14

Nonequilibrium issues in RHICs and DCCs

14.1 Relativistic heavy ion collisions (RHICs)

14.1.1 In the beginning

The goal of this chapter is to provide a short summary of the main points where nonequilibrium field theory may contribute to our understanding of relativistic heavy ion collisions. We skip over details of strong interaction processes, but focus on those aspects which are directly related to the nonequilibrium features of the (collective) dynamics.

The relevant experiments are the Super Proton Synchroton (SPS) (CERN) and the Relativistic Heavy-Ion Collider (RHIC) (Brookhaven), with the Large Hadron Collider (LHC) coming on line soon. SPS accelerates lead ions ($Z = 82$, $A = 207$) to energies of 17 GeV per nucleon in the center-of-mass frame; RHIC collides gold ($Z = 79$, $A = 197$) at energies of 130 to 200 GeV per nucleon. The RHIC experiments are described in detail in the so-called "white papers," which are possibly the most reliable source on the subject [BRAHMS05, PHOBOS05, STAR05, PHENIX05]. Other basic references are [Cse94, Won94, Gyu01, Shu88].

We shall work in natural units, the characteristic scale for strong interactions being $1\ \mathrm{fm} = 10^{-15}\,\mathrm{m} = (200\,\mathrm{MeV})^{-1}$. The strength of the interaction is measured by the structure constant $\alpha_S = g^2/4\pi$, where g is the coupling constant. (We assume that the symmetry group is $SU(3)$ with eight gluons.) In the perturbative regime $E \gg \Lambda_{\mathrm{QCD}} \sim 200\,\mathrm{MeV}$, the structure constant runs with scale as

$$\alpha_S(E) = \frac{12\pi}{(33 - 2n_f)\ln\left[\frac{E^2}{\Lambda_{\mathrm{QCD}}^2}\right]} \tag{14.1}$$

where n_f is the number of flavors (6) and Λ_{QCD} is the QCD energy scale. This means that for scales of the order of the proton mass $m_p \sim 1\,\mathrm{GeV}$, $\alpha_S \sim 0.5$. Because of the logarithmic fall off, it will not get much smaller in the relevant range of energies.

The most abundant product from the heavy ion collisions are the lightest mesons, the pions $\pi^{\pm}$ and π^0 with masses $m_\pi \sim 140\,\mathrm{MeV}$. Pions are pseudoscalars, so they do not have different polarization states. The proton, on the other hand, comes in two different spin states – this will be important in what follows.

One of the goals of the RHIC program was to probe into possible new phases of nuclear matter at higher energies such as a conjectured deconfined phase. In such a high-energy phase, matter would most likely be a plasma of gluons and

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(massless) quarks (quark–gluon plasma, QGP). Remember that for relativistic particles each bosonic degree of freedom contributes

$$\epsilon_B = \frac{\pi^2}{30} T^4 \tag{14.2}$$

to the energy density in equilibrium, while (neglecting chemical potentials) each massless fermionic degree of freedom contributes $\epsilon_F = (7/8)\,\epsilon_B$. We have eight different gluons with two polarization degrees of freedom each, and four effectively massless quarks (u, d and their antiparticles) coming in three colors and two spin states each. Therefore the energy density in the deconfined phase is

$$\epsilon_{\text{plasma}} = \frac{37}{30}\pi^2 T^4 \tag{14.3}$$

and the pressure is $p_{\text{plasma}} = \epsilon_{\text{plasma}}/3$.

In the low-temperature phase, only the pions are effectively massless. These pions live on a quark condensate which enforces confinement. Therefore the energy density is $\epsilon_{\text{hadron}} = \epsilon_{\text{pions}} + \epsilon_{\text{condensate}}$, where

$$\epsilon_{\text{pions}} = \frac{3}{30}\pi^2 T^4 \tag{14.4}$$

and $\epsilon_{\text{condensate}} \equiv -B$, where $B \sim \Lambda_{\text{QCD}}^4$ is known as the *bag constant*. The pressure of the confined phase is $p_{\text{hadron}} = \epsilon_{\text{pion}}/3 + B$.

At the coexistence point, both phases have the same pressure, and so the critical temperature obeys

$$\frac{34}{90}\pi^2 T_{\text{c}}^4 = B = \Lambda_{\text{QCD}}^4 \tag{14.5}$$

namely $T_{\text{c}} = \left(90/34\pi^2\right)^{1/4} \Lambda_{\text{QCD}} \sim 0.72\Lambda_{\text{QCD}} \sim 150\,\text{MeV}$. This means that to enter the deconfined phase, we need a mininal energy density of $\epsilon_{\text{crit}} = (3 \cdot 37/34)\,\Lambda_{\text{QCD}}^4 = 650\,\text{MeV/fm}^3$.

Of course, this is the transition point at zero chemical potential only; in general, we have a coexistence curve in the (μ, T) plane, so that the critical temperature may be lowered by increasing the baryon number density.

Nevertheless, evidence seems to suggest that the QGP has not been created at RHIC [BRAHMS05, PHOBOS05, STAR05, PHENIX05]. The high-energy collisions have created what seems to be a new state of dense nuclear matter, whose description in terms of purely hadronic degrees of freedom seems inadequate. This suggests the presence of unscreened color charges over distances larger than the size of a nucleon. However, the system seems to be strongly interacting throughout, with properties closer to a liquid than to a plasma.

14.1.2 The Bjorken scenario

Virtually all the field-theoretic analyses of RHICs assume a spacetime picture of collision provided by the Bjorken model [Bjo83]. The colliding nuclei are seen as

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slabs of quark and gluon matter. In the center-of-mass frame, both slabs approach each other at near light speed. Upon collision, the two slabs of matter will mostly go through each other, leaving behind a wake of hot plasma. We may then distinguish three different regions: the two fragmentation regions corresponding to the receding slabs, and the central region corresponding to the plasma in between. We are interested in phenomena in the central region.

At the time of crossing a number of hard scattering processes will occur, whose products will directly reach the detectors. These hard processes are unrelated to the nonequilibrium dynamics of the plasma; and may presumably be predicted on perturbative QCD grounds. In what follows, we will assume this hard component has been isolated despite great difficulty to achieve this in reality.

The hot plasma will expand and cool, and eventually fragment into ordinary particles in flight intercepted by the detectors. We wish to predict the number of particles of each species to be detected, as a function of the angle θ between the direction of flight and the direction z of the beam. It is remarkable that with this simple picture we can state a first observable prediction already.

Indeed, because of Lorentz contraction, we may think of the approaching slabs as infinitely thin in the direction of motion z, and in a first approach to the problem, as infinite and homogeneous in the transverse directions x and y. This picture is invariant under boost in the z direction, and so is the final distribution of particles. So if we parameterize the momentum of an out-going particle as $p^0 = E$, $p^3 = p$ and $\left(p^1, p^2\right) = p_\perp$, then the distribution of particles may depend only upon the transverse momentum and $E^2 - p^2 = m^2 + p_\perp^2$. In particular, it must be independent of θ, since $\cos\theta \sim p/E$ is not invariant. It is conventional to plot the yield of the collision in terms of the *rapidity* y, defined by $p/E \equiv \tanh y$, or rather the pseudo-rapidity $\eta = -\ln\tan[\theta/2]$, $\tanh\eta = p/|\mathbf{p}|$. Rapidity and pseudo-rapidity agree at momenta which are large compared to the mass of the particle. Then the prediction in this picture is that there is a *plateau* in the (pseudo) rapidity distribution, at least for small rapidity ($|\eta| \to \infty$ corresponds to the fragmentation rather than the central region).[1]

We may elaborate on the Bjorken picture further. Let us assume that the plasma is formed on the plane $z = 0$ at the time $t = 0$ of the collision, and then expands along the z direction. A given plasma element will cool according to its own proper time τ. Now, as in the twin paradox, the proper time lapse will be less for those elements which move faster, which are also those which reach farther if we compare the relative positions at a given fixed time, as measured, say, in the center-of-mass frame. Thus the plasma will be hotter in the outer layers than in the center. This situation resembles the dessert known as *baked Alaska*, made by briefly putting an ice-cream ball in the oven, thereby the outer crust heats up while the center remains frozen.

[1] This prediction is not clearly borne out by the RHIC data [PHOBOS05]. Therefore, it remains a possibility that analyses based on the Bjorken model are not so relevant to current experiments compared to future higher energy collisions.

Eventually, at some given constant τ surface, the plasma will be cold enough (and/or dilute enough) to break up into hadrons. Assuming that the product hadrons are thermally distributed, massless and at zero chemical potential, the Bose–Einstein distribution predicts that the energy per particle is $\epsilon/n = 2.7\ T$. Since temperature is constant on the break-up surface, this means that in all collisions particles should have the same average energy. Indeed, it is observed that the energy per particle is about 0.8 GeV, regardless of the center-of-mass energy and impact parameter.

Another important observation is that for transverse momenta higher than 2 GeV, the number of emitted protons is actually higher than pions. This can be explained as a consequence of hydrodynamic behavior [HeiKol02a], or else, at very large momenta, through a recombination mechanism [MulNag06]. If no chemical potentials were involved, then equality would obtain (at $p_T = 2$ GeV) for a temperature of about 340 MeV. In reality, pions do not have chemical potential, but protons do, associated with baryon number conservation. Adding a chemical potential $\mu \sim 40$ MeV for the protons reduces the crossing temperature to 280 MeV [HeiKol02a].

To obtain a more quantitative description of the process, we may describe the plasma as a relativistic ideal fluid [BelLan56, CarDuo73, CarZac83]. The assumption of a homogeneous plasma in the transverse direction is too simplistic, and a full four-dimensional solution must be sought, which requires numerical methods [KoSoHe00, MolGyu00, Hir01, MMNH02, HeiWon02, TeLaSh01, HirTsu02, KolRap03, HeiKol02b, HKHRV01]. To close the hydrodynamic system of equations we must provide the equation of state. The central feature of this is the "softening" near the critical point, meaning that the speed of sound $c_{\mathrm{s}}^2 = \partial p/\partial\epsilon \to 0$ as we approach the transition point. The softening of the equation of state affects the evolution of the fireball, which then becomes a signal of whether the transition point has been reached or not.

Since perfect fluids conserve entropy, the total entropy within the fireball remains constant, and T scales as $V^{-1/3}$. So, if the expansion is one-dimensional, and we consider the volume enclosed between two fixed rapidities, then $T \sim \tau^{-1/3}$, where τ is the proper time. In particular, the energy density scales as $\tau^{-4/3}$ rather than τ^{-1}, as in our earlier estimate. This leads to a slight increase in the estimate of the initial plasma temperature.

For treatment of RHICs beyond perfect fluids, see [GyRiZh96a, GyRiZh97, Ris98, Tea03].

14.1.3 Break-up

We now consider more closely the phenomenon of break-up [CooFry74, SiAkHa02]. Assume this occurs on a three-dimensional surface Σ defined by some equation $\Sigma(x^\mu) = 0$. If x_0 is a solution, then the normal vector at x_0 is $n_\mu = (-\alpha)\Sigma_{,\mu}$, $\alpha = \left(-\Sigma_{,\mu}\Sigma^{,\mu}\right)^{-1/2}$. We shall assume that n_μ is time-like. For a more realistic scenario where the surface has both time-like and space-like regions, see [Bug03]. The invariant measure on Σ is given by $d^3\sigma = d^4x\, \delta(\Sigma)\, \alpha^{-1}$.

Let us assume that both before and after break-up, we can describe matter as a perfect relativistic fluid. Let $K_a = \partial/\partial x^a$ be the four Killing vectors of Minkowski space. Then Gauss' theorem shows that the quantities $n_\mu K_{a\nu} T^{\mu\nu}$ and $n_\mu N^\mu$ are continuous across the break-up surface (we shall consider only one conserved current, corresponding to, say, the baryon number). These conditions plus the equation of state of the hadronic phase define the energy density, pressure, baryon number density (or equivalently, the temperature and chemical potential) and the 4-velocity of the hadrons at break-up. The detailed spectrum is found by assuming that the hadrons are thermally distributed.

In principle we could distinguish between matter having a thermal distribution of momenta (kinetic equilibrium) and in chemical equilibrium. Correspondingly, there is a kinetic freeze-out, and a chemical freeze-out, which are not necessarily simultaneous. This permits some extra freedom in matching models to data.

The total number of emitted particles is

$$\int d^3\mathbf{x}\; K_{0\mu} N^\mu_{\mathrm{had}} \tag{14.6}$$

where the integral is over some $t =$ constant surface well to the future of the collision. Because of Gauss' theorem, we may replace the integral by an integral over the break-up surface (we may have to complete this surface to get a Cauchy surface, but the particle density flux will vanish on these additions anyway). But then we may use the matching conditions to express this integral in terms of the particle current before break-up. We obtain the total number of emitted particles as

$$\int d^4x\; \delta(\Sigma)\, \Sigma_{,\mu} N^\mu_{\mathrm{hydro}} \tag{14.7}$$

In practice, we may wish to smear a little the position of the break-up surface, thus writing the total number of emitted particles as

$$\int d^4x\; \left[\frac{e^{-\Sigma^2/2(\Delta\Sigma)^2}}{\sqrt{2\pi}\,(\Delta\Sigma)}\right] \Sigma_{,\mu} N^\mu_{\mathrm{hydro}} \tag{14.8}$$

The total number of particles of species i with momentum p^μ is

$$g_i \int \frac{d^4x}{C(x)} \left[\frac{e^{-\Sigma^2/2(\Delta\Sigma)^2}}{\sqrt{2\pi}\,(\Delta\Sigma)}\right] \Sigma_{,\mu} N^\mu_{\mathrm{hydro}} \delta\left(p_i^2 - m_i^2\right) \frac{U^{\mathrm{had}}_\lambda p_i^\lambda}{\left[\exp\left(-\beta_\nu p_i^\nu - \mu b_i\right) - \varepsilon_i\right]} \tag{14.9}$$

where

$$C(x) = \sum_i g_i \int \frac{d^4p_i}{(2\pi)^4} \delta\left(p_i^2 - m_i^2\right) \frac{U^{\mathrm{had}}_\mu p_i^\mu}{\left[\exp\left(-\beta_\nu p_i^\nu - \mu b_i\right) - \varepsilon_i\right]} \tag{14.10}$$

The two basic observables are the total number of particles with transverse (with respect to the beam axis) momentum $p_\perp$, which is usually given in terms of the

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transverse mass $m_\perp^2 = m^2 + p_\perp^2$, and the elliptic flow coefficient v_2, which results from fitting the particle spectrum in the transverse plane to a second harmonic $(1 + 2v_2(p_\perp)\cos 2\phi)$, where ϕ is the angle measured from the reaction plane. This is equivalent to considering an elliptic fireball, in which case v_2 measures the eccentricity of the ellipse. The first harmonic is called directed flow, and would represent a shifted spherical fireball in the transverse plane [VolZha96].

In our simplified discussion we have not considered the possibility that some particles produced at break-up may actually decay before reaching the detectors, so that the one-to-one correspondence we have assumed is not strictly valid. Also, because of long-range interactions, the propagation of charged particles from break-up to detection is not quite free. Both phenomena must be considered for a meaningful contrast between theory and experiment. Finally, observe that the form of the distribution function we have used is not a solution of the transport equation if there are gradients of the hydrodynamical variables. If these gradients are important, one may consider using an improved distribution function [Sin99].

The agreement of predictions from hydrodynamical simulations with experimental data is good, provided the simulation is started very early (earlier than 1 fm/c after the collision). If one believes that the validity of hydrodynamics preassumes (local) equilibration, this very short time is somewhat of a puzzle. However, as we shall see presently, not all is well with hydro simulations. This is the main area where NEqQFT may be relevant to understanding RHICs.

14.1.4 Measuring the fireball

We shall describe a method of data analysis from heavy ion collisions which, in principle, yields direct information on the geometry of the fireball at break-up. It pertains to studying the simultaneous detection of pairs of identical particles, rather than individual ones [GyKaWi79, Hei96, WieHei99].

Let us make the simplifying assumption that the only particles produced at break-up are pions, and that these may be treated as a free Klein–Gordon field. The Heisenberg pion field operator obeys a wave equation

$$\partial^2 \Phi(x) - m^2 \Phi(x) = -J(x) \tag{14.11}$$

where the external c-number source $J(x)$ represents the particle creating current at break-up. Under the action of this source, the pion vacuum state $|0\rangle$ evolves (in the interaction picture) into

$$|J\rangle = T\left\{\exp\left[i\int d^4x\; J(x)\,\Phi_0(x)\right]\right\}|0\rangle \tag{14.12}$$

where $\Phi_0(x)$ is a free Klein–Gordon field. $\Phi_0(x)$ may be expanded into positive and negative frequency parts

$$\Phi_0(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,\frac{e^{i\mathbf{px}}}{\sqrt{2\omega_p}}\left\{e^{-i\omega_p t}a_{\mathbf{p}} + e^{i\omega_p t}a^\dagger_{-\mathbf{p}}\right\} \tag{14.13}$$

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where $\omega_p^2 = \mathbf{p}^2 + m^2$. The state $|J\rangle$ is a coherent state

$$a_{\mathbf{p}} |J\rangle = \frac{iJ_{(\mathbf{p},\omega_p)}}{\sqrt{2\omega_p}} |J\rangle \tag{14.14}$$

where

$$J_p = \int d^4x \, e^{-ipx} J(x) \tag{14.15}$$

The number of particles with momentum p in the final state is then

$$N_p = \frac{|J_p|^2}{2p^0} \tag{14.16}$$

Let us introduce the emission function

$$S(x,p) = \int d^4y \, e^{-ipy} J^* \left(x - \frac{y}{2}\right) J \left(x + \frac{y}{2}\right) \tag{14.17}$$

whence

$$|J_p|^2 = \int d^4x \, S(x,p) \tag{14.18}$$

Comparing (14.9) and (14.16), one may be strongly tempted to write

$$S(x,p) = \frac{g}{C(x)} \left[\frac{e^{-\Sigma^2/2(\Delta\Sigma)^2}}{\sqrt{2\pi}(\Delta\Sigma)}\right] \Sigma_{,\mu} N^{\mu}_{\text{hydro}} \delta\left(p^0 - \omega_p\right) \frac{U^{\text{had}}_{\lambda} p^{\lambda}}{[\exp(-\beta_\nu p^\nu - \mu) - 1]} \tag{14.19}$$

The number of pairs of particles, one with spatial momentum $\mathbf{p}$ and another with spatial momentum $\mathbf{q}$, is

$$N_{\mathbf{pq}} = \langle J| a^\dagger_{\mathbf{p}} a^\dagger_{\mathbf{q}} a_{\mathbf{q}} a_{\mathbf{p}} |J\rangle \tag{14.20}$$

For a coherent source, such as discussed so far, $N_{\mathbf{pq}} = N_{\mathbf{p}} N_{\mathbf{q}}$, which is not terribly interesting.

However, let us consider the case in which the source is an incoherent superposition of elementary sources

$$J(x) = \sum_i e^{i\theta_i} J_i(x), \qquad J_i(x) = e^{ip_i(x-x_i)} J_0(x - x_i) \tag{14.21}$$

meaning that the identical elementary sources J_0 are translated, boosted and phased in different ways, with the x_i, p_i, θ_i all random mutually independent variables. In this case, the emission function reads

$$S(x,p) = \sum_{i,j} \int d^4y \, e^{-ipy} e^{i(\theta_i - \theta_j)} J^*_j \left(x - \frac{y}{2}\right) J_i \left(x + \frac{y}{2}\right) \tag{14.22}$$

Averaging over the unknown phase of each source, we get

$$S(x,p) = \sum_i S_i(x,p) \tag{14.23}$$

Let us consider again the average number of pairs

$$N_{pq} = \frac{1}{4\omega_p\omega_q} \left\langle J_p^* J_q^* J_q J_p \right\rangle$$
$$= \frac{1}{4\omega_p\omega_q} \sum_{ijkl} e^{i(\theta_i+\theta_j-\theta_k-\theta_l)} \left\langle J_{k,p}^* J_{l,q}^* J_{i,q} J_{j,p} \right\rangle \tag{14.24}$$

The average over phases vanishes unless $i = l$, $j = k$ or $i = k$, $j = l$ (we neglect the possibility of $i = j = k = l$ simultaneously). Therefore

$$N_{pq} = N_p N_q + \frac{1}{4\omega_p\omega_q} \left| \sum_i \left\langle J_{i,q}^* J_{i,p} \right\rangle \right|^2 \tag{14.25}$$

The second term shows the existence of correlations among the created particles. This is the so-called pion bunching, or HBT (for Hanbury-Brown/Twiss) correlations. In the real world, the sources are neither totally coherent nor totally incoherent; we may account for this by adding a fudge factor to the second term in (14.25) (for a more sophisticated treatment, see [AkLeSi01]). A similar factor may arise from the superposition of particle emission from a collision *core* and a *halo* of long-lived resonances [NiCsKi98].

Introducing

$$P = \frac{p+q}{2}, \qquad \xi = p - q \tag{14.26}$$

then

$$\sum_i \left\langle J_{i,q}^* J_{i,p} \right\rangle = \int d^4x \; e^{-i\xi x} S(x, P) \tag{14.27}$$

and we see that it is possible to express the HBT correlations in terms of the emission function, for which we already have the ansatz (14.19). In practice, this is too involved to attempt a direct comparison with data. Rather, the usual procedure is, for a given P, to evaluate the moments of the emission function

$$\int d^4x \; S(x, P) = 2\omega_P N_P \tag{14.28}$$

$$\bar{x}^\mu = \frac{1}{2\omega_P N_P} \int d^4x \; x^\mu S(x, P) \tag{14.29}$$

$$R^{\mu\nu} = \frac{1}{2\omega_P N_P} \int d^4x \; x^\mu x^\nu S(x, P) - \bar{x}^\mu \bar{x}^\nu \tag{14.30}$$

Let us assume the source is axisymmetric and $\mathbf{P}$ points in the x-direction (z being the beam direction). In the center-of-mass frame we have $R^{0i} = R^{ij} = 0$ for $i \neq j$. The values of the momenta suggest the approximation

$$S(x, P) = \frac{2\omega_P N_P}{(2\pi)^2 \left(\det\left[R^{\mu\nu}\right]\right)^{1/2}} \exp\left\{ -\frac{1}{2}\left[\frac{(t-\bar{t})^2}{R^{00}} + \frac{x^2}{R^{11}} + \frac{y^2}{R^{22}} + \frac{z^2}{R^{33}} \right] \right\} \tag{14.31}$$

It is important to realize that the $R^{\mu\nu}$ are not the moments of the source as a whole, since the emission function is weighted by P dependent factors. We may think of the emission functions as the probability of a particle with momentum P being emitted at point x. The $R^{\mu\nu}$ then measure the size of the region where emission is most likely. This expression for the emission function is simple enough that we may compute the HBT correlations.

One last point: If the p and q momenta in N_{pq} are on-shell, the components of P and ξ are not independent. We have $P^2 = m^2 - \xi^2/2$, so we may consider P on-shell when ξ is small, and $P\xi = 0$, meaning that $\xi^0 = (P/\omega_P)\,\xi^1$. Therefore

$$\begin{aligned}&\left|\int d^4x\, e^{-i\xi x} S\,(x,P)\right|^2 \\ &\quad = (2\omega_P N)_P^2\, \exp\left\{-\left[\left(\frac{R^{00}P^2}{\omega_P^2} + R^{11}\right)\xi_1^2 + R^{22}\xi_2^2 + R^{33}\xi_3^2\right]\right\} \qquad (14.32)\end{aligned}$$

We see that the HBT correlations may be parameterized in terms of three "radii," with z corresponding to the "longitudinal" direction, x to the "out" direction, and y to the "side" direction.

$$\left|\int d^4x\, e^{-i\xi x} S\,(x,P)\right|^2 \sim \exp\left\{-\left[R_{\text{out}}^2\xi_1^2 + R_{\text{side}}^2\xi_2^2 + R_{\text{long}}^2\xi_3^2\right]\right\} \qquad (14.33)$$

($R_{\text{out}}^2 = R^{00}P^2/\omega_P^2 + R^{11}$, $R_{\text{side}}^2 = R^{22}$, $R_{\text{long}}^2 = R^{33}$). Observe that, in general, we expect $R^{11} \sim R^{22}$, and so the out radius, which is sensitive also to the duration of the emission process (in terms of laboratory time) is predicted to be larger than the side radius. This prediction is not borne out by the data, which show $R_{\text{out}}/R_{\text{side}} \sim 1.25$–$1.5$ [BRAHMS05, PHOBOS05, STAR05, PHENIX05]. This disagreement constitutes the so-called *HBT puzzle*.

This suggests that the emission process occurs early, which reinforces the need for an early onset of the hydrodynamic regime, or else for some new thinking [SoBaDi01, Hum06]. In principle the HBT puzzle is a puzzle only within the framework of hydrodynamical models.

14.1.5 Insights from nonequilibrium quantum field theory

We see from the above analysis that the clue to understanding the physics of RHICs lies in the first fermi/c or so after the collision. This is the point where nonequilibrium field theory methods may have an impact on the theory of RHICs.

The first input for any field theoretic modeling is of course some well-defined initial condition. The basic idea is that each colliding nucleus is not just a bunch of nucleons marching in step, but a rather complex array of gluons and partons. In fact, a naive perturbative calculation yields the result that the number of gluons with a given momentum diverges as the momentum becomes light-like. It is believed that this divergence is cut off at some scale by nonperturbative effects (parton saturation) [Mue01, KhaLev01, KhLeNa01].

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A sophisticated model built on this premise is the so-called color glass condensate [IaleMc02, BjoVen01, KrNaVe02, McLVen94a, McLVen94b, McLVen94c].

The basic framework to understand the early evolution of the plasma is the so-called *bottom-up* scenario [BMSS01, MuShWo05]. The hard gluons released from the color glass condensate take part in both elastic and inelastic collisions. Elastic collisions broaden a little the initial gluon distribution (see below) while inelastic collisions contribute to the creation of a soft gluon background. It may be observed that the emission of ultrasoft gluons is suppressed by destructive interference between multiple collision events, the so-called Landau–Pomeranchuk–Migdal effect [BaiKAt03, ArMoYa01a, ArMoYa01b, ArMoYa02, BBGM06]. On the other hand, nearly collinear events are amplified by the small denominators in the transition amplitude [Won04].

The soft gluons thermalize very efficiently. Eventually they become the dominant species, and we have a picture of a few very energetic gluons on a thermalized soft gluon background. The remaining hard gluons decay (through gluon branching, which is a specific form of wave splitting for a non-abelian plasma). The decay of the hard gluons heats up the soft gluons over and above the cooling from the longitudinal expansion of the plasma, and so we may enter the fully hydrodynamic stage at a conveniently high temperature.

The key question in the bottom-up scenario is how fast the soft fields build up from the initial hard quanta. The natural approach would seem to be to write a kinetic equation for those hard gluons [Mue00a, Mue00b], taking into account both elastic and inelastic processes (see also Chapter 11). The result seems to be that the build up of soft fields is too slow to meet the demands of hydrodynamical RHIC models.

At the time of writing, much effort is being spent on elucidating a proposal by S. Mrowczynski which would result on a much faster growth rate [Mro94a, Mro94b] (see [Mro05] for a recent review). Mrowczynski's insight is that the initial gluon distribution must be highly anisotropic. Since gluons with a substantial longitudinal momentum will stream out of the central region, the momentum distribution in the local rest frame is squeezed along the beam. Under these conditions, the so-called filamentation or Weibel instability sets in. Suppose the initial hard gluon distribution results in alternating currents along a transverse direction. These currents create magnetic fields, and the corresponding Lorentz force accelerates particles along the longitudinal direction. Moreover, particles are redistributed in such a way that the initial currents are amplified, thus setting up a positive feedback loop. While the instability lasts, the soft fields are found to increase exponentially. Instabilities do not directly equilibrate the system but rather isotropize it and thus speed up the process of thermalization [Mro07].

Current efforts are aimed at a precise estimate of the growth rates that may be achieved by this mechanism, and to identify possible effects which may knock off the instability. At the time of writing, the most important limiting factor seems to be that the growing soft modes will in turn excite a turbulent ultraviolet cascade [ArLeMo03, ALMY05, Moo05, ArnMoo05, ArMoYa05, ArnMoo06, MuShWo07]. The energy extracted from the hard gluons through the magnetic fields is returned to them through the cascade. The growth of the soft modes turns from exponential to linear, and eventually ceases altogether. It is not clear whether this effect will rule out fast enough thermalization through Weibel instabilities. In principle, it ought to be possible to obtain an answer by coupling the Yang–Mills classical equations for the soft fields to the Wong kinetic equations for the hard fields (see Chapter 11 and [ManMro06, Mro06, RomReb06, DuNaSt07, Str06, RomVen06, Ven07]), but it is hard to carry out numerical simulations within a realistic parameter range.

14.2 Disoriented chiral condensates (DCCs)

Besides deconfinement, other exotic events are thought to lie just above the QCD phase transition. Among these, one of the best researched is the possibility of chiral symmetry restoration. More concretely, the idea is, if it were possible to heat strongly interacting matter above the chiral restoration temperature, and then quenching it again below the critical point, there exists the possibility that the second time around the system will settle into a different vacuum than the one we are familiar with. That would create a new form of matter, the so called "disoriented chiral condensate" (DCC). When brought into contact with the ordinary vacuum, the DCC would decay with a characteristic burst of particles, whose detection would provide a signature of its existence.

Theoretical and experimental interest in DCCs had a strong surge in the early 1990s [KowTay92], further motivated by the unexplained Centauro events seen in cosmic ray experiments [MohSer05]. After several searches both in an *ad hoc* experiment [MINIMAX03] and as a part of larger RHIC program, no clear detection has been reported. However, this null result is actually in agreement with theoretical estimates. New probes are being suggested which could lead to a positive result [AgSoVi06]. We refer the reader to [Bjo97] and [MohSer05] for reviews.

With these experimental perimeters delimited, let us describe in slightly more detail what a DCC is expected to look like. According to the standard models of particle interactions, the fundamental constituents of hadrons are quarks. There are six flavors of quarks, organized into three isospin doublets (u, d), (c, s) and (t, b). The quark masses increase as we go from one doublet to the next; for the (u, d) pair they are of a few MeV, about a GeV for (c, s) and a few GeV for (t, b). In a first approximation, the (u, d) quarks may be taken as massless.

Now, a theory with a massless isospin doublet would be invariant under *independent* global isospin rotations of the left and right quark components. Thus the isospin group should have been $SU(2) \times SU(2)$, rather than the observed $SU(2)$. In particular, for each hadronic state there would be a partner with opposite parity. This is not even approximately observed, and therefore the $SU(2) \times SU(2)$ symmetry must be broken down to the physical isospin $SU(2)$.

The idea is that the quark vacuum is not invariant under $SU(2) \times SU(2)$. Since the algebra of this group is isomorphic to $SO(4)$, it is natural to take the order parameter for this transition (*chiral* symmetry breaking) as a vector in a four-dimensional internal Euclidean space. The symmetric state corresponds to a vanishing order parameter. A nonzero order parameter picks up a definite direction in four-dimensional internal space, therefore breaking the symmetry down to $SO(3)$, with covering group $SU(2)$. From the microscopic point of view, the components of the order parameter express the formation of quark pair condensates, in a mechanism which resembles the formation of Cooper pairs (with a breaking of the $U(1)$ symmetry) in a BCS superconductor.

According to Goldstone's theorem, the breaking of a global symmetry must be followed by the apparition of one massless particle for each broken symmetry. In our case there are three, one for each $SU(2)$ generator, while the Goldstone bosons are the pions. Of course, quarks are not really massless, $SU(2) \times SU(2)$ is not an exact symmetry, and pions are therefore not quite massless, but their masses are small enough, certainly in comparison with the quarks themselves.

In this picture, pions are viewed as the lowest energy excitations of the quark vacuum, and at low energy the standard model is a pion theory. In the broken-symmetry phase, the modulus of the pion vector is fixed by the symmetry-breaking condition, and so pions are represented by a vector living in the unit sphere of Euclidean 4-space. This is the *nonlinear sigma model.* At higher energy, the modulus also becomes dynamical, and we may represent pions as a 4-vector self-interacting via a $SO(4)$-invariant potential. This is the *linear sigma model*, which will be the starting point for our discussion below.

As we have seen, the Bjorken scenario of a RHIC leads to the "baked Alaska" picture of the collision, where the edge of the expanding central region is hotter than its center. The hot plasma layer shields the cool center from interaction with the outer world, and therefore makes it possible for cooling the pion field to develop in a direction (in internal isospin space) different from the (cosmologically chosen) direction outside.

At some point the outer layer will be cool enough that causal contact will be restored, and the "disoriented" pion condensate will register as "ordinary" pions. Suppose that we call z the direction corresponding to neutral pions in isospin space, and that the disoriented pion condensate points in a direction z' at an angle Θ with respect to z. Upon decay into ordinary matter, the ratio of neutral to total number of pions will go roughly as $f = \cos^2\Theta$. Assuming that all directions in the unit sphere in isospace are equivalent, and recalling that the same f results from angles Θ and $\pi - \Theta$, then the probability to find a ratio between f and $f + df$ would go like $df/\sqrt{f}$. This characteristic distribution is another remarkably simple prediction of the "baked Alaska" scenario. Other

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signatures of DCC formation involve the nonequilibrium emission of photons [BVHK97, CNLL02].

Going beyond this qualitative picture, we now wish to introduce a microscopic perspective based upon nonequilibrium quantum field theory to provide a more detailed description of the chiral phase transition in the aftermath of the collision. We will largely follow the treatment by Cooper and collaborators [CKMP95, CoKlMo96, LaDaCo96]. To the best of our knowledge, this was also one of the first attempts to apply NEqQFT to a realistic experimental situation. Mean field models have also been investigated [MroMul95, Ran97, AmBjLa97], and there is a proposal to study DCC evolution within a Langevin framework [BeRaSt01].

14.2.1 Self-consistent mean fields in the large N approximation

Adopting the above qualitative picture we now study the evolution of the mean field in a $O(4)$ symmetric theory assumed to describe the low-energy excitations of the QCD vacuum. We shall make one further simplification, namely, instead of $O(4)$ we work with an $O(N)$ theory under the *large* N approximation. We have studied the large N (LN) approximation in Chapter 6. Unlike there, now we have to account for the possibility of symmetry breaking. To avoid misunderstandings, we shall develop the relevant formulae from scratch.

The $O(N)$ invariant action, allowing for spontaneous symmetry breaking, reads

$$S = \int d^4x \left\{ -\frac{1}{2}\partial_\mu \Psi^A \partial^\mu \Psi^A - \frac{\lambda}{8N}\left(\Psi^A\Psi^A - Nv^2\right)^2 \right\} \tag{14.34}$$

We scale $\Psi^A = \sqrt{N}\Phi^A$ to get

$$S = N\int d^4x \left\{ -\frac{1}{2}\partial_\mu \Phi^A \partial^\mu \Phi^A - \frac{\lambda}{8}\left(\Phi^A\Phi^A - v^2\right)^2 \right\} \tag{14.35}$$

To make the perturbative expansion more manageable, we use the Coleman–Jackiw–Politzer trick of including an auxiliary field $\chi = \lambda\left(\Phi^A\Phi^A - v^2\right)/2$, by adding a term to the action, which becomes

$$\begin{aligned} S = N\int d^4x \Bigg\{ & -\frac{1}{2}\partial_\mu \Phi^A \partial^\mu \Phi^A - \frac{\lambda}{8}\left(\Phi^A\Phi^A - v^2\right)^2 \\ & + \frac{1}{2}\left(\frac{\chi}{\sqrt{\lambda}} - \frac{\sqrt{\lambda}}{2}\left(\Phi^A\Phi^A - v^2\right)\right)^2 \Bigg\} \end{aligned} \tag{14.36}$$

Expanding out, we get

$$S = N\int d^4x \left\{ \frac{-1}{2}\partial_\mu \Phi^A \partial^\mu \Phi^A + \frac{\chi^2}{2\lambda} + \frac{1}{2}v^2\chi - \frac{1}{2}\chi\Phi^A\Phi^A \right\} \tag{14.37}$$

In this new action, strings of fish graphs beyond two loops are no longer 2PI. The next nontrivial graph is the three-pointed star, Fig. 6.10 in Chapter 6, which scales as N^{-1}. Thus, once again, we obtain a closed form for NLO large N.

To obtain this explicit expression, we begin by shifting the field $\Phi^A \to f^A + \varphi^A$, $\chi \to K + \bar{\kappa}$. As usual, we discard linear terms, so

$$S = S\left[f^A, K\right] + N \int d^4x \times \left\{ \frac{-1}{2}\partial_\mu \varphi^A \partial^\mu \varphi^A + \frac{\bar{\kappa}^2}{2\lambda} - \frac{1}{2}K\varphi^A\varphi^A - f^A\bar{\kappa}\varphi^A - \frac{1}{2}\bar{\kappa}\varphi^A\varphi^A \right\} \quad (14.38)$$

It is convenient to eliminate the quadratic cross-term, shifting $\bar{\kappa} = \kappa + \lambda f^A \varphi^A$. We get

$$S = S\left[f^A, K\right] + N \int d^4x \left\{ -\frac{1}{2}(\partial\varphi)^2 + \frac{\kappa^2}{2\lambda} - \frac{1}{2}M^2_{AB}\varphi^A\varphi^B - \frac{1}{2}\kappa\varphi^A\varphi^A - \frac{1}{2}\lambda f^A\varphi^A\varphi^B\varphi^B \right\} \quad (14.39)$$

$(M^2_{\alpha\beta} = K\delta_{AB} + \lambda f_A f_B)$, where the 2PIEA is

$$\Gamma^{\mathrm{NLO}} = S\left[f^A, K\right] + \frac{N}{2}\left\{ \left[\nabla^2\delta_{AB} - M^2_{AB}\right]G^{AB} + \frac{H}{\lambda} \right\} - \frac{i\hbar}{2}\left\{\mathrm{Tr}\,\ln H + \mathrm{Tr}\,\ln G\right\} + \Gamma_Q^{\mathrm{NLO}} + \mathrm{O}\left(N^{-1}\right) \quad (14.40)$$

$$\Gamma_Q^{\mathrm{NLO}} = \frac{iN^2}{4\hbar}\int d^4x d^4x' \left\{ H(x,x')\left[G^{AB}(x,x')\right]^2 + \lambda^2 f^A(x) f^B(x') \Delta^{AB}(x,x') \right\} \quad (14.41)$$

$$\Delta^{AB}(x,x') = G^{AB}(x,x')\left[G^{CD}(x,x')\right]^2 + 2G^{AD}(x,x')G^{CD}(x,x')G^{CB}(x,x') \quad (14.42)$$

Let us write the equations of motion leaving the CTP indices implicit

$$\nabla^2 f^A - K f^A - \lambda G^{AB}(x,x) f_B(x) + \frac{i\lambda^2 N}{2\hbar}\int d^4x'\; f^B(x')\Delta^{AB}(x,x') = 0 \quad (14.43)$$

$$\frac{K}{\lambda} + \frac{1}{2}v^2 - \frac{1}{2}f^A f^A - \frac{1}{2}G^{AA}(x,x) = 0 \quad (14.44)$$

$$\mathbf{1} - \frac{i\lambda\hbar}{N}H^{-1} + \frac{i\lambda N}{2\hbar}\left[G^{AB}(x,x')\right]^2 = 0 \quad (14.45)$$

$$\begin{aligned} &\left[\nabla^2\delta_{AB} - M^2_{AB}\right] - \frac{i\hbar}{N}G^{-1}_{AB} + \frac{iN}{\hbar}H(x,x')G^{AB}(x,x') \\ &+ \frac{i\lambda^2 N}{2\hbar}f^A(x)f^B(x')\left[G^{CD}(x,x')\right]^2 \\ &+ \frac{i\lambda^2 N}{\hbar}f^C(x)f^D(x')G^{CD}(x,x')G^{AB}(x,x') \\ &+ \frac{i\lambda^2 N}{\hbar}f^C(x)f^D(x')G^{CB}(x,x')G^{AD}(x,x') \\ &+ \frac{i\lambda^2 N}{\hbar}f^A(x)f^D(x')G^{CD}(x,x')G^{CB}(x,x') \\ &+ \frac{i\lambda^2 N}{\hbar}f^C(x)f^B(x')G^{CD}(x,x')G^{AD}(x,x') = 0 \end{aligned} \quad (14.46)$$

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It is clear from these equations that the propagators are $O\left(N^{-1}\right)$, and therefore some of the terms we have included are actually of higher order. In removing them, however, we must be careful that we compute factors of N which may arise when summing over internal indices. The resulting equations are

$$\nabla^2 f^A - K f^A = 0 \tag{14.47}$$

$$\frac{K}{\lambda} + \frac{1}{2}v^2 - \frac{1}{2}f^A f^A - \frac{1}{2}G^{AA}(x,x) = 0 \tag{14.48}$$

$$\mathbf{1} - \frac{i\lambda\hbar}{N}H^{-1} + \frac{i\lambda N}{2\hbar}\left[G^{AB}(x,x')\right]^2 = 0 \tag{14.49}$$

$$\left[\nabla^2\delta_{AB} - M_{AB}^2\right] - \frac{i\hbar}{N}G_{AB}^{-1} + \frac{i\lambda^2 N}{2\hbar}f^A(x)\,f^B(x')\left[G^{CD}(x,x')\right]^2 = 0 \tag{14.50}$$

We observe that the H propagator does not feed back on the mean fields, so we will not consider its evolution. For the other propagators, it is convenient to discriminate between the “pion” propagators $G_{\perp}^{AB}$ (which is defined by the property that $G_{\perp}^{AB} f_B \equiv 0$) and the “sigma” propagator, that is, the propagator for fluctuations along the mean field. Since there are $N-1$ “pions” and only one “sigma,” only the former feed back on K. Writing only the equations for the mean field and the pion propagator, we obtain

$$\nabla^2 f^A - K f^A = 0 \tag{14.51}$$

$$\frac{K}{\lambda} + \frac{1}{2}v^2 - \frac{1}{2}f^A f^A - \frac{1}{2}G_{\perp}^{AA}(x,x) = 0 \tag{14.52}$$

$$\left[\nabla^2 - K\right]\delta_{\perp AB} - \frac{i\hbar}{N}G_{\perp AB}^{-1} = 0 \tag{14.53}$$

where $\delta_{\perp AB}$ is the projector orthogonal to the mean field. These are the equations which determine the mean field evolution. Clearly, they admit a solution where $f^1 = f,\ f^A = 0\ (A \neq 1)$ and $G_{\perp}^{AB} = G(x,x')\,\delta_{\perp}^{AB}/N$. For such a solution, we obtain

$$\nabla^2 f - K f = 0 \tag{14.54}$$

$$\frac{K}{\lambda} + \frac{1}{2}v^2 - \frac{1}{2}f^2 - \frac{1}{2}G(x,x) = 0 \tag{14.55}$$

$$\left[\nabla^2 - K\right] - i\hbar G^{-1} = 0 \tag{14.56}$$

where we have used the result that $(N-1)/N = 1 - O(1/N)$.

14.2.2 The quantum pion field

The equations for the mean fields and the pion propagator are simplified by the observation that the latter are identical to the equations for the propagators of free fields with a position-dependent mass K. Thus we may introduce a “Heisenberg” pion field Φ, decompose it in modes, and then compute the propagators by summing over modes in the usual way. It is natural to choose a set of modes adapted to the boost symmetry of the baked Alaska scenario. That is,

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we introduce, instead of the usual Minkowski coordinates t and x, *Rindler* coordinates τ and η, defined by $t = \tau \cosh \eta$ and $x = \tau \sinh \eta$. In these coordinates, the Minkowski metric reads $ds^2 = -d\tau^2 + \tau^2 d\eta^2 + dx_\perp^2$ ($\mathbf{x}_\perp = (y, z)$), and the D'Alembertian $\nabla^2 = -\tau^{-1}\partial_\tau \tau \partial_\tau + \tau^{-2}\partial_\eta^2 + \nabla_\perp^2$. We therefore write

$$\Phi(\tau, \eta, \mathbf{x}_\perp) = \int \frac{d^2k_\perp dp}{(2\pi)^{3/2}} e^{ik_\perp x_\perp} e^{ip\eta} \left\{ \phi_{pk_\perp}(\tau)\, a_{pk_\perp} + \phi^*_{pk_\perp}(\tau)\, a^\dagger_{-p-k_\perp} \right\} \tag{14.57}$$

where the mode functions obey

$$\left\{ \frac{1}{\tau} \frac{d}{d\tau} \tau \frac{d}{d\tau} + \frac{p^2}{\tau^2} + k_\perp^2 + K \right\} \phi_{pk_\perp}(\tau) = 0 \tag{14.58}$$

and the destruction operators $a_{pk_\perp}$ have the usual commutation relations

$$\left[a_{pk_\perp}, a^\dagger_{p'k'_\perp} \right] = \delta(p - p')\, \delta^2(k_\perp - k'_\perp) \tag{14.59}$$

To obtain the usual ETCCRs for the field operators, we must normalize the modes as

$$\phi^*_{pk_\perp}(\tau) \frac{d}{d\tau} \phi_{pk_\perp}(\tau) - \phi_{pk_\perp}(\tau) \frac{d}{d\tau} \phi^*_{pk_\perp}(\tau) = -\frac{i}{\tau} \tag{14.60}$$

If we make the reasonable assumption that the initial state, defined on some surface $\tau = \tau_0 =$ constant, is an incoherent superposition of states with well-defined occupation numbers as defined from the $a_{pk_\perp}$ particle model, then the coincidence limit in the equation for K reads

$$G = 2 \int \frac{d^2k_\perp dp}{(2\pi)^3} \left| \phi_{pk_\perp}(\tau) \right|^2 \left\{ \frac{1}{2} + n^0_{pk_\perp} \right\} \tag{14.61}$$

where $n^0_{pk_\perp}$ is the occupation number for the corresponding mode in the initial state.

14.2.3 Adiabatic modes and renormalization

At this point, we have reduced the problem of computing the mean field evolution to a system of $n+2$ coupled ordinary differential equations, where n is the number of modes we care to include in our numerical solution (already this problem is too complex to attempt a closed analytical solution). Since the number of modes is necessarily finite, in effect we are imposing a momentum cut-off on the theory. This means that the coincidence limit (14.61) is *de facto* finite, but, since it diverges as the cut-off is removed, it is strongly cut-off dependent.

Physics, on the other hand, is supposed to be cut-off *in*dependent, so we should be able to absorb the dependence on the cut-off by renormalizing the parameters in the equation for K, namely λ and v^2, which implies, as a previous necessary condition, that the cut-off dependent part of the coincidence limit depends on the instantaneous value of K, but not on its derivatives.

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To analyze the ultraviolet behavior of the mode amplitudes, let us write $\phi_{pk_\perp}(\tau) = \tau^{-1/2}\varphi_{pk_\perp}(\tau)$, whereby

$$\frac{d^2}{d\tau^2}\varphi_{pk_\perp}(\tau) + \left[\Omega^2_{pk_\perp}(\tau) + \frac{1}{4\tau^2}\right]\varphi_{pk_\perp}(\tau) = 0, \qquad \Omega^2_{pk_\perp}(\tau) = \frac{p^2}{\tau^2} + k_\perp^2 + K \tag{14.62}$$

and $\varphi^*_{pk_\perp}\varphi'_{pk_\perp} - \varphi_{pk_\perp}(\varphi'_{pk_\perp})^* = -i$, where a prime stands for a τ derivative. In this regime $\Omega^2_{pk_\perp}(\tau)$ becomes a slowly varying function of τ. This suggests trying a WKB-type solution

$$F_{pk_\perp}(\tau) = \frac{e^{-iS(\tau)}}{\sqrt{2w_{pk_\perp}(\tau)}}, \qquad S = \int^\tau d\tau'\, w_{pk_\perp}(\tau') \tag{14.63}$$

$F_{pk_\perp}$ is well normalized by construction, and the mode equation becomes

$$w^2_{pk_\perp} = \Omega^2_{pk_\perp}(\tau) + \frac{1}{4\tau^2} - \frac{1}{4}\frac{\left(w^2_{pk_\perp}\right)''}{w^2_{pk_\perp}} + \frac{5}{16}\left[\frac{\left(w^2_{pk_\perp}\right)'}{w^2_{pk_\perp}}\right]^2 \tag{14.64}$$

The hypothesis of slow variation allows us to seek an adiabatic solution, namely, an iterative solution starting from the zeroth order approximation $w^2_{pk_\perp} = \Omega^2_{pk_\perp}(\tau)$. Let us write this solution as a formal series

$$w^2_{pk_\perp} = \sum_{n=0}^{\infty} W^{(n)}\left[\frac{p^2}{\tau^2}, k_\perp^2, \tau\right] \tag{14.65}$$

where $W^{(n)}$ is a homogeneous function of p^2/τ^2 and $k_\perp^2$ of degree $1-n$. It follows that

$$\frac{1}{w_{pk_\perp}} = \frac{1}{\left[W^{(0)}\right]^{1/2}} - \frac{1}{2}\frac{W^{(1)}}{\left[W^{(0)}\right]^{3/2}} + R \tag{14.66}$$

where R vanishes at large momentum as $(\text{momentum})^{-5}$. It is clear that only $W^{(0)}$ and $W^{(1)}$ may contribute to the cut-off dependence. Replacing (14.65) into (14.64), we obtain

$$W^{(0)} = \frac{p^2}{\tau^2} + k_\perp^2 \tag{14.67}$$

$$W^{(1)} = K + \frac{1}{4\tau^2} - \frac{1}{4}\frac{\frac{6p^2}{\tau^4}}{\frac{p^2}{\tau^2} + k_\perp^2} + \frac{5}{16}\left(\frac{\frac{2p^2}{\tau^3}}{\frac{p^2}{\tau^2} + k_\perp^2}\right)^2 \equiv K + \frac{k_\perp^2\left(\frac{-4p^2}{\tau^2} + k_\perp^2\right)}{4\tau^2\left(\frac{p^2}{\tau^2} + k_\perp^2\right)^2} \tag{14.68}$$

The potentially cut-off dependent terms in the coincidence limit of the propagator are

$$\frac{1}{\left(\frac{p^2}{\tau^2} + k_\perp^2\right)^{1/2}} - \frac{K}{2\left(\frac{p^2}{\tau^2} + k_\perp^2\right)^{3/2}} - \frac{k_\perp^2\left(\frac{-4p^2}{\tau^2} + k_\perp^2\right)}{8\tau^2\left(\frac{p^2}{\tau^2} + k_\perp^2\right)^{7/2}} \tag{14.69}$$

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However, the third term vanishes upon integration (this is easiest to see in polar coordinates). In conclusion, we obtain the same cut-off dependence as from the simple approximation $w^2_{pk_\perp} = \Omega^2_{pk_\perp}(\tau)$, and in passing we have proved that the cut-off dependent terms are functions of the instantaneous value of K, as required.

To complete the renormalization procedure, we write

$$v^2 = v_r^2 + \frac{\Lambda^2}{4\pi^2}; \qquad \frac{1}{\lambda} = \frac{1}{\lambda_r} - \frac{1}{8\pi^2}\ln\left(\frac{\Lambda}{\kappa}\right) \tag{14.70}$$

where κ defines the renormalization point. The finite equations of motion now read

$$\nabla^2 f - Kf = 0 \tag{14.71}$$

$$\frac{K}{\lambda_r} + \frac{1}{2}v_r^2 - \frac{1}{2}f^2 - \frac{1}{2}M^2 = 0 \tag{14.72}$$

$$\frac{d^2}{d\tau^2}\varphi_{pk_\perp}(\tau) + \left[\Omega^2_{pk_\perp}(\tau) + \frac{1}{4\tau^2}\right]\varphi_{pk_\perp}(\tau) = 0 \tag{14.73}$$

where $\Omega^2_{pk_\perp}(\tau) = \frac{p^2}{\tau^2} + k_\perp^2 + K$ and

$$M^2 = \frac{1}{\tau}\int^\Lambda \frac{d^2k_\perp dp}{(2\pi)^3}\left\{ |\varphi_{pk_\perp}(\tau)|^2\left(1 + 2n^0_{pk_\perp}\right) - \frac{1}{\left(\frac{p^2}{\tau^2} + k_\perp^2\right)^{1/2}} + \frac{K\theta\left(\frac{p^2}{\tau^2} + k_\perp^2 - \kappa^2\right)}{2\left(\frac{p^2}{\tau^2} + k_\perp^2\right)^{3/2}}\right\} \tag{14.74}$$

In a typical collision, the initial occupation numbers $n^0_{pk_\perp}$ will be high enough to ensure a large positive M^2, and therefore also K will be positive; in this regime, the symmetric point $f = 0$ is stable. As the system expands and cools, M^2 will go down, and eventually K becomes negative. This event marks the start of the chiral symmetry-breaking transition, and the formation of the disoriented condensate.

For negative K and large enough τ, not only f but also some of the long-wavelength modes will grow exponentially. This will shift the particle spectrum towards the infrared, which becomes the basic signal for DCC formation.

In summary, we have depicted DCC formation as a spinodal decomposition process in an expanding geometry. Since we have already discussed a similar process in Chapter 4, we will not discuss further the evolution of this model. The size and duration of the ordered domains determine the prospective sizes of the DCCs, and therefore the probability of their detection.