

Merging black holes of any size and hierarchy

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Abstract. With the hundreds of merging binary black holes (BHs) expected to be detected by LIGO, LISA, and other upcoming instruments, the modelling of astrophysical channels that lead to the formation of compact BH binaries has become of crucial importance. BHs of any size can form bound systems in every astrophysical environment, from the field to galactic nuclei. If a binary is too wide, it needs a catalysis process to harden and merge, as in the case a third objects orbiting the BH binary on a distant orbit. In this case, Kozai-Lidov cycles can pump up the binary eccentricity, thus driving it to a merger thanks to efficient energy dissipation at the pericenter. Some remarkable scenarios where the Kozai-Lidov mechanism operates are in triple and quadruple systems of stellar BHs, and in intermediate-mass BH-stellar BH binaries in orbit around a central supermassive BH in galactic nuclei.

Keywords. stars: kinematics and dynamics – galaxies: star clusters: general – stars: black holes – stars: neutron – Galaxy: centre – Galaxy: kinematics and dynamics

1. Introduction

The recent discovery of gravitational waves (GWs) has opened new horizons for black-hole physics. Supermassive black holes (SMBHs) with masses from a million up to a billion times the mass of the Sun are known to be ubiquitous in galactic nuclei, and their masses correlate with the properties of the host galaxy in which they reside (Kormendy & Ho 2013). Black holes less massive than hundred solar masses are believed to be the end-product of massive stars (stellar-mass black holes; SBHs), and have recently been directly observed via GW emission by LIGO (LIGO Collaboration 2018). Very little is known about intermediate-mass black holes (IMBHs; $100 \text{ M}_\odot \lesssim \text{mass} \lesssim 10^6 \text{ M}_\odot$), but recent analyses suggest the possible presence of IMBHs in the centers of a few globular clusters and dwarf galaxies (Mezcua 2017). At present, only SBHs and neutron stars have been detected through GWs. However, current and upcoming missions, as LIGO-Virgo, LISA, and ET, vow to observe hundreds black holes of any size, and promise to shed light on their formation, evolution, and host galaxy demographics.

Black holes of any size can form bound systems in almost every astrophysical environment, from the field to galactic nuclei. We illustrate this schematically in Fig. 1, as a black hole merger tree. Some of these binaries can merge within a Hubble time, thus producing an observable GW signal. If a binary is too wide, it needs a catalysis process to harden and merge. For example, binary hardening can be catalyzed by dynamical encounters in star clusters (Rodriguez *et al.* 2018). A remarkable case is when the catalyst is a third objects orbiting the black hole binary on a distant orbit.

2. Kozai-Lidov mechanism in hierarchical systems

In the case of a triple system made up of an inner binary (of mass $m_{12} = m_1 + m_2$; semi-major axis a_{12}) that is orbited by an outer companion (of mass m_3 ; semi-major axis a_3 ; eccentricity e_3), the inner eccentricity (e_{12}) and inclination oscillate due to the

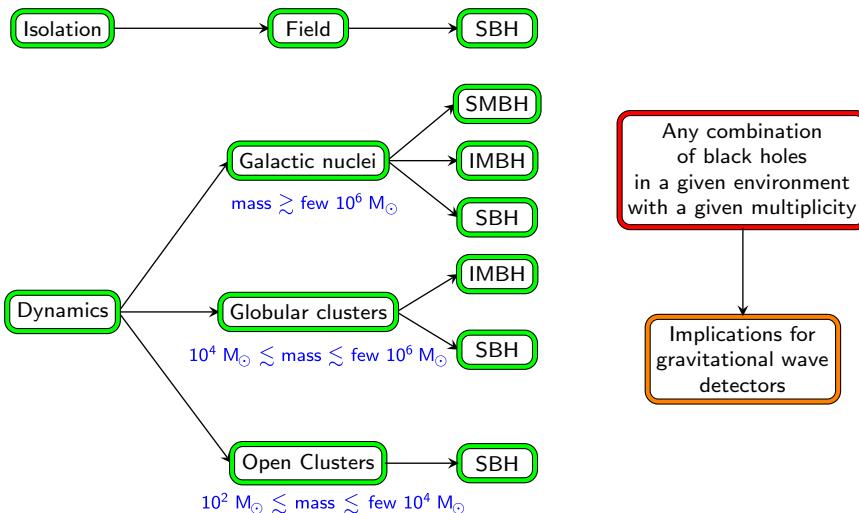


Figure 1. Black hole merger tree.

quadrupole moment of the tidal potential of the third body via the Kozai-Lidov (KL) mechanism, whenever the initial mutual orbital inclination is in the window $i_0 \sim 40^\circ - 140^\circ$ (Kozai 1962; Lidov 1962). At the secular quadrupole order of approximation, KL oscillations occur on a timescale

$$T_{\text{KL}} = \frac{8}{15\pi} \frac{m_{\text{tot}}}{m_3} \frac{P_3^2}{P_{12}} (1 - e_3^2)^{3/2}, \quad (2.1)$$

where $m_{\text{tot}} = m_{12} + m_3$, P_{12} and P_3 are the orbital periods of the inner and outer orbit, respectively, and e_3 is the eccentricity of the third companion. The maximum eccentricity of the inner binary due to the KL mechanism essentially depends on the initial mutual inclination

$$e_{12}^{\max} = \sqrt{1 - \frac{5}{3} \cos^2 i_0}. \quad (2.2)$$

As i_0 approaches $\sim 90^\circ$, the inner binary eccentricity approaches almost unity. At the octupole order, the eccentricity excitation becomes more prominent and e_{12} can reach almost unity even if i_0 is outside of the KL-angle window. This occurs on a timescale

$$T_{\text{oct}} = \left(\frac{m_1 + m_2}{m_1 - m_2} \right) \left(\frac{a_3}{a_{12}} \right) \left(\frac{1 - e_3^2}{e_3} \right) T_{\text{KL}}. \quad (2.3)$$

In the case the third companion is itself a binary star ($m_{34} = m_3 + m_4$, a_{34} , e_{34}), each binary acts as a distant perturber inducing KL cycles on the other binary. The evolution of such quadruple systems differs from a combination of two uncoupled three-body KL processes. Both binaries can experience coherent eccentricity oscillations and excursions to very high eccentricity, which take place over a much larger fraction of the parameter space compared to triple systems, also thanks to additional resonances (Hamers & Lai 2017).

3. Black hole mergers in isolated triples and quadruples

In Fragione & Kocsis (2019), we have used high-precision simulations to study the mergers of SBHs in quadruple systems. The majority of the merging systems has initially high inclinations and the distributions peak at $\sim 90^\circ$, but with broader tails compared to the SBH mergers in triples, as shown in Fig. 2 (left panel). The merger fraction can be

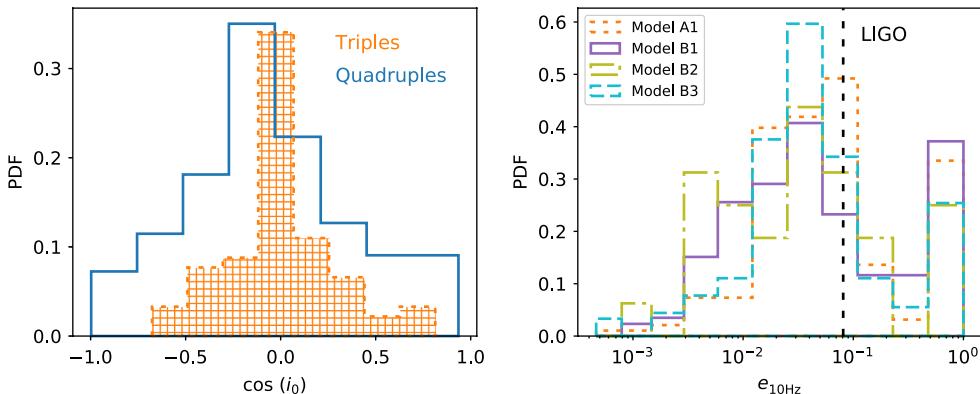


Figure 2. Left: initial inclination distribution of SBH binaries that merge in triple and quadruple systems. In quadruples, excursions to very high eccentricity can take place over a much larger fraction of the parameter space and the distribution is broader. Right: distribution of eccentricities at the moment the SBH binaries enter the LIGO frequency band (10 Hz) for mergers produced by quadruple models in [Fragione & Kocsis \(2019\)](#). The vertical line shows the minimum $e_{10\text{Hz}} = 0.081$ where LIGO-Virgo and the KAGRA network may distinguish eccentric sources from circular sources ([Gondán & Kocsis 2019](#)).

up to $\sim 3\text{--}4\times$ higher for quadruples than for triples (rate $\sim 1 \text{ Gpc}^{-3} \text{ yr}^{-1}$). Thus, even if the number of quadruples is 20%–25% of the number of triples, the quadruple scenario can represent an important contribution to the events observed by LIGO-Virgo and the upcoming GW instruments.

One of the main signatures of SBHs merging in hierarchical systems is a large eccentricity at the moment the peak GW frequency enters the LIGO frequency band (10 Hz), compared to other channels. Fig. 2 (right panel) shows the distribution of eccentricities at the moment the SBH binaries enter the LIGO frequency band (10 Hz) for mergers produced by the quadruple BH models presented in [Fragione & Kocsis \(2019\)](#). Typically, the eccentricity distribution presents a peak at $\sim 10^{-2} - 10^{-1}$, with a secondary peak at ~ 1 . Eccentric binaries emit a GW signal with a broad spectrum of frequencies, which can be observed. Moreover, these high eccentricities imply that a fraction of these systems could emit their maximum power at higher frequencies, potentially observable by LISA.

4. Intermediate-mass ratio inspirals in galactic nuclei

GW astronomy will help significantly in the hunt for IMBHs. Present and upcoming facilities, such as LISA and ET, will be able to detect IMBH-SBH binaries of different masses. The detection of the inspiral of an SBH onto an IMBH (intermediate-mass ratio inspiral; IMRI) would be the final proof of the existence of this elusive black hole population. In [Fragione & Leigh \(2018\)](#), we have investigated the dynamical evolution of binaries composed of an IMBH (M_{IMBH}) and an SBH, in orbit (semi-major axis a_{out} ; eccentricity e_{out}) around a central SMBH (M_{SMBH}). Such black hole triplets can form via the inspiral of globular clusters toward galactic nuclei due to dynamical friction, or even major/minor galaxy mergers.

Mass segregation combined with KL oscillations induced by the primary SMBH can effectively merge IMBH-SBH binaries on timescales (T_{merger}) much shorter than GW emission alone (T_{GW}). Most of the mergers happens when the initial inclination is $i_0 \sim 90^\circ$, since the KL cycles are maximal, eccentricity oscillates up to unity, the IMBH-SBH binary experiences efficient gravitational energy dissipation near the pericenter,

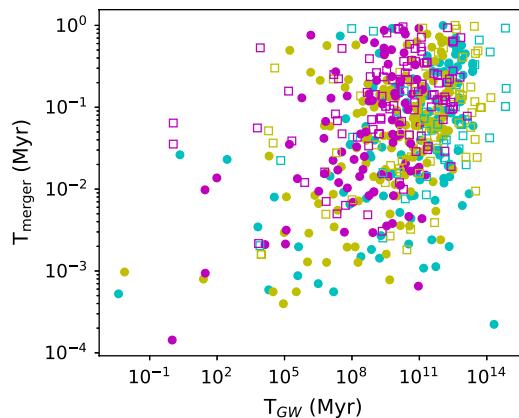


Figure 3. Merger time T_{merger} as a function of the nominal GW merger time-scale T_{GW} (Peters 1964), for IMBH-SBH binaries that end up as IMRIs. The IMBH has mass $M_{\text{IMBH}} = 5 \times 10^3 M_{\odot}$ and orbits an SMBH of mass $M_{\text{SMBH}} = 4 \times 10^6 M_{\odot}$ (filled circles) or mass $10^8 M_{\odot}$ (void squares), with semi-major axis $a_{\text{out}} = 0.1$ pc and eccentricity $e_{\text{out}} = 0.0$ (cyan), 0.4 (yellow), 0.7 (magenta), respectively.

and finally merges. The smaller $M_{\text{IMBH}}/M_{\text{SMBH}}$ and a_{out} , the more rapid the merger. The typical rate is $\sim 1 \text{ Gpc}^{-3} \text{ yr}^{-1}$.

Conclusions

- present and upcoming GW missions promise to shed light on black holes of every size
- black holes merge in different environments in different hierarchies
- most of the scenarios predicts similar rates in the range $\sim 0.01 - 100 \text{ Gpc}^{-3} \text{ yr}^{-1}$
- the elusive IMBH population can be revealed in the next few years
- still many viable pathways to be investigated

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