THE SOLAR DYNAMO AND PLANETARY DYNAMO

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ABSTRACT. A dynamo driven by flows of differential rotation and global convection in a rotating sphere is reviewed as a model of the solar and planetary dynamos. The flows can amplify a magnetic field from an infinitesimal level and thus can generate a magnetic field. The flows periodically reverse the polarity of the field and force the generated field system to propagate along iso-rotation surface in the sphere in form of a wave during the generation process. The flows can also generate the field without reversing its polarity depending on the structure of the flows of the differential rotation. The basic dynamo process with and without polarity reversals is explained in terms of topological deformation of field lines by the flows in the sphere. The oscillatory and steady dynamos are interpreted as corresponding to the solar and planetary dynamos respectively.

1. Introduction

A fundamental question of the cosmic dynamo is whether or not there is any kind of fluid motions in a highly conducting electrically neutral medium that can amplify a magnetic field from an infinitesimal level without wires or rods which serve as guiding routes of electric currents in the medium. This historic problem in astrophysics has been answered positively by flows of differential rotation and global convection under the effects of rotation in form of Coriolis force (Yoshimura, 1972, 1975a, b, 1983a, b). We review in this paper physical processes of the dynamo mechanism. The behavior of the magnetic field in a highly electrically conducting fluid or plasma can be described by movement of magnetic field lines that represent the magnetic field vector in space and time. The behavior is governed by an ordinary magnetohydrodynamic (MHD) induction equation with the divergence free equation for the magnetic field vector, neglecting terms second and higher order in (v/c) where v is representative order of velocity of flows and c is velocity of light. The essential aspect of dynamo process is stretching and deformation of magnetic field lines in three-dimensional space governed by the induction equation and the divergence free equation and the associated creation of magnetic field energy. The essence of the dynamo process is time evolution of geometry of the magnetic field lines. We describe in the following how these processes can be understood visually, particularly how the dynamo reverses polarity of the field and forces the amplified field

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system to propagate along iso-rotation surface as a wave to make an oscillatory dynamo and how the same dynamo can also generate a magnetic field without reversing its polarity. We proceed then to the problem how the oscillatory and steady dynamos can be interpreted as corresponding to the solar and planetary dynamos.

2. Formulation

The first attempt to follow the movement of the magnetic field lines deformed by the flows of the differential rotation and the global convection in three-dimensional space was done by averaging the induction equation over longitude in order to reduce the mathematical structure of the governing equations to that of two-dimensional space (Yoshimura, 1972). The resulting equation was called the dynamo equation. The philosophy underlying this procedure is similar to that of mean-field magnetohydrodynamic (MMHD) formulation of the action of turbulence on magnetic field by Steenbeck, Krause, and Rädler (Steenbeck et al., 1963, 1966; Krause, 1976; Rädler, 1976). The averaging procedures in both cases were necessary to reduce the governing equations to those that were solvable in those times. The same concept of averaging was also used in the formulation of dynamo by Parker (1955). The philosophy underlying the longitudinal averaging procedure, however, was different from that of MMHD. The concept of MMHD was applied to the problem of dealing with turbulence with helical twists due to Corilois force without knowing its detailed structure. The concept of the longitudinal averaging, on the other hand, was applied to the problem for the purpose of formulating a solvable equation in two-dimensional space that captured essential aspects of the dynamo process in three-dimensional space. Detailed structure of differential rotation and global convection was taken into account in the formulation. In the MMHD case, the space over which the averaging is to be done must be larger than the scale of the turbulence but coud be much smaller than the system under consideration which is a sphere in the present case. Hence the resulting equation could still be in three-dimensional space. The averaging procedure of MMHD was done to extract the bulk effect of the turbulent twisting of the magnetic field lines in three-dimensional space. The resulting effect was expressed by a parameter α or Γ, which corresponds to regeneration term R in our case. The concept of the turbulence as a medium to drive the dynamo was devised to provide a kind of nonaxisymmetric component in the flows of the dynamo. The cosmic dynamo hypothesis was once questioned seriously by the Cowling's anti-dynamo theorem (Cowling, 1934). He showed that if flows consisted of only axisymmetric component, the dynamo could not work. To overcome the anti-dynamo theorem, Elsassar (1947) suggested that if there was a nonaxisymmetric component in the flows with helical twists due to Coriolis force, Cowling's anti-dynamo theorem needed not hold true. The first candidate for such flows of nonaxisymmetric component was the turbulence with twists due to Coriolis force. For the case of the Earth, global scale flows were considered in the context of explaining the westward propagation of the geomagnetic field. For the case of the Sun, small scale convective flows observed on the surface as granulation were considered. The problems for both cases were how to treat the turbulence and how to specify what kinds of flows in the Earth and the Sun correspond to the turbulence in reality. For the case of the Earth, the structure of the flows and of the magnetic fields was expressed by expansion of the structures by series of spherical hamonics (Bullard and Gellman, 1954). But in this case there were always the question of convergence of the expansion. The geometry of deformation process of the field lines was not clear. In the beginning phase of the development of dynamo theories, the flows were not required to satisfy the Navier-Stokes equation since the principal concern was to prove that the dynamo can work. Any kind of flows was sufficient at that time as far as it could be shown that the dynamo can work. For the case of the Sun, and later for the case of the Earth, the turbulence was treated by the formulation of MMHD. Only one aspect of the structure of the turbulence, the helical twist of the turbulence was considered. However, the net effect of the twist, which is related to the sign of the parameter, α or Γ, was ambiguous. The sign of the corresponding term R, on the other hand, could be determined definitively by the formulation since the structure of the flows explicitly came into the formulation (Yoshimura, 1972). Also, there were criticisms against the MMHD formulation for the case of the Sun mainly due to the fact, in my opinion, that the scale of the observed turbulence on the Sun in form of granulation or even supergranulation and the observed scale of magnetic field elements are similar to each other. We could not be sure whether such a system can be treated with confidence by a simple linear concept of the MHD action of flows on the magnetic field. The interaction could be highly nonlinear so that we could not be sure whether the resulting effect works as a dynamo to generate the magnetic field energy or works as a diffusive medium to dissipate the magnetic field energy as observed on the surface of the Sun (Leighton, 1964). However, the nonaxisymmetric flows need not be such small-scale flows for the case of the Sun too.

3. A Linear Dynamo Driven by Differential Rotation and Global Convection

In the case of global convection as the nonaxisymmetric flows that could drive the dynamo, scale of the flows is comparable to scale of a system under consideration, a sphere in the present case of the

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planetary dynamos. Hence the scale of the flows is so large that the scale of the magnetic field elements in form of small flux tubes on the surface of the Sun or even in form of flux tubes of sunspots are much smaller than the scale of flows of the global convection. In this case, the flux tubes can be regarded as being carried by the flows of the convection rather passively as if they were magnetic field lines which do not exist in reality but whose concept is important to visualize magnetic vector field in space and time. This is the basic philosophy underlying the treatment of the behavior of the magnetic field inside a sphere or a spherical shell by a linear magnetohydrodynamic equation with the divergence free equation.

4. Structure of Differential Rotation and Global Convection

In the present model, structure of differential rotation and global convection is given. The structure of differential rotation is completely arbitrary. This aspect of the formulation gives us a kind of flexibility and universality in the solutions of the problem. We do not have a universal theory yet that can determine internal structure of differential rotation in the Sun and planets and stars in general. However, once we have a tool to follow response of a magnetic field to difference of structure of the rotation, we could have a tool to conversely infer structure of the rotation from behavior of the magnetic field. Results of the present formulation show that dynamo generation of the magnetic field and behavior of axisymmetric component of solutions is sensitive to difference of structure of the rotation but rather insensitive to difference of structure of the convection or modes of the convection as far as the structure has two basic properties, i.e., (i) helical twists, and (ii) wave-like propagation of convective pattern. Both are results of action of Coriolis force of the rotation on the flows of the convection. The structure of global convection which is used in the present formulation is a solution of a linearized Navier-Stokes equation in a thin spherical shell. The solution describes two basic aspects of the effects of rotation on the flows of global convection in a spherical geometry. The helical structure of the flows appears in latitude-radius, longitude-latitude, and longitude-radius planes. The propagation of the convective pattern appears around the rotational axis in longitudinal direction. Whether the pattern propagates prograde (in the direction of rotation) or retrograde (in the direction opposite to the rotation) depends on the magnitude of rotation and modes of convection (e.g., Yoshimura, 1974). Both aspects are universal properties of global flows in a spherical geometry and are at the same time two vital factors for operation of the dynamo. Preserving these two basic properties, the structure of global convection is deformed topologically so that it can also represent flows of global convection in a deep shell or in a whole sphere. The structure of global convection is characterized by a spherical harmonic that represents a mode of the convection when the rotation of the system is reduced to null. When the rotation is not null, more than one spherical harmonics are needed to represent the flows of the convection. The mode is then represented by longitudinal wave number m and order n of the spherical harmonic when the rotation is fictitiously reduced to null. The order n with a given m represents latitudinal structure of the flows of the convection. These are concepts of a linear theory. But these concepts are necessary and sufficient for understanding basic aspects of operation of the dynamo.

5. Operation of the Dynamo Wave

The first attempt to follow the movement of the magnetic field lines under the action of the flows of the differential rotation and the global convection was, as described in section 2, done by formulating a dynamo equation in latitude-radius space by averaging the MHD induction equation over longitude. The divergence free equation can be satisfied automatically by using a vector potential for the magnetic field vector. The field vector is a curl of the vector potential. Then the dynamo equation was solved numerically with a given set of structure of the differential rotation and the global convection. It was found that the oscillatory and growing solutions can be obtained as natural solutions (Yoshimura, 1975a). A new reversed magnetic field system appeared within the pre-existing system and then the whole system propagates along isorotation surface in latitude-radius space (Yoshimura, 1975b). This was the dynamo wave found by Parker (1955). But in those times of Parker (1955), it was not noticed that the dynamo wave propagates along iso-rotation surface. The theorem that the dynamo wave propagates along iso-rotation surface is vital to understand the operation of the solar and possibly stellar dynamos which are oscillatory. The concept that the Butterfly diagram, which shows that the sunspot appearance zones start from mid-latitudes in the beginning phase of the solar cycle and then move toward the equator along with the progress of the solar cycle, represents a cross-section of the dynamo wave at the surface and does not represent directly the propagation of the dynamo wave cannot be understood without the concept of the dynamo wave propagation along iso-rotation surface.

6. Topology of the Magnetic Field Line Deformation of the Dynamo Operation

The second attempt to follow the movement of the magnetic field lines was done by directly solving the MHD induction equation in three-

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dimensional space numerically by a computer (Yoshimura, 1983a, b). The graphic display of the solutions showed that the basic characteristics of the solutions of the dynamo equation in the first formulation in two-dimensional mathematical space were reproducible in the solutions of the induction equation in three-dimensional space. The basic dynamo problem was answered definitively that there exist flows in a continous fluid system that can amplify a magnetic field from an infinitesimal level. Moreover, the longitudinal structure of the magnetic field can be studied by solutions of the three-dimensional MHD equation. We do not re-display here all the same diagrams that show the solutions. Interested readers are referred to the papers. However, an important aspect of the solutions is displayed in Figure 1 to show the dynamo wave reversal process in latitude-radius plane for later purpose. The diagrams shows the axisymmetric zonal component of the three-dimensional solution of the MHD equation in three-dimensional space (Yoshimura, 1983a). The zonal component of the three-dimensional solution in this case is similar to those of the two-dimensional dynamo equation (Yoshimura, 1978a, b, c). Both are for the case when the upper part of the convective cell is much larger than the lower part of the cell. The solution of the two-dimensional dynamo displayed in Yoshimura (1975a) is for the case in which the lower part of the convection zone is much larger than the upper part. In the latter case, the dominant part of the dynamo wave propagates toward the bottom layer while the weaker upper part of the dynamo wave propagates toward the surface. The three-dimensional numerical integration experiments of the dynamo problem showed that, as far as behavior of the zonal component is concerned, the solutions of the two-dimensional dynamo equation are equivalent to those of the three-dimensional MHD equation. Figure 2 shows the time series of the same solution as in Figure 1 at the surface to demonstrate that the Butterfly Diagrams of the toriodal and poloidal components of the magnetic field in latitude are a manifestation of time-series of cross-section of the dynamo wave propagating along iso-rotation surface in latitude-radius space in Figure 1. The rotation rate for this case increases inward and equatorward. However, the graphic display of the solutions at that time was not adequate enough to unambigously show the movement of the field lines in three-dimensional space to demonstrate the generation and reversal process of the dynamo. The basic difficulty came from the fact that, when the magnetic vector field was displayed uniformly in space, the field structure in three-dimensional space was difficult to visualize. To overcome this difficulty, we have recently devised a new kind of graphic display. To display the time evolution of a vector field, we use a concept which is similar to that of a test particle whose motion shows structure of a field in space and time. It cannot show all the details of the field in the whole system under consideration. Similarly we could use a test field line. But then the three-dimensional perception that is essential for understanding dynamo process is not easy. So we use a test deformable rod to display the time evolution of the field in space and time. Figures in the following show the evolution of the magnetic field vector associated with the oscillatory dynamo. The purposes of these figures are to show the following basic aspects of the dynamo process. (i) The helical structure of the magnetic field lines deformed by the flows of the differential rotation and the global convection is different in the upper and lower layers of the convective cell in the radial direction. This different helical structure is related to the propagation of the dynamo wave. (ii) The helical structure derived by the three-dimensional MHD equation is equivalent to that derived by the two-dimensional dynamo equation when it is seen in latitude-radius space. (iii) The combined effects of the differential rotation and the global convection stretch the magnetic field lines and thus amplify the field and reverse the polarity of the field. The first stage of stretching of the magnetic field lines in the longitudinal direction or of creation of the toroidal component from any amount of poloidal component by flows of the differential rotation is well known both for the Earth's case (Elsasser, 1947) and for the solar case (Babcock, 1961). Figure 3 shows the toroidal field created by this process and the geometry and position of the hexahedrons of following diagrams in the context of the spherical geometry.

The second stage is deformation of the field lines by the convective flows. This was studied by Weiss (1966) in two-dimensional space. In the case of Weiss (1966), the convective flows with cell-like structure wind up the field lines and concentrate the field in the boundaries between different convective cells. The field would eventually be destroyed by the diffusion. In the present case, the convective pattern propagates along the rotational axis so that, in a reference frame in which the convective pattern is stationary, the rotation looks like a mean flow passing by the convective pattern. In this case, the resulting stationary flow pattern in the reference frame looks like an ocean surface wave. The magnetic field lines represented by the rod are deformed by the convection but are not wound up at the boundaries. This stage is shown in Figure 4. The two rods in the upper and lower parts of the convection cells are displayed. The boundary between the two parts is where the horizontal flows change their direction. When the rotation is differential, there would be latitudes where mean flows pass by the pattern in the direction of rotation as well as in the opposite direction depending on the relative angular velocity of the pattern with respect to the angular velocity of the rotation. The digrams shown here are for the case when the propagation angular velocity of the pattern is smaller than that of rotation so that the mean flows of rotation pass by the pattern in the direction of rotation.

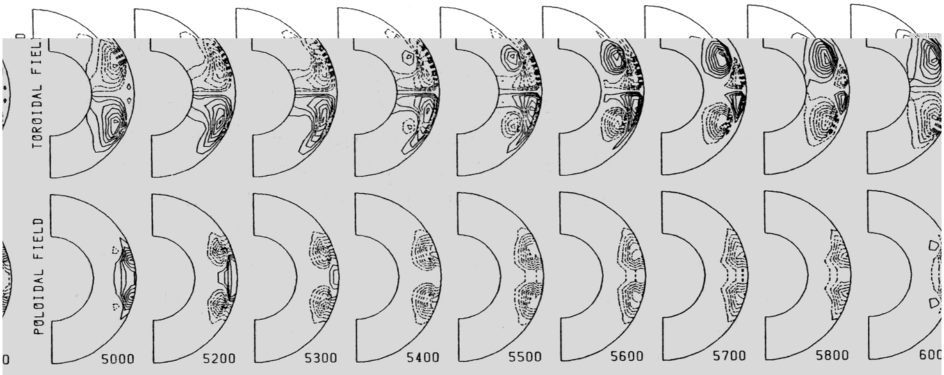


Fig. 1. Time evolution of zonal components of a solution of the three-dimensional MHD equation in latitude-radius space showing dynamo wave propagation driven by flows of differential rotation and global convection (Yoshimura, 1983a).

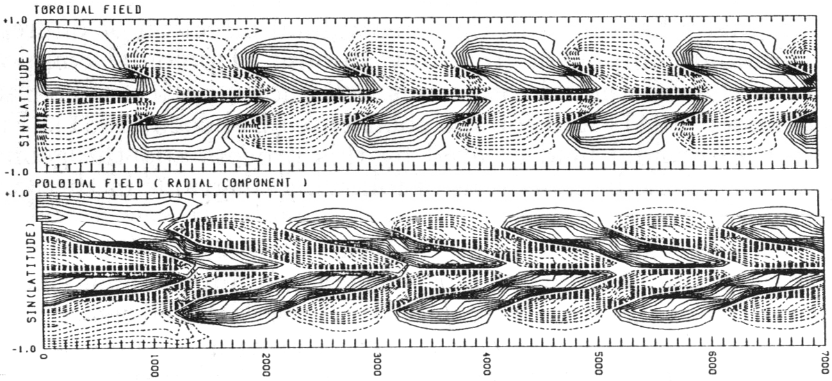
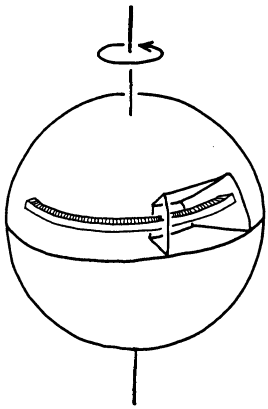


Fig. 2. Time series of cross-section of the dynamo wave of Fig. 1 at the surface. Abscissa is time step. Ordinate is sin (latitude). The patterns reflect the dynamo wave propagation along iso-rotation surface, which increases both downward and equatorward.

Fig. 3. The first stage of the dynamo process showing creation of the toroidal field from any poloidal field by shearing flows of differential rotation. Geometry of a sphere and a hexahedron of the dynamo system in the following figures are shown. The magnetic field line is represented by the deformable rod.



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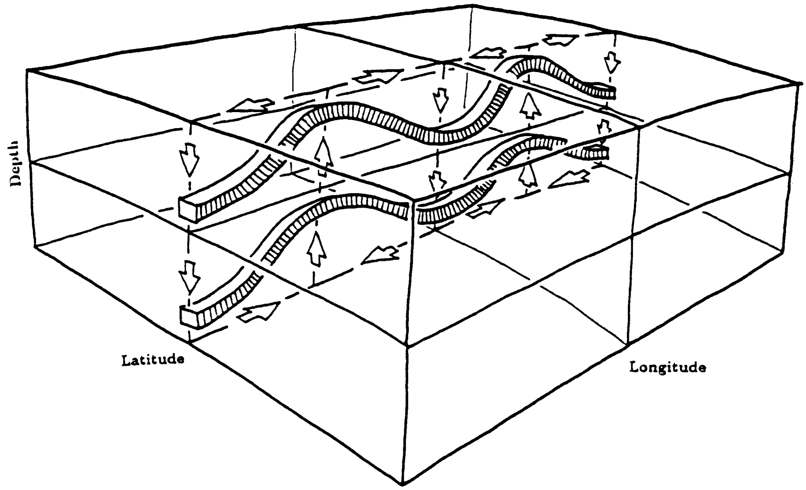

Fig. 4. The second stage of the dynamo process showing deformation of the field line rod by the convective flows. Since convective pattern propagates in the opposite direction of rotation, mean flows toward the direction of rotation are superposed on the convective flows. As a result of this, the field line rod is not wound up around the convective cells. The horizontal axes are for longitude and latitude coordinates. The vertical axis is for radial coordinate. The direction of the field line rods is along the longitudinal coordinate toward the direction of rotation.

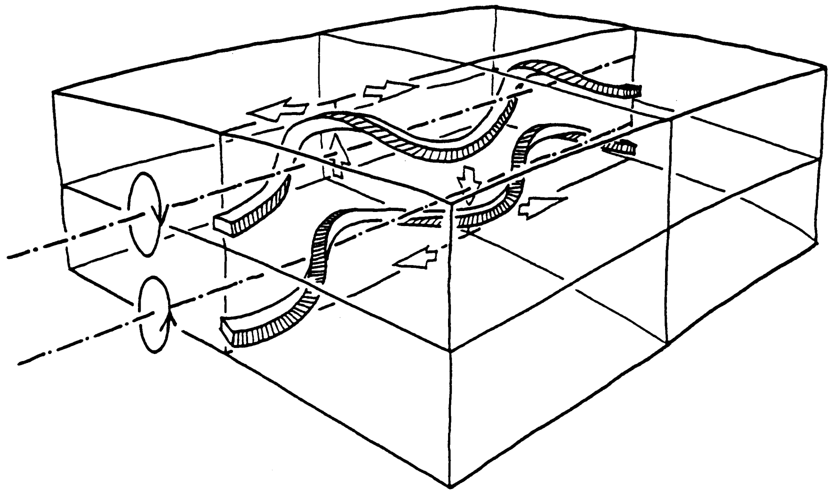

Fig. 5. The third stage of the dynamo process showing the helical twists of the field line rod by the convective flows affected by Coriolis force. This case is for the northern hemisphere. The effect is equivalent to the clockwise (anti-clockwise) twist in the upper (lower) layer when the field line rods are seen in latitude-radius space toward the direction of rotation.

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The third stage is the helical twist of the convective flows and associated twist of the magnetic field lines. The helical structure of the flows due to the Coriolis force makes the flow pattern twist. Figure 5 shows the case for the northern hemisphere. The Coriolis force in the upper part acts on the flows in a clockwise direction in the place where the upflows reach the surface and hence twists the flow pattern and the magnetic field lines clockwise when they are seen in the latitude-radius plane toward the direction of rotation. In the lower part on the other hand, the Coriolis force acts in the same clockwise direction but in the place where the downflows reach the bottom boundary and hence twists the flow pattern and the magnetic field lines anti-clockwise in the latitude-radius direction. This is the basic mechanism how the sign of the regeneration term R, which is equivalent to the parameter α or Γ of MMHD, is determined and this is why the sign is different in the upper and lower parts in the convection zone. It is positive (negative) in the upper part of the convection zone and negative (positive) in the lower part in the northern (southern) hemisphere.

The fourth stage of the action of the flows of the differential rotation on the field lines is shown in Figures 6 and 7. The behavior of the field lines is different for different structures of differential rotation. Figure 6 (Figure 7) shows the case of pure latitudinal (radial) differential rotation. The rod is shown only for the upper layer of Figures 4 and 5. In Figure 6 (Figure 7) of the case of latitudinal (radial) differential rotation, equatorial low latitudes (deeper layers) rotate faster than high latitudes (shallower layers). The action of the latitudinal (radial) differential rotation is equivalent to the rotational twist around the vertical (horizontal) axis denoted by the thick dotted lines in the figures. A new reversed field system appears in the deeper (higher latitude) zone for the case of Figure 6 (Figure 7).

The fifth stage is the appearance of the dyamo wave and propagation of the wave. By the winding and twisting actions of the flows of the convection and the differential rotation, a reversed field system is created in one place. In the opposite side along the iso-rotation surface, the field is strengthened. As a result of this process, the whole magnetic field system with two polarities propagates as a dynamo wave along iso-rotation surface. The process for the case of the latitudinal differential rotation of Figure 6 is shown in Figure 8. When the stretching and twisting process shown in Figure 6 continues, the dynamo wave appears in the radial direction. The process creates a new reversed field system in the deeper layer and at the same time strengthens the field system in the shallower layer. The net effect is creation of two field systems with opposite polarities that propagate toward the surface as a wave. We call this wave the dynamo wave, which is equivalent to the dynamo waves of Parker (1955) and of Yoshimura (1975b) though the governing equations of the waves in

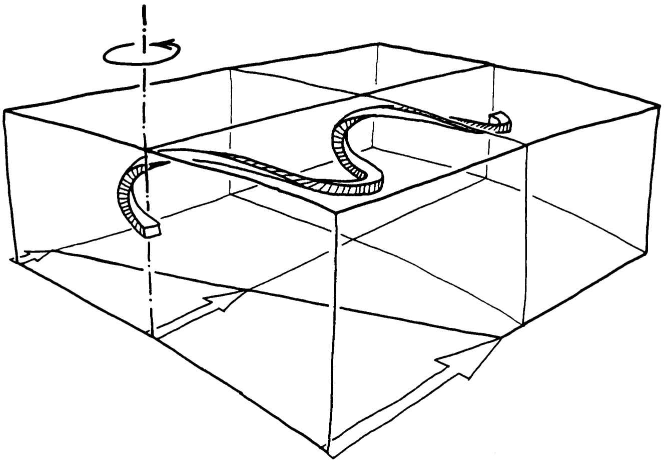

Fig. 6. The fourth stage of the dynamo process for the case of the latitudinal differential rotation.

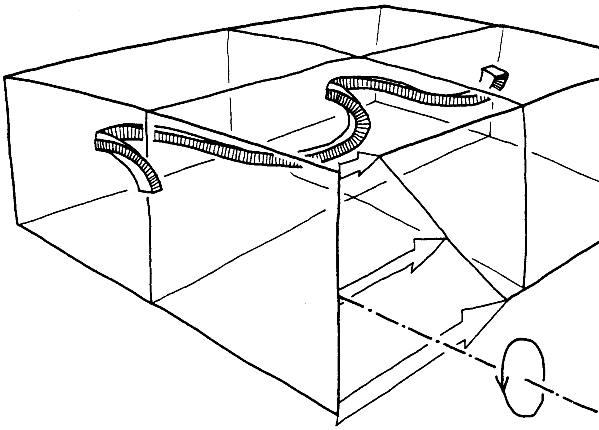

Fig. 7. The fourth stage of the dynamo process for the case of the radial differential rotation.

the present case and the latter cases are different.

In the case of the lower layer with the opposite helical twist shown in Figure 5, the wave propagates toward the bottom boundary. Similarly, when the stretching and twisting process continues for the case of the radial differential rotation in Figure 7, the field systems propagate toward the equator as a wave. The generalization of the situations is the theorem that the dynamo wave propagates along isorotation surface in the direction determined by the sign of R or α or Γ that represents direction of the effect of the helical twist of the flows

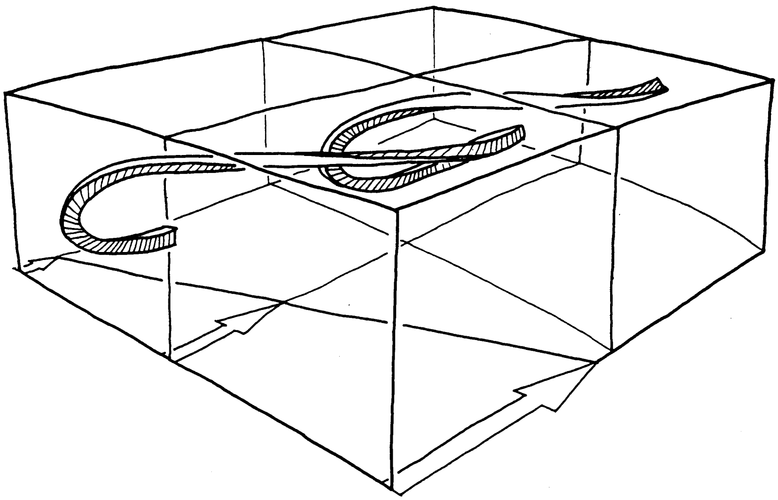

Fig. 8a. The fifth stage of the dynamo process showing the first step of reversal of the field line rod from the diagram of Fig. 6 of the case of the latitudinal differential rotation.

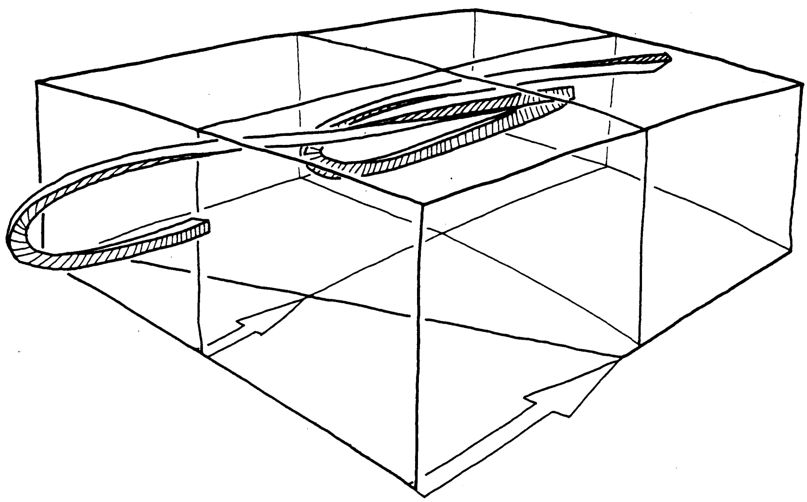

Fig. 8b. The second step of reversal of the field line rod. The reversed field line rod in the lower part is strengthened. The upper part is stretched and is strengthened.

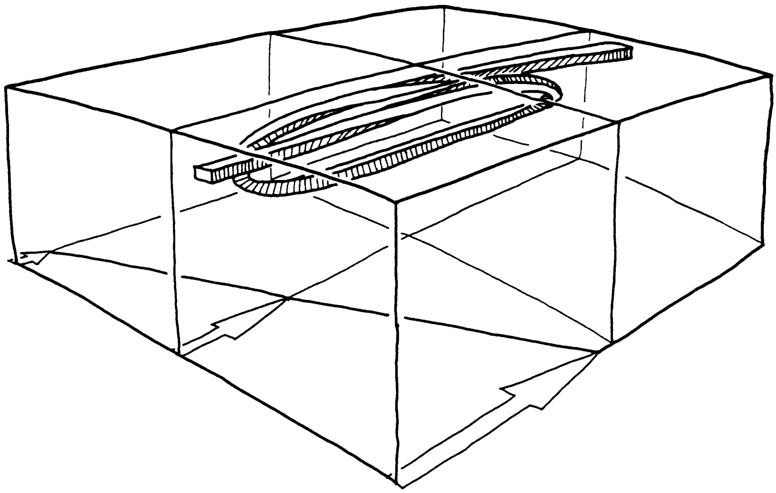

Fig. 8c. The third step of reversal of the field line rod. The field line rod is folded to give rise to the reversed field in the lower part and the strengthened field in the upper part.

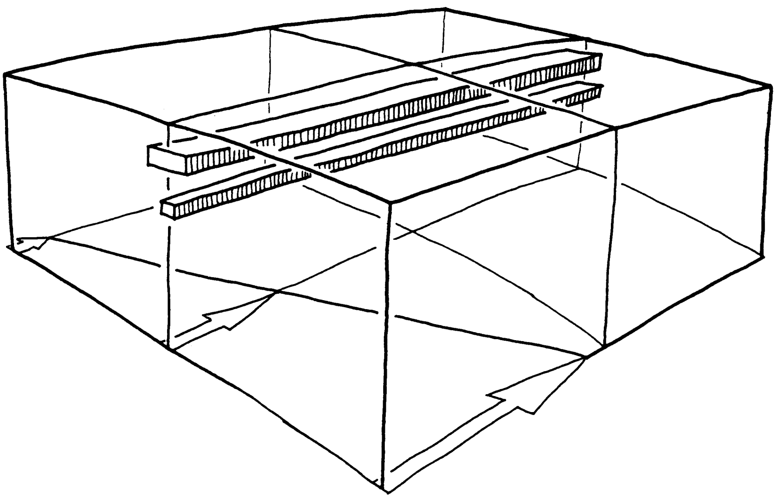

Fig. 8d. The fourth step of reversal of the field line rod. Reconnection makes the upper straight strengthened field line rod in the same direction as in Fig. 3 and the lower straight reversed field line rod. In reality, the whole stages and steps of Figs. 3 - 8 take place simultaneously.

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and the magnetic field lines by Coriolis force (Yoshimura, 1975b). This theorem, which was found in the two-dimensional dynamo formulation, holds true also for the three-dimensional MHD dynamo formulation. During the process, the field lines are stretched and the magnetic field energy is created and thus the dynamo can work. The reversal is a natural result of the dynamo mechanism and the oscillatory dynamo is universal.

7. The Solar Dynamo and Planetary Dynamo

Then a question naturally arises. How can a steady dynamo without polarity reversals be achieved by the same mechanism as that of the oscillatory dynamo? The solar dynamo as the generation mechanism of the solar magnetic field and as the driver of the solar cycle can naturally be interpreted by the oscillatory dynamo. Many observable characteristics were reproduced and predicted by the model. A prediction, for example, was later confirmed by observation (Yoshimura, 1976a, b). The Earth's dynamo is generally regarded as an example of the steady dynamo. The polarity of the Earth's magnetic field does not change for a long time though it reverses a few times in a million years. By similarity arguments, planetary dynamos are likely to be steady dynamos. These concepts are, however, to be confirmed by observations. When we do not have them, we need to explore properties of theoretical dynamo models in order to examine how general or how accurate these inferences could be. The first steady dynamo solution in the present model was found by chance in an effort to determine the level of solar cycle by a nonlinear model (Yoshimura, 1978b). The level is determined by the balance between the strength of the dynamo and the diffusion of the magnetic field. The strength of dynamo is weakened by modification of the flows of the dynamo by Lorentz force of the generated magnetic field. When the dynamo weakens, a balance is achieved between generation and diffusion processes. At this stage, the solution can either be oscillatory or steady. In a nonlinear model of the dynamo, a steady solution was found in which the balance is achieved without polarity reversals. In an effort to understand this phenomenon, a systematic exploration of nature of solutions of an eigen-problem of linear dynamos as a function of strength of the dynamo was done within the context of the two-dimensional formulation of Yoshimura (1972, 1975a). The regeneration factor R was assumed to be constant in radial direction corresponding to the upper layer of the convective cell. We have found that the steady dynamo can easily be achieved when the dynamo is weak and the structure of the differential rotation is such that the dynamo waves propagate inward to the narrower part of the deep spherical shell (Yoshimura et al., 1984a, b, c). These situations can be achieved when latitudinal differential rotation is dominant

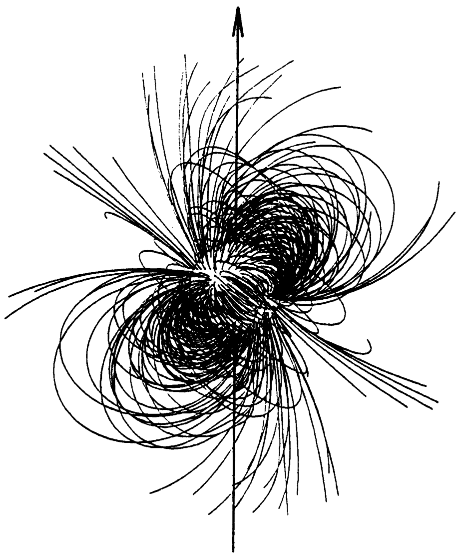

Fig. 9. An example of the steady growing solution with a tilted and offset dipole axis as a model of the magnetic field of planets Uranus and Neptune.

and the poles rotate faster than the equator for the upper part of the convective cell. When the dynamo is strong or when the dynamo is in an infinite free space without boundaries, the dynamo always reverses the polarity of the field as Figures 6, 7, and 8 demonstrate. However, when the dynamo is weak, the diffusion makes the field system expand in space. Then, if there is no space for the dynamo wave to propagate within boundaries of the dynamo system, a spherical shell in the present case, there could be situations when the dynamo generates the field against the diffusion process and yet cannot reverse the polarity of the field. These situations were achieved in our numerical integration experiments of the dynamo equation in two-dimensional space as well as in numerical integration experiments of the MHD induction equation in three-dimensional space when either upper part or lower part is dominant and the direction of progagation of a potential dynamo wave under a fictitiously stronger dynamo action is toward the narrower bottom zone, or when spacial scales of the upper and the lower parts are similar and the potential dynamo waves from the two zones propagate toward the boundary between the two zones and collide at the boundary. Whether or not a planet has such a convective or fluid shell with such flows must be seen by solving dynamics of flows in the planet. In any case, we can say from the above results that a planet has likely a steady dynamo when the poles rotate faster than the equator, which is opposite to the case of the solar dynamo.

8. Longitudinal Structure of the Generated Magnetic Field and a Case of Steady Dynamo with a Tilted and Offset Dipole Axis

The dynamo process as a generation mechanism of a magnetic field is sensitive to the difference of the differential rotation structure but not much to the difference of the global convection structure. Even when the convection consists of different kinds of modes, the basic features of the dynamo do not change. One exception to this general statement is the longtitudinal structure of the magnetic field. The longitudinal structure reflects directly the longitudinal structure of the global convection which depends on the mode of the convection (Yoshimura, 1971). Coronal holes as a manifestation of the global surface magnetic field are likely an example of this case and reflect the structure of the global convection. An aspect of the three-dimensional MHD induction formulation of the dynamo which the two-dimensional dynamo equation formulation does not have is its capability to resolve and represent the longitudinal structure of the magnetic field. One example that demonstrates this aspect is its capability to reproduce the tilted and offset dipole fields of Uranus and Neptune. These fields arise because the nonaxisymmetric component of the magnetic field that is caused by the nonaxisymmetic global flows of longitudinal wave number 1 is to be superposed onto the axisymmetric field. The axisymmetric field has north-south mirror-symmetry. One example is shown in Figure 9 which is a steady solution with the differential rotation of polar acceleration and the global convection of wave number 1. Whether such flows are achieved in Uranus and Neptune must be pursued further both from a dynamical point and an observational point of view. We need to send a series of spacecrafts to these planets to explore their internal dynamics and dynamical evolution.

9. Conclusions

The dynamo process reviewd here is for the case of a cosmic dynamo in which flows of differential rotation and global convection amplify a magnetic field from an infinitesimal level and reverse its polarity in form of a propagating dynamo wave. This time-dependence of the magnetic field arises despite the fact the flows are time-independent or stationary in a reference frame. This simple model works as a model to prove that the cosmic dynamo can work against the Cowling's anti-dynamo theorem. Beyond this simple application, the model can also work as a model of the oscillatory solar dynamo and the steady planetary dynamo. Depending on the structure of the flows, the planetary dynamo could also be an oscillatory dynamo. We have proposed a model, as a matter of fact, in which the reversal of the Earth's magnetic field takes place by a sporadic and temporary transition to an oscillatory dynamo during a short interval (Yoshimura, 1980). The flows also need not be time-independent. When there are more than one mode of global convection, then the problem becomes time-dependent since the propagation angular velocities of different modes are different. There is no reference frame in which the convective flow pattern with mixed modes looks stationary. This is a generalization of the present dynamo to a more complex system of flows. We have to keep in mind, however, that when we apply this model to a real case in nature, we need to make sure that such flows exist in reality. The driving force of the flows need not be convective. As long as there are such flows with properties described here and the flows are strong enough against diffusion, the dynamo works.

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