convection and oscillation, which has been neglected in the work of Ando and Osaki (1975) because of lack of the definitive theory. The damping due to turbulent viscosity of convection is thought to have a stabilizing tendency for otherwise unstable p-modes. Goldreich and Keeley (1977) have made a rough estimate of its effect and found that it is as important as the thermodynamic excitation of oscillations. However, they have failed to reach the definitive conclusion about the overstability of p-modes of the sun because of uncertainty in convection theory.

(2) Cepheids.

Cepheids are pulsationally unstable against radial modes due to the κ-mechanism in the hydrogen- and helium-ionization zones. Since this mechanism works also for non-radial p-modes, it will be natural to ask whether or not Cepheids are vibrationally unstable against non-radial modes.

There is, however, an important difference between radial pulsations and non-radial p-mode oscillations. Dziembowski (1971) first noticed that, in the case of non-radial oscillations of giant stars, even high-frequency envelope p-modes behave as internal gravity waves of extremely short wavelength in the deep interior, and he once concluded that the excitation of non-radial oscillations would be prevented by strong radiative dissipation in the core of these evolved stars. However, there exists another important character of non-radial oscillations, that is the wave-trapping phenomenon, and this effect was not taken into account in Dziembowski's (1971) discussion. Shibahashi and Osaki (1976) have shown that non-radial modes with high-order spherical harmonics can be very clearly divided into two types in evolved stars, one type being a gravity mode trapped in the core and the other a mode trapped in the envelope. There exist, therefore, envelope p-modes with high ℓ that are almost completely free from the influence of the core.

By taking into account both the effect of dissipation in the core and the non-adiabatic effect in the envelope, Osaki (1977) has examined vibrational stability against non-radial modes in a Cepheid model. It is found that non-radial modes with lower-order spherical harmonics (i.e., $\ell\lesssim5$)

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are stable because of heavy leakage of wave energy from the envelope to the core, but that those of higher ℓ (i.e., $\ell \gtrsim 6$) are trapped well within the envelope and some of them (i.e., f- and p_1-modes) are unstable due to the negative dissipation in the hydrogen- and helium-ionization zones. The growth rates of unstable non-radial modes are found to be of the same order as those of radial modes. A similar result was obtained independently by Dziembowski (1977) who found that the overstability of non-radial modes with very high-order spherical harmonics extends far beyond the boundary of the classical Cepheid instability strip.

(3) Other stars.
The vibrational stability of non-radial p-mode oscillations has been investigated for various stellar envelope models in the wide range of the HR diagram by Ando (1976) and by Dziembowski (1977).

Ando (1976) examined the stability of stellar envelopes in late-type dwarfs, giants, and super-giants against non-radial p-modes with high-order ℓ which are well trapped near the surface. He found many of non-radial p-modes are overstable due to the κ-mechanism of the hydrogen ionization zone in stars that lie to the right of the Cepheid instability strip in the HR diagram. He then suggested that these overstable acoustic modes might be responsible for the formation of the chromosphere and the corona and for the Wilson-Bappu effect in late type stars. However, the biggest uncertainty of this result is again the problem of coupling between convection and oscillation, whose effect has been ignored in this investigation.

Dziembowski (1977) made a similar study for stars lying within and to the left of the Cepheid instability strip. He found that δ Scuti stars are unstable both for radial and for low ℓ non-radial modes, but that the maximum of instability occurs for non-radial modes with very high ℓ (i.e., $\ell \simeq 500$). He also studied two models of an early type supergiant corresponding to α Cyg (A2Ib) and of a medium type supergiant ($T_e \sim 5000°K$). The overstability was found in the case of the early-type supergiant only for non-radial f-modes with very high ℓ-values (i.e., $\ell = 32 \sim 63$). On the other hand, the overstability of low-order non-radial modes ($\ell \simeq 3-5$) was found in the case of the medium-type supergiant.

5. Summary and Discussion

We have considered the nature of oscillations in stars from the local and the global standpoints. Overstability of non-radial oscillations has then been discussed. Results of numerical analysis show that many of non-radial p-modes are very likely unstable in various stars of the HR diagram. Observationally, non-radial oscillations of high tesseral harmonics (i.e., excepting the low-ℓ modes) do not give rise to neither the stellar light variability nor the stellar radial velocity variability, but they show up as unresolved velocity fields responsible to spectral line broadening and the mechanical source of energy to heat the upper atmosphere (i.e., the non-thermal velocity fields in the stellar atmospheres).

We have so far discussed the linear stability of non-radial modes, but we have not mentioned non-linear effects which are thought to be responsible for limiting amplitudes of overstable oscillations. Non-linear effects that are important in limiting amplitudes in pulsating variables are;

1. The saturation of the excitation mechanism.

The saturation of the κ-mechanism in the second helium ionization zone is thought to be the most important in determining the final amplitude of Cepheids and RR Lyrae stars.

2. The enhancement of dissipation by shock waves.

When the amplitude of oscillations becomes sufficiently large, shock waves are generated in the atmosphere, which greatly enhances the dissipation. Besides these,

3. Non-linear mode coupling, will be important, in the case of non-radial oscillations. This will redistribute kinetic energy of oscillations between various non-radial modes.

Even in the linear stability analysis, there remain several unresolved problems. The problem of coupling between convection and oscillation was mentioned previously. We have not yet succeeded in finding the instability mechanism of pulsations in β Cephei stars (early-type pulsating variables). This means that our stability analysis is not still accurate enough. Possible insufficiencies in our knowledge are suspected to exist with respect to the opacity and the effect of radiation pressure. Much theoretical investigation is thus needed.

References

Ando, H. 1976, Publ. Astron. Soc. Japan, 28, 517.
Ando, H. and Osaki, Y. 1975, Publ. Astron. Soc. Japan, 27, 581.
Christy, R. F. 1962, Astrophys. J., 136, 887.
Deubner, F.-L. 1975, Astron. Astrophys., 44, 371.
Deubner, F.-L. 1977, in The Energy Balance and Hydrodynamics of the Solar Chromosphere and Corona, ed. R.-M. Bonnet and Ph. Delache (G. de Bussac, Clermont-Ferrand), p.45.
Deubner, F.-L., Ulrich, R. K., and Rhodes, E. J., Jr. 1979, Astron. Astrophys., 72, 177.
Dziembowski, W. 1971, Acta Astron., 21, 289.
Dziembowski, W. 1977, Acta Astron., 27, 95.
Eckart, C. 1960, Hydrodynamics of Oceans and Atmospheres (Pergamon Press, London).

Goldreich, P. and Keeley, D. A. 1977, Astrophys. J., 211, 934.
Lucy, L. B. 1976a, Astrophys. J., 206, 499.
Lucy, L. B. 1976b, Paper read at IAU Colloq. No.29 (Budapest, Sept. 1975).
Moore, D. W. and Spiegel, E. A. 1966, Astrophys. J., 143, 871.
Osaki, Y. 1977, Publ. Astron. Soc. Japan, 29, 235.
Shibahashi, H. and Osaki, Y. 1976, Publ. Astron. Soc. Japan, 28, 199.
Smith, M. A. 1977, Astrophys. J., 215, 574.
Unno, W. 1977, in Problems of Stellar Convection, ed. E. A. Spiegel and J.-P. Zahn (Springer-Verlag, Berlin), p.315.
Unno, W., Osaki, Y., Ando, H., and Shibahashi, H. 1979, Nonradial Oscillations of Stars (University of Tokyo Press, Tokyo).
Unno, W. and Spiegel, E. A. 1966, Publ. Astron. Soc. Japan, 18, 85.

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