J. Austral. Math. Soc. Ser. B 30(1989), 389–413

MATHEMATICAL RESEARCH AT THE AERONAUTICAL RESEARCH LABORATORIES 1939–1960

D. G. HURLEY[1]

(Received 3 August 1988; revised 9 September 1988)

Abstract

The Aeronautical Research Laboratories were established in Australia in 1939 as the CSIR Division of Aeronautics. Mathematicians were amongst the first staff employed, and their number reached a peak in the mid 1950s. They were an able group: in their subsequent careers 12 became Professors, 5 obtained higher doctorates, 6 became Fellows of the Australian Academy of Science and 6 Fellows of the Royal Society. They published over 100 papers, and these are discussed here under 11 separate headings.

The length of discussion given here to the various areas of research is not uniform. I have emphasised those with which I am familiar and those that interest me personally. Nevertheless, I believe the present paper provides an accurate picture of the mathematical research that was carried out at ARL during the period under review, and makes it clear that mathematicians at ARL made substantial contributions to many areas of theoretical aeronautics.

1. Introduction

On Australia Day 1939, L. P. Coombes, from the Royal Aircraft Establishment, Farnborough, U. K. arrived in Melbourne to organise and take charge of the newly established Division of Aeronautics of C.S.I.R.O., Australia (then just "C.S.I.R."). The objectives of the Division were to serve the needs of the local aircraft industry, the R.A.A.F. and civil aviation. By April 1940 the first group of buildings was completed on the present 15-acre site at Fishermens Bend, Melbourne, and the first wind-tunnel (a low-speed one having an octagonal working section 9 ft $\times$ 7 ft) began operation two days before Pearl Harbor.

Mathematicians were amongst the first staff employed by A.R.L. Their number gradually increased and reached a peak of about 16 in the mid 1950s and

[1]Mathematics Department, University of W.A., Nedlands, W.A. 6009.

then decreased to about 6 in the early 1960s. Most of the mathematicians who left took up university appointments. In all, about 26 mathematicians were employed full time by A.R.L. during the period 1939-1960, and about another 6 part-time or as vacation students. They were an able group: in their subsequent careers 12 became Professors, 5 obtained higher doctorates, 6 became Fellows of the Australian Academy of Science, and 6 Fellows of the Royal Society. The last group includes G. K. Batchelor, R. H. Dalitz, W. J. Strang, A. A. Townsend, W. H. Whittrick (Sydney University and part-time A. R. L.) and very recently A. K. Head, but does not include T. M. Cherry, who did not work at A. R. L. but who exerted a considerable influence on the mathematical activities there, as we shall see. It could be argued that there was a greater concentration of mathematical talent at A. R. L. during the period under review than anywhere else in Australia, and at any other time during the past 200 years.

Throughout the period, but particularly during the early years, mathematicians contributed to the solution of *ad hoc* problems originating from the aircraft industry and the R.A.A.F. The results of most of this work were not published externally and will not be reviewed here. Another omission is work of a classified nature, such as that done by the highly regarded Operations Research Group. In the preparation of the present article, the lists of external publications that form part of A.R.L. Annual Reports were examined and over one hundred papers that had at least one mathematician (an undefined term) as an author were noted. These papers have been classified under the following headings, and their titles are included in the list of references at the end of this paper:

Wind-tunnel design and wind-tunnel corrections.

Wood in aircraft construction.

Boundary layer flow and turbulence.

Heat transfer at low Reynolds numbers.

The stalling of thin wings.

Transonic Aerofoils.

Supersonics.

Vibrations.

Fatigue.

Mathematical Methods.

Miscellaneous.

The length of discussion given to these various areas of research is not uniform. I have emphasised those with which I am familiar and those that interest me personally. Nevertheless I believe that the present paper including its titled

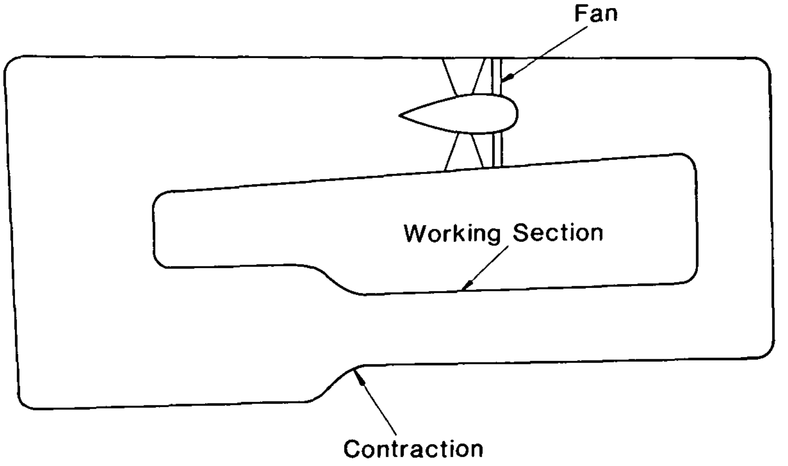


FIGURE 1. Plan form of Division of Aeronautics 9 ft × 7 ft wind tunnel

references, provides an accurate picture of the mathematical research that was carried out at A.R.L. during the period under review.

2. Work on wind-tunnel design and wind-tunnel corrections

Both the 9 ft × 7 ft low speed tunnel (Figure 1) and the variable pressure transonic one are of the closed return-circuit type. Each has a contraction immediately upstream of the working section, a main purpose of which is to achieve uniform flow in the working section. To a good approximation the flow in the contraction may be taken to be potential apart from the flow in the boundary layers on the surrounding walls. Batchelor and Shaw [12] used relaxation methods to design a contraction for the variable pressure tunnel so as to have only small adverse pressure gradients near its entry, after having found that flow separation did occur there in a model of the 9 ft × 7 ft tunnel.

The fan system driving a closed circuit wind-tunnel is an example of a ducted fan, and in a notable series of papers [81]–[84], [97], [98], Patterson and Scholes developed a design procedure for such fans and also conducted experiments to determine the correctness of their theories. Their work was used in the design of the drive system of the variable pressure tunnel.

The influence of the walls of a wind-tunnel of rectangular, circular or elliptic cross-section on the behaviour of a model in the tunnel is well-known. However, the 9 ft × 7 ft tunnel has the octagonal cross-section shown in Figure 2, and the influence of walls of this shape had not been determined. Batchelor [1] considered

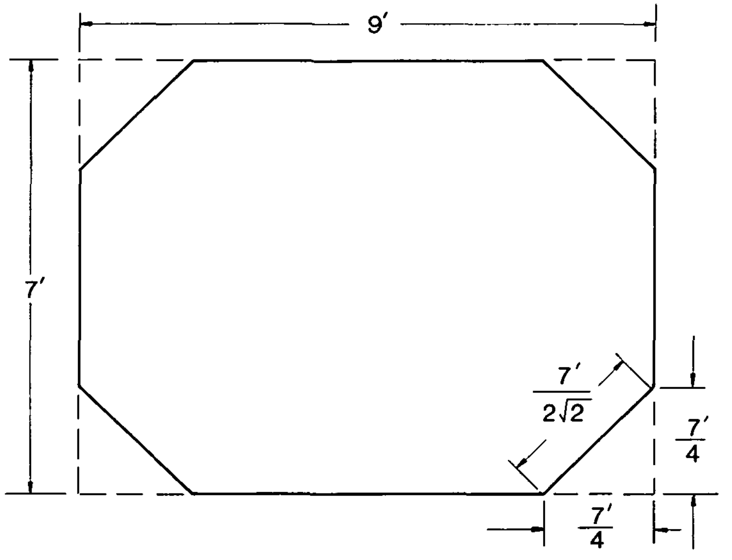


FIGURE 2. Cross-section of 9 ft × 7 ft wind-tunnel

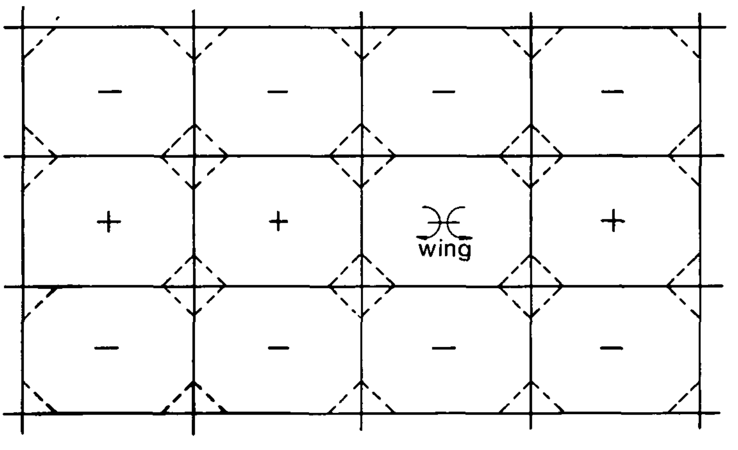


FIGURE 3. Images for a rectangular section

first the interference experienced by a model of small span mounted on the centre-line of such a tunnel. Considering the flow in a two-dimensional cross-section far downstream, it follows from the work of Prandtl that the problem reduces to that of finding the flow due to a dipole at the centre of the octagon, the sides of the octagon being stream-lines.

By introducing a doubly-infinite series of positive and negative image dipoles, the walls of the circumscribed rectangle can be made streamlines. (Figure 3).

https://doi.org/10.1017/S0334270000006342 Published online by Cambridge University Press

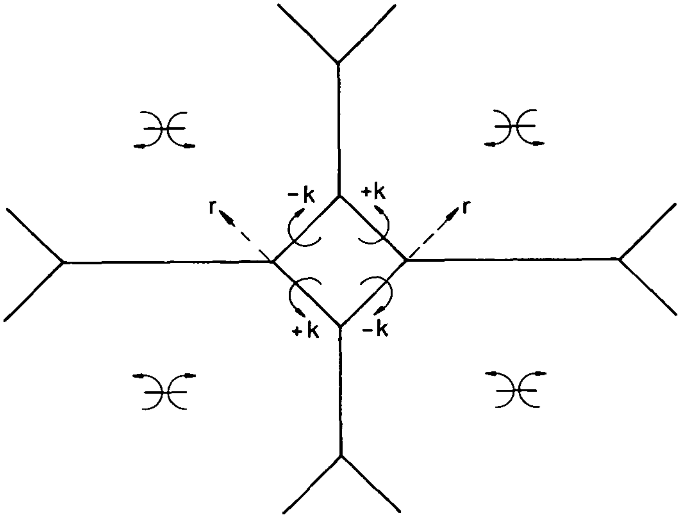


FIGURE 4. Polarities of replacement vortices

To make the sides of the square that the fillets form into stream-lines, Batchelor replaced them by distributions of vortices of strength $k(r)$, where r is the distance along an edge, having the polarities shown in Figure 4. Since the sides of the square are symmetrically placed, the sides of the rectangle are still stream-lines. He took $k(r) = K\,r(a-r)$ where a is the edge length of the square and determined K so that the normal velocities induced on the edges of the square by the vortices as nearly as possible cancelled those due to the dipoles. The effect of filling in the corners of the rectangle to form the octagon increased the interference factor by approximately 5%.

Other work on interference in tunnels of octagonal cross-section is described in [2] and [32], and [13] describes work related to wind-tunnel design.

3. Wood in aircraft construction

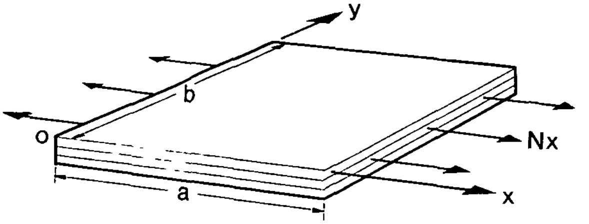
Early in the war, Australia was faced with only two weeks supply of aluminium for aircraft construction, and A.R.L. in conjunction with the C.S.I.R. Division of Forest Products undertook an investigation of the suitability of Australian timbers for aircraft construction.

Wood is an anisotropic material and has three axes of elastic symmetry– along the grain, in the direction tangential to the growth rings and in the radial direction from the axis of symmetry of the log. Six independent elastic constants are needed to specify its properties, whereas two suffice for an isotropic medium.

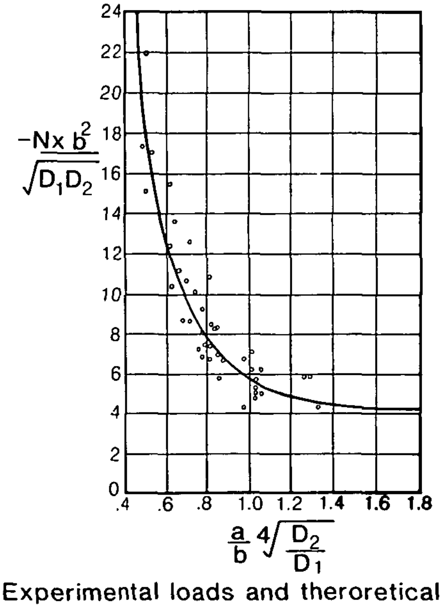
Smith [105] investigated the buckling of three-ply wooden plates in compression, the plates being formed from three veneers, the direction of the grain in the outer ones being perpendicular to the grain direction of the inner one. He generalised Timishenko's treatment of an isotropic plate to obtain the equation

$$D_1 \frac{\partial^4 w}{\partial x^4} + 2D_{12} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} = N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \tag{1}$$

where Oxy are axes parallel to the edges of the plate (and parallel to the grain in one of the veneers), $w(x, y)$ is the plate displacement, N_x, N_{xy}, N_y are the resultant forces per unit length of plate, D_1 and D_2 the bending stiffnesses and D_{12} another elastic constant. (Figure 5(a)).



(a) Notation



(b) Experimental loads and theroretical curve

FIGURE 5. Plywood plate in compression

https://doi.org/10.1017/S0334270000006342 Published online by Cambridge University Press

The case of compressive buckling that was considered is that for which N_{xy} and N_y are zero and N_x=constant, and the problem was to determine the smallest value of this constant for which equation (1) has a nontrivial solution. Experimental results are compared with the theory in Figure 5(b) for the case when the edges to which the load is applied are clamped and the other two simply supported.

In a later paper [106] Smith used the Rayleigh-Ritz energy method to discuss buckling due to shear for the case $N_x = N_y = 0$ and N_{xy} = constant.

Silberstein [103] described experimental work on the buckling of curved panels in compression, and produced recommended design curves.

Other work on the elastic properties of wood is described in [23], [30] and [93].

4. Boundary-layer flow and turbulence

The maintenance of large areas of laminar flow on the surface of a body moving through air results in substantial reductions in its drag and the rate of heat transfer to it, an important consideration today in the design of re-entry vehicles such as the shuttle. Consequently intense effort has been devoted to understanding both the mechanism of transition and the subsequent development of the turbulent boundary layer.

In 1944 there were two different theories for the cause of transition: one claimed that it resulted from the amplification of Tollmien-Schlicting waves in the laminar boundary-layer and the other that it was due to the turbulent pressure fluctuations in the flow outside the boundary-layer producing intermittent separation. Batchelor [3] reviewed these two theories of transition and this was followed by a review of hydrodynamic stability by Pillow [85] and a review of turbulence theories by Cumming [22].

For wind-tunnel experiments to reproduce flight conditions, the velocity distribution in the working section should be uniform and the turbulence level there low. These can be achieved by locating wire screens in the settling chamber upstream of the contraction, and Batchelor [4], [5] investigated the properties of such screens and also the sound level in the tunnel which must be kept low. He also published his first papers [7], [8] on turbulence whilst at A.R.L.

Subsequent work at A.R.L. on laminar flow concentrated on the suction-wing glider project, the mathematical input to which was slight.

5. Heat transfer at low Reynolds numbers

Collis and Williams [21] conducted careful experiments on both the free and forced convection of heat from the very thin wires used in hot-wire anemometry and found that the widely used formula due to King [57] was in serious error.

These experiments prompted Mahony [73] to investigate free convection at small Grashof numbers (G = buoyancy forces/viscous forces) for bodies of general shape, and Levey [67] to investigate heat transfer in slip flow. The latter investigation was appropriate because the diameter of the smallest wire used, 2.6 $\times$ 10^{-4}cm, was comparable to the mean free path in air at a pressure of 2cm of mercury and at room temperature. Mahony showed that his theoretical results for both a sphere and a circular cylinder were in close agreement with Langmuir's [58] correlation of data based on the concept of a stagnant film surrounding the body provided a minor correction to the latter for temperature difference was made. Mahony's method involved determining the convection velocities in the plume above the body and he was the first to determine the correct solution for a sphere.

Levey showed that at the body surface a temperature jump condition had to be imposed if the Knudsen number (K = mean free path/wire diameter) was not small and found a simple expression for the heat-transfer rate for a circular cylinder valid over a wide range of Knudsen numbers.

More recently Wood [127] calculated the heat loss from a circular cylinder due to forced convection when the Reynolds number is small and obtained excellent agreement with the measurements of Collis and Williams. In a later paper [128] he investigated the effects of buoyancy when the free-stream is inclined at an acute angle to the vertical.

6. The stalling of thin wings

When a thin two dimensional wing is at a small incidence, a small laminar separation bubble may be formed close to the leading edge. As the incidence is increased, a critical value, called the nose-stalling incidence, is reached when a large region of separation extending downstream from the leading-edge is formed. Since this nose stall is associated with the separation of a laminar boundary layer, an attempt was made to induce transition artificially in the laminar boundary layer before it separated; either a spanwise row of air jets (Wallis [116]), or a narrow strip of surface roughness (Hurley and Ward [56]) was located on the lower surface between the leading-edge and the front dividing streamline. The nose stall was delayed to a higher incidence and then occurred due to the turbulent boundary layer separating a small distance downstream of the bubble,

https://doi.org/10.1017/S0334270000006342 Published online by Cambridge University Press

as was first proposed by Wallis. The most effective roughness was composed of relatively large grains located close to the dividing streamline, where the velocity outside the boundary-layer was low; smaller roughness placed closer to the leading edge produced a greater thickening of the boundary layer at the separation point and was not as effective. Hurley [51] used a simple theoretical model to confirm these findings, and Hurley [50] and Levey [62] did other theoretical investigations that were relevant to this problem.

A more radical solution to the problem by exploiting rather than suppressing this leading-edge separation was proposed by Hurley [54]. The flow would be permitted to separate from the leading edge and boundary layer control would be used to make it re-attach to a forward facing flap mounted above the wing. Thus a flow would be established about a very thick pseudo-wing which would have good high-lift characteristics.

Free streamline theory was used to calculate the flow about an idealised wing consisting of two flat plates joined at the trailing edge, and a typical solution is shown in Figure 6(a). The speed on the free streamline is 2.73 times the free-stream speed and the lift coefficient has the large value of 4.6.

Experiments were carried out using the model shown in Figure 6(b). The upper plate had a rounded leading edge that incorporated a blowing slot through which high-speed air could be ejected. Experiments in a smoke-tunnel, Figure 6(c) showed that, with blowing, a flow that closely resembled the theoretical one could be established.

Now the theory showed that for a particular trailing-edge angle there was only a flow of the type envisaged for a particular value of the incidence. Recently Saffman and Tanveen [95] (at Caltech) have generalised the theory to allow the vorticity in the re-circulating region, which was taken to be constant, (Batchelor [9]) to be non-zero. The mathematical problem (Figure 7) then consists of solving $\nabla^2\psi_2 = \omega =$constant, in the re-circulating region II and $\nabla^2\psi_1 = 0$ in the remaining (outer) region, where ψ is the stream-function. Also the jump in the square of the velocity across the vortex-sheet that joins the points A and B must be constant. They found that in general the trailing-edge angle and the incidence could be prescribed independently and that their specification determined ω.

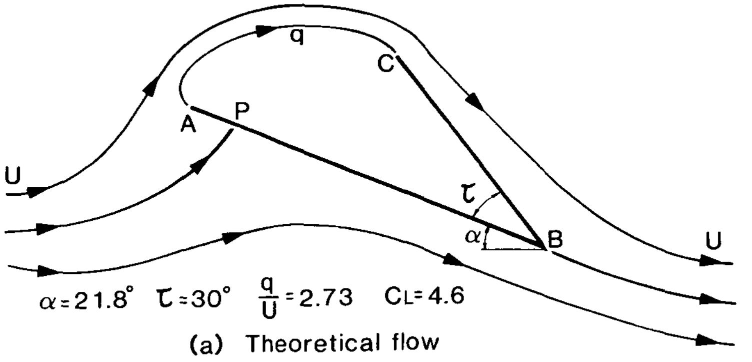
In a later paper [96] the same authors proposed that the flap be extended beyond its junction with the lower surface (Figure 8) and argued that then the flow would be less likely to separate from the flap surface because the stagnation point at the trailing edge would be eliminated. They also suggested a combination of suction and blowing to establish reattachment.

The circulating flow in the region ahead of the flap is an example of one having closed stream-lines, and both Pillow [88] and Wood [123], [125] have made contributions to the theory of such (Prandtl-Batchelor) flows.

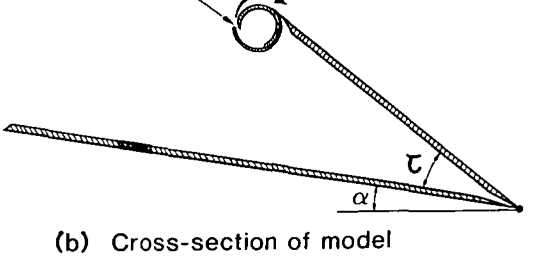
Other work on low speed aerodynamics is described in references [55], [65] and [68].

7. Transonic aerofoils

Consider a symmetric aerofoil at zero incidence in a stream of air whose Mach number at large distances from the aerofoil is $M_\infty < 1$. When M_∞ is small, the fluid velocity everywhere is less than the local speed of sound (i.e. it is subsonic). As M_∞ is increased, patches develop above and below the aerofoil where the fluid speed is supersonic (Figure 9), and in the early 1950s there was considerable interest in the development of wing sections for which the recompression to subsonic flow at the rear of the supersonic patches was either shock free or contained only shocks that were sufficiently weak so that boundary-layer separation with its consequent adverse effects did not occur.

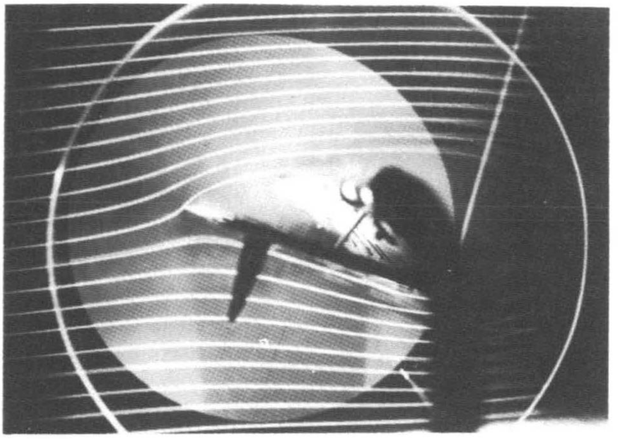


(a) Theoretical flow

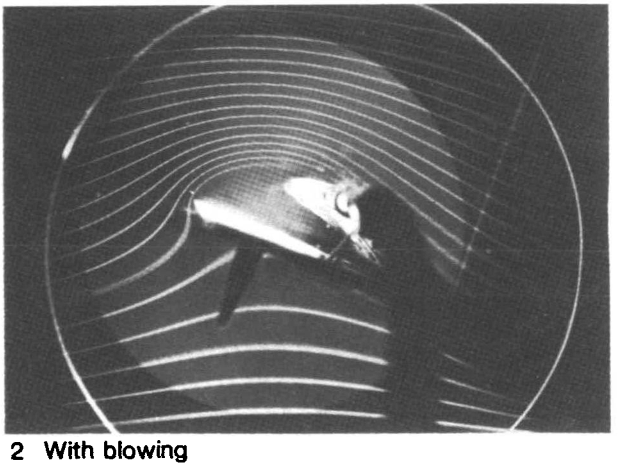


(b) Cross-section of model

FIGURE 6. Free stream-line flap (continues)



1 Without blowing



2 With blowing

FIGURE 6(c). Experimental flow

Wind tunnel investigations in this speed range were fraught with uncertainties due to the effects on the flow of the wind-tunnel walls. These effects were known to be large, and it was uncertain if the flows observed in the wind tunnel were even qualitatively similar to those that would occur in an unbounded stream.

Also the theoretical analysis of the problem was exceedingly difficult. The governing differential equation for the velocity potential $\varphi(x, y)$, namely

$$(C^2 - \varphi_x^2)\varphi_{xx} - 2\varphi_x\varphi_y\varphi_{xy} + (C^2 - \varphi_y^2)\varphi_{yy} = 0, \tag{2}$$

where C is the local speed of sound, is nonlinear so little progress in its solution could be made. However, some progress can be made if the velocity components (u, v) are taken as independent variables. For then it can be shown that

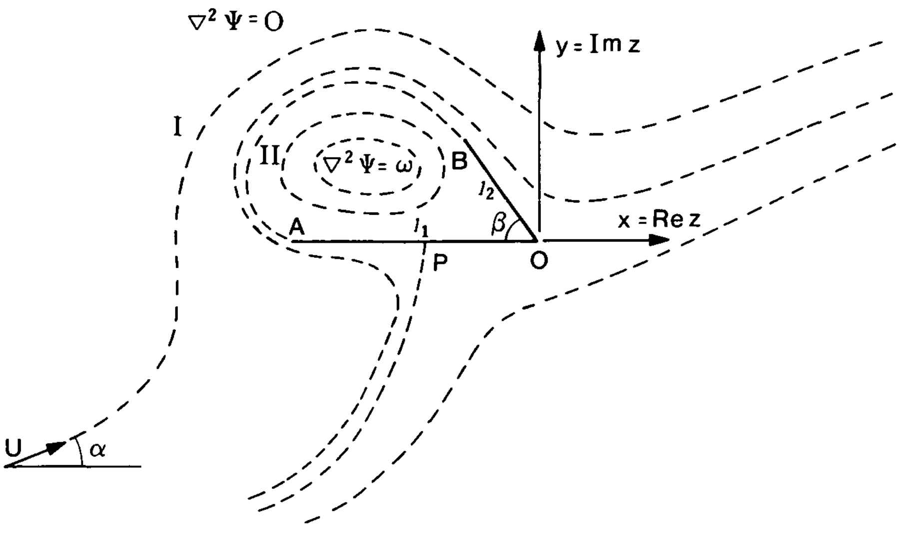


FIGURE 7. Prandtl-Batchelor flow considered by Saffman and Tanveer

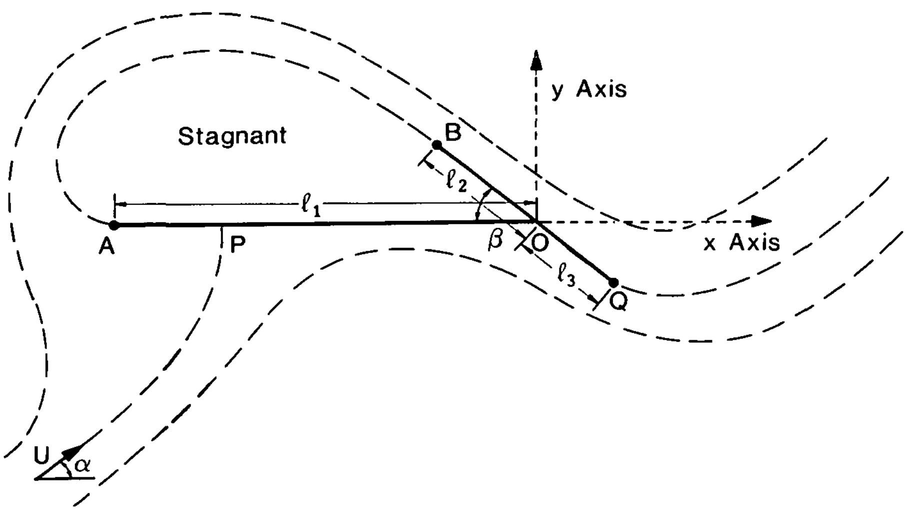


FIGURE 8. Saffman's and Tanveer's generalised free stream-line flap

$\Omega = \varphi - x\varphi_x - y\varphi_y$ satisfies the linear hodograph equation

$$\begin{aligned}\left\{a_0^2 - v^2 - (\gamma-1)(u^2+v^2)/2\right\}\Omega_{uu} + 2uv\Omega_{uv} \\ + \left\{a_0^2 - u^2 - (\gamma-1)(u^2+v^2)/2\right\}\Omega_{vv} = 0 \qquad (3)\end{aligned}$$

where a_0 is the stagnation speed of sound and γ the ratio of the specific heats.

https://doi.org/10.1017/S0334270000006342 Published online by Cambridge University Press

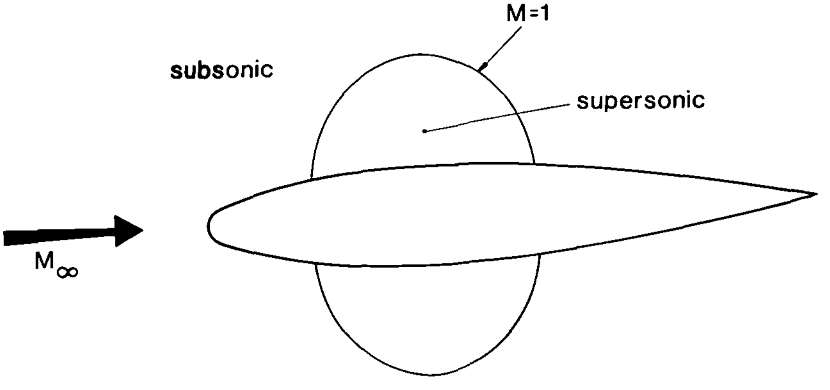


FIGURE 9. Supercritical flow past symmetrical aerofoil

Work on this equation in Australia was initiated by Cherry [16]–[18] and continued by Levey [59], [60], [63], first as a graduate student of Cherry and later at A.R.L. Two methods of solutions were developed.

In the first method, the known solution for the flow of an incompressible liquid is generalised to give the compressible flow about a similar but not identical body. Polar velocity components given by

$$u - iv = qe^{-i\theta}$$

are introduced and it can be shown that a general solution of (3) is

$$\Omega = \sum_n (A_n e^{in\theta} + B_n e^{-in\theta}) F_n(\tau) q^n \tag{4}$$

where $\tau = q^2/q_m^2$, q_m = vacuum speed, and $F_n(\tau)$ are certain hypergeometric functions. The limit $\tau \to 0$ of (4) is, since $F_n(0) = 1$,

$$\Omega = \sum_n (A_n e^{in\theta} + B_n^{-in\theta}) q^n, \tag{5}$$

which is appropriate to an incompressible flow. Now if the complex position coordinate $Z = (x + iy)$ can be expressed as an explicit function of the complex velocity $(= Qe^{-i\theta})$ of a known incompressible flow, a number of series of the type (5), each valid for a certain region of the hodograph plane, and which are analytic continuations of each other, may be found. However, the corresponding series of type (4) which describe compressible flows are in general not analytic continuations of each other, and the main problem in obtaining the solution to the compressible problem is to determine these analytic continuations.

This method was successfully applied by Cherry to a circular cylinder and by Levey to an ellipse and a particular aerofoil, although no numerical results were given in the last application.

In the second method for tackling (3), its linearity is exploited by adding together a number of particular solutions to obtain the flow about a body of desired shape in a stream having a particular Mach number.

Now for a physically significant solution, $u(x, y)$ and $v(x, y)$ must be single-valued functions, but in many of the problems of practical interest the inverse functions $x(u, v)$, $y(u, v)$ are multiple-valued. Cherry showed that if a parameter ϕ is defined by

$$\theta - \theta_0 = \phi - \left[\left(\frac{\gamma+1}{\gamma-1}\right)^{1/2} - 1\right] \arctan \frac{q \sin \phi}{1 - q \cos \phi} \tag{6}$$

(where now $q = 1$ is the vacuum speed), certain solutions for which x and y are multiple-valued functions of q and θ, are single-valued functions of q and ϕ, applicable over the whole domain of interest.

Cherry [18] used this procedure to obtain results for two aerofoil-type flows. Figure 10 gives results for a low value of M_∞ which are entirely satisfactory. However, results for $M_\infty = 0.660$ which are given in Figure 11 are not satisfactory because the profile has cusps at P and Q and there is a small region PQR in

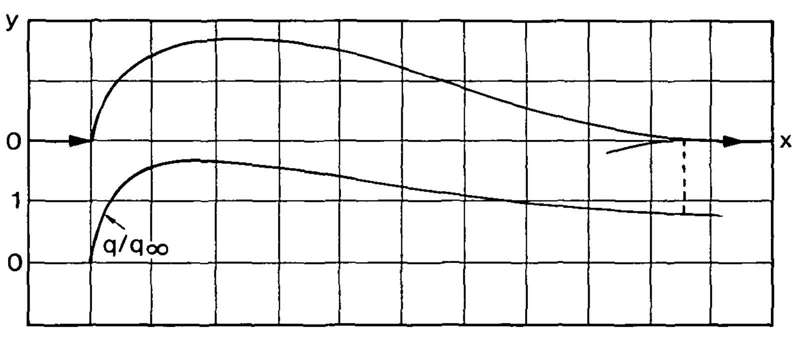


FIGURE 10. Aerofoil profile and speed on it for M_∞ small

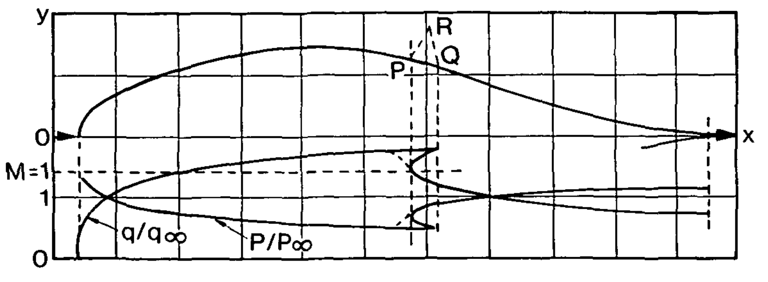


FIGURE 11. The streamline $\Psi = 0$, with speed and pressure on it for $M_\infty = 0.660$

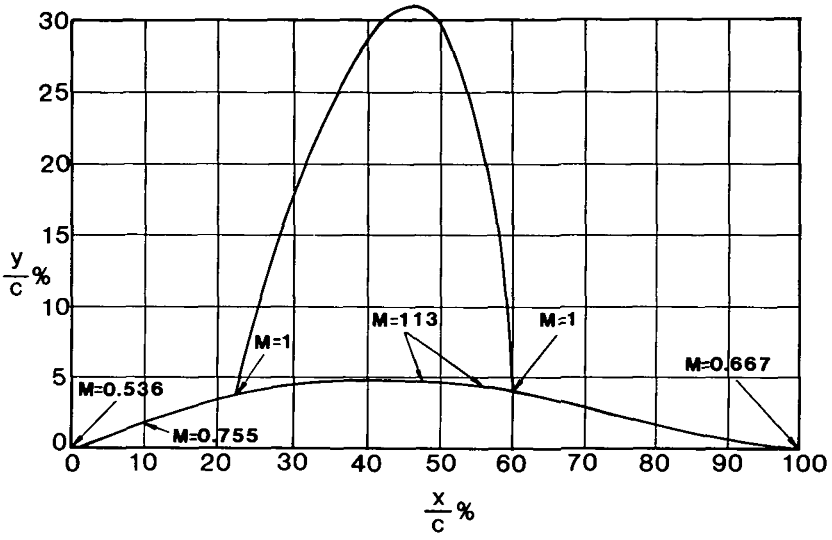


FIGURE 12. Levey's shock-free aerofoil, $M_\infty = 0.8$

which the functions $u(x, y)$ and $v(x, y)$ are three-valued. Cherry remarks that a small reduction in M_∞ should shift this region to within the aerofoil surface and then the solution will be satisfactory. The dotted line indicates the conjectured surface speed for $M_\infty = 0.6$ and the aerofoil shape would be nearly as shown but without the cusps.

Levey [63] developed the method and used it to obtain the flow about an aerofoil with $M_\infty = 0.80$ for which $u(x, y)$ and $v(x, y)$ were single-valued everywhere, with the minor drawback that the aerofoil had a cusped leading edge. Figure 12 gives the profile shape, the boundary of the supersonic region and the Mach number distribution on the aerofoil surface. The references show that the algebra involved in deriving this solution was horrendous and no attempt will be made to describe it.

Now Morawitz [80] had shown that the problem of calculating flows of this type was ill-posed. She showed that to retain shock-free flows, variations in the profile $y = \pm Y(x)$ can at most be prescribed outside some finite area containing the point of maximum surface Mach number. Hence it was uncertain if the theoretical shock-free solution would occur in practice.

In an attempt to resolve this important issue, a very accurate model of Levey's aerofoil was constructed and tested [99] in 1959 in the high-speed variable pressure tunnel at A.R.L. This tunnel has "slotted walls" which reduce greatly wind-tunnel interference and whose effectiveness has been investigated by Wood [126].

Figure 13 compares the theoretical surface Mach number distribution for $M_\infty = 0.8$ with measured values for $M_\infty = 0.80$ and $M_\infty = 0.82$, the value of

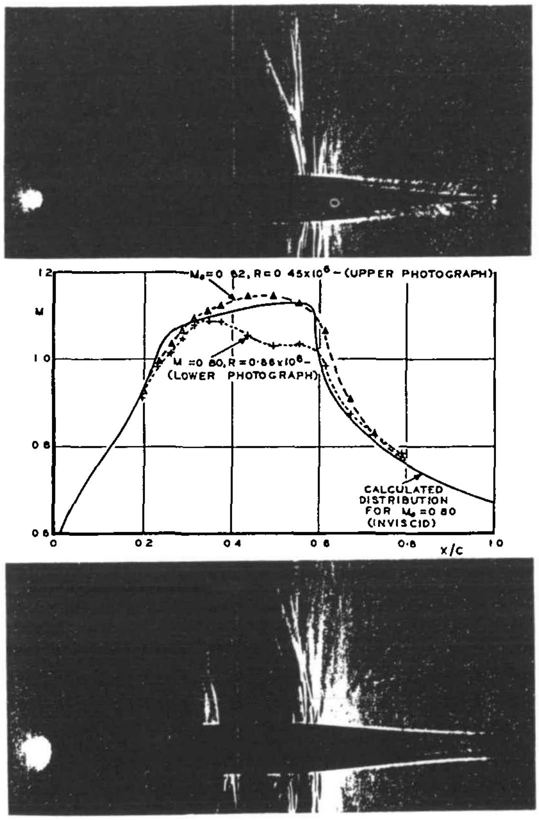


FIGURE 13. Experiments on Levey's Transonic Aerofoil

M_∞ that gave closest agreement with the theory. Schlieren photographs of the flow are also shown. The results suggest that a low-loss transonic compression does occur from a local Mach number of about 1.14, and that unsteady wavelets are present in the compression region but boundary layer separation does not occur. Noting that no account has been taken of the displacement effect of the boundary-layer, the agreement with the inviscid theory is reasonable.

It is interesting to note that a procedure similar to Levey's was subsequently used by Bauer, Garabedian and Korn [14] for the design of super-critical wing-sections. The hodograph method is used to calculate a super-critical wing-section which will be free from shocks at a specified Mach number and angle of incidence. A finite difference scheme is then used to obtain the performance at off-design conditions. In those calculations the effects of weak shocks are included; the variation in entropy across them is ignored so that irrotational flow may still be assumed. Good agreement between theory and experiment is obtained.

https://doi.org/10.1017/S0334270000006342 Published online by Cambridge University Press

Earlier work in this area using a different approach is described by Batchelor [6] and the subject was reviewed by Dalitz [25].

8. Supersonics

The work on shock waves was focussed on basic problems that are easily stated but which are very difficult to solve.

Pillow [86] investigated the breakdown of the continuous one-dimensional unsteady flow that results if a piston is accelerated into an inviscid gas that is initially at rest. A shock wave of variable strength is formed so that the flow downstream of it is not homentropic, and this leads to severe mathematical complications. However, these were overcome and the paper describes the first successful solution of the problem.

Levey [61] considered the steady two-dimensional source-type flow of a viscous, heat conducting perfect gas. If viscosity is neglected and r_s is the radial distance at which the local Mach number is one, there is no solution for $r < r_s$ and two solutions, one supersonic and one subsonic for $r > r_s$ (see Figure 14). When viscosity is included, he determined the structure of the shock wave where the radial flow changes from the supersonic to the sub-sonic branch, for a particular value of the Prandtl number, that simplifies the analysis. A surprising result was that the entropy rises to a maximum within the shock even when the viscosity tends to zero, and this result carries over to the one-dimensional problem. In a later paper [66] he showed that previous investigations of the problem using the P.L.K. method were in error.

This work was started whilst Levey was a research student at Manchester University, as was Mahony's work on supersonic flow. The latter ([72], Part I) derived a solution for the general wave interaction problem of two-dimensional, steady, homentropic flow of a perfect gas. Later, ([72], Part II) he extended the method to flows with shocks provided these are weak enough for entropy changes to be ignored. He also investigated the accuracy of shock-expansion theory [71] and also internal axi-symmetric flow [74].

Strang started work on unsteady supersonic aerofoil theory at King's College London and continued working in this area on joining A.R.L. In his first paper [108] he determined the solution for transient point and line sources in linearised supersonic flow and used them to solve the problem of an aerofoil (a) entering a sharp edged gust and (b) having its incidence suddenly changed. In a later paper [109] he extended his theory to cover three-dimensional wings in unsteady purely supersonic flow.

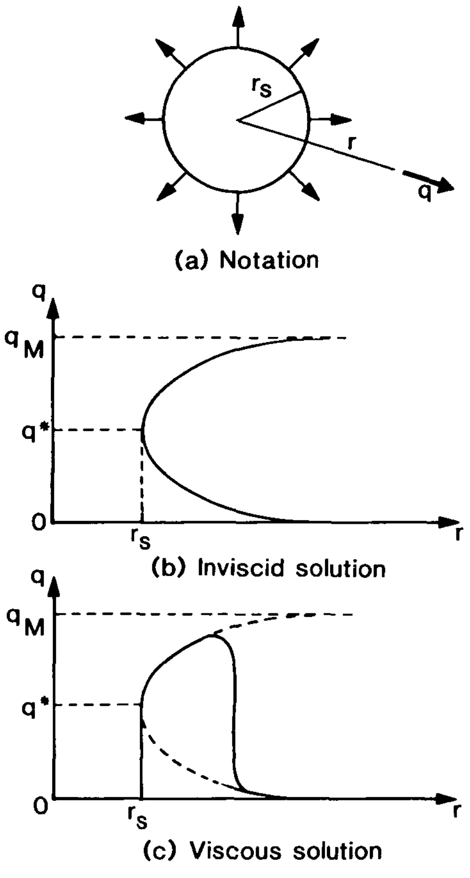


FIGURE 14. Two-dimensional source flow of a viscous fluid

9. Vibrations

Love (University of Melbourne and part-time A.R.L.) and Silberstein [69] calculated the impedance of a fan as a function of the frequency of torsional vibration of the driving shaft on which it is mounted. The impedance is the ratio of the applied oscillating couple to the angular amplitude of vibration. The fan was modelled by elastic blades mounted on the outer of three concentric rigid rims that are joined by elastic webs. Eight generalised coordinates are used to specify the configuration of the model and methods of classical mechanics are used to derive the impedance-frequency relation. Detailed results were obtained for a particular fan and it was concluded that three degrees of freedom (but no less) were needed to give reasonably accurate results.

In a later paper [70] the same authors with the assistance of Radok considered the vibration of propellers. Existing theories were reviewed and a new more general one was developed; it covers propeller blades that are straight and slender but whose section and twist may vary along the blades. Detailed results were given for a particular propeller.

Other work on vibrations is described in references [111], to [114].

10. Fatigue

Australia places a greater fatigue demand on aircraft than most other countries. The average flying time of some civil aircraft exceeds 12 hours per day. Consequently Australian airlines have found themselves in the unenviable position of experiencing fatigue troubles long before their colleagues in other countries, and Wills [117], [118] initiated work at A.R.L. on this topic.

The stresses produced in aircraft structure during flight that lead to fatigue arise from atmospheric turbulence and are of an irregular random nature. Head and Hooke [46] developed a random-noise fatigue machine and found that the fatigue life of a specimen was only about a third of that predicted using Miner's [79] rule that is based on the results of applying sinusoidal stresses.

The deformation and the stresses resulting from atmospheric disturbances were investigated by Radok and Stiles [89], [94].

A.R.L. established a materials research laboratory in which metallurgists and mathematicians conducted long range research programmes on problems relevant to fatigue. Head [42] discussed theoretical suggestions as to how a fatigue crack may form and grow and he and co-workers made notable contributions in this area ([15], [20], [39]–[44] and [47]). Mann [78] developed a theory for the internal stresses set up by slip that generalised that due to Volterra [115]. She assumed that only the three stress components that act across a dislocation need be continuous so that more general types of dislocation were covered by her theory.

11. Mathematical methods

11.1 Relaxation methods

Both Green and Shaw were introduced to the relaxation method at Oxford [19], [33], [34] and whilst at A.R.L. they and others applied the method to a variety of engineering problems including the design of wind tunnel contractions [12], the flow of a compressible fluid through a two-dimensional nozzle [34], past a circular cylinder [10] and a Joukowski aerofoil [11], and to a problem in elasticity [100]. Later, Shaw wrote a book on the subject [101].

11.2 Singular perturbation theory

Many of the problems in theoretical aerodynamics are of the singular perturbation type and members of the Theoretical Fluid Mechanics Group at A.R.L., as well as solving particular physical problems, made substantial contributions to the general theory both whilst at A.R.L. and during their subsequent careers.

Levey's interest in this class of problem began with his work in Manchester [61] and in later papers [64], [66] he developed perturbation techniques for obtaining analytic approximations to such solutions.

Mahony's work on heat transfer [73] whilst at A.R.L. and his subsequent work on the deflection of cantilevered plates [75] involved singular perturbation techniques and led to his proposing [76] a general method for the solution of nonlinear partial differential equations that involved either a small or a large parameter.

Pillow [88] and Wood [124] studied problems of this type whilst at Cambridge and remained active in this area [87] and [128].

Miscellaneous

Other work on mathematical methods is described in references [35], [36], [91].

12. Miscellaneous

Work carried out that does not fall under any of the above headings is described in references [24], [26]–[29], [31], [37], [38], [45], [48], [49], [52], [53], [77], [102] and [119]–[123].

13. Conclusions

The above review makes it clear that mathematicians at A.R.L. made substantial contribution to many areas of theoretical aeronautics. Due to the small design activity of the Australian aircraft industry there was little incentive for them to work on particular long range problems; consequently they were free to spend much of their time on problems of their own choice. They thus enjoyed considerable freedom, but on the other hand they lacked the satisfaction of seeing much of their effort put to practical use.

Acknowledgements

I am very grateful to Dr. G. L. Brown, Director, A.R.L. and Mr. Neil Pollock of the Aerodynamics Division, for their help in supplying me with the source material on which this paper is largely based.

References

[1] G. K. Batchelor, "Interference in a wind tunnel of octagonal section", *Australian Council for Aeronautics; Report ACA-1* (1944).

[2] G. K. Batchelor, "Interference on wings, bodies and airscrews in a closed tunnel of octagonal section", *Australian Council for Aeronautics; Report ACA-5* (1944).

[3] G. K. Batchelor, "Transition to turbulence within the boundary layer", *Austral. J. Sci.* **6** (1944) 108–112.

[4] G. K. Batchelor, "On the concept and properties of idealized hydrodynamic resistance", *Australian Council for Aeronautics; Report ACA-13* (1945).

[5] G. K. Batchelor, "Sound in wind-tunnels", *Australian Council for Aeronautics; Report ACA-18* (1945).

[6] G. K. Batchelor, "Power series expansion of the velocity potential in compressible flow", *Quart. Appl. Math.* **2** (1945) 318–328.

[7] G. K. Batchelor, "Theory of axisymmetric turbulence", *Proc. Roy. Soc. London A* **186** (1946) 480–502.

[8] G. K. Batchelor, "Recent deductions concerning the double velocity correlation function in turbulent motion", *Nature* **158** (1946) 883–884.

[9] G. K. Batchelor, "On steady laminar flow with closed stream-lines at large Reynolds number", *J. Fluid Mech.* **1** (1956) 177–190.

[10] G. K. Batchelor and A. F. Pillow, "A numerical method of determining subsonic compressible flow past an obstacle I: Circular cylinder", *A.R.L. Aero. Note No. 61* (1944).

[11] G. K. Batchelor and F. S. Shaw, "A numerical method of determining subsonic compressible flow past an obstacle II: Joukowski aerofoil", *A. R. L. Aero Report A 21 and SM 28* (1944).

[12] G. K. Batchelor and F. S. Shaw, "A consideration of the design of wind tunnel contractions", *Australian Council for Aeronautics; Report ACA-4* (1944).

[13] G. K. Batchelor and A. A. Townsend, "Singing corner vanes", *Nature* **155** (1945) 236.

[14] F. Bauer, P. Garabedian and D. Korn, "Supercritical wing sections", *Lecture notes in economics and mathematical systems 66* (Springer-Verlag, 1972).

[15] F. P. Bullen, A. K. Head and W. A. Wood, "Structural changes during the fatigue of metals", *Proc. Roy. Soc. London A* **216** (1953) 332–343.

[16] T. M. Cherry, "Flow of a compressible fluid about a cylinder", *Proc. Roy. Soc. London A* **192** (1947) 45–79.

[17] T. M. Cherry, "Flow of a compressible fluid about a cylinder. II Flow with circulation", *Proc. Roy. Soc. London A* **196** (1949) 1–36.

[18] T. M. Cherry, "A transformation of the hodograph equation and the determination of certain fluid motions", *Phil. Trans. Roy. Soc. London A* **245** (1953) 583–624.

[19] D. R. Christopherson, L. Fox, J. R. Green, F. S. Shaw and R. V. Southwell, "Relaxation methods applied to engineering problems. VII B The elastic stability of plane frameworks and of flat plating", *Phil. Trans. Roy. Soc. London A* **239** (1945) 461–487.

[20] L. M. Clareborough, M. E. Hargraves, A. K. Head and G. W. West, "Energy stored during fatigue of copper", *J. of Metals* **7** (1955) 99.

[21] D. C. Collis and M. J. Williams, "Two-dimensional convection from heated wires at low Reynolds numbers", *J. Fluid Mech.* **6** (1959) 357–384.
[22] B. L. Cumming, "Review of turbulence theories", *Australian Council for Aeronautics; Report ACA-27* (1946).
[23] F. A. Dale and R. C. T. Smith, "Grid sandwich panels in compression", *Australian Council for Aeronautics; Report ACA-16* (1945).
[24] R. H. Dalitz, "The effect of a sharp-nosed leading edge on the boundary layer of a flat plate", *A.R.L. Report A40* (1945).
[25] R. H. Dalitz, "Some mathematical aspects of compressible flow", *Australian Council for Aeronautics; Report ACA-20* (1946).
[26] W. Freiberger, "The uniform torsion of an incomplete tore", *Austral. J. Sci. Res. Ser. A* **2** (1949).
[27] W. Freiberger, "On the solution of the equilibrium equations of elasticity in general curvilinear coordinates", *Austral. J. Sci. Res. Ser. A 2* (1949).
[28] W. Freiberger, "A problem in dynamic plasticity: the enlargement of a circular hole in a flat sheet", *Proc. Camb. Phil. Soc.* **48** (1952) 135–148.
[29] W. Freiberger, "The uniform torsion of a perfectly plastic circular ring", *A.R.L. Report SM213* (1953).
[30] W. Freiberger, F. S. Shaw, J. P. O. Silberstein and R. C. T. Smith, "Plywood panels in end compression. Flat panels with grain at various angles to direction of loading", *Australian Council for Aeronautics; Report ACA-30* (1947).
[31] W. Freiberger and R. C. T. Smith, "The uniform flexure of an incomplete tore", *Austral. J. Sci. Res. Ser. A* **2** (1949).
[32] B. L. Gent, "Interference in a wind tunnel of regular octagonal section", *Australian Council for Aeronautics; Report ACA-2* (1944).
[33] J. R. Green and R. V. Southwell, "Relaxation methods applied to engineering problems. VII. A Problems relating to large transverse displacements of thin elastic plates", *Phil. Trans. Roy. Soc. London C* **1** (1944) 137–176.
[34] J. R. Green and R. V. Southwell, "Relaxation methods applied to engineering problems. IX High-speed flow of compressible fluid through a two-dimensional nozzle", *Phil. Trans. Roy. Soc. London A* **239** (1944) 367–386.
[35] J. Guest, "The solution of linear simultaneous equations by matrix iteration", *Austral. J. Appl. Phys.* **8** (1955).
[36] J. Guest, "A short note on the checking of contractants", *J. Roy. Aero. Soc.* **60** (1956) 685–686.
[37] J. Guest, "The compressive buckling of a clamped parallelogram plate with a longitudinal stiffener", *Austral. J. Appl. Sci.* **7** (1956) 191–198.
[38] J. Guest, "The buckling of a clamped parallelogram plate under combined bending and compression", *Austral. J. Appl. Sci.* **8** (1957) 27–34.
[39] A. K. Head, "The mechanism of fatigue of metals", *J. Mech. Phys. Solids* **1** (1953) 134–141.
[40] A. K. Head, "Edge dislocation in inhomogeneous media", *Proc. Phys. Soc. B* **66** (1953) 793–801.
[41] A. K. Head, "The interaction of dislocations and boundaries", *Phil. Mag. Ser. 7* **44** (1953) 92–94.
[42] A. K. Head, "Fatigue", *J. Austral. Inst. Met. 1* (1956) 148–154.
[43] A. K. Head, "The effect of frequency on the fatigue of metals", *J. Phys. Soc. of Japan* **11** (1956) 468.
[44] A. K. Head, "The propagation of fatigue cracks", *J. Appl. Mech.* **23** (1956) 407.
[45] A. K. Head, "A class of aplanatic optical systems", *Physical Soc. Proc.* **71** (1958) 546–551.
[46] A. K. Head and F. H. Hooke, "Fatigue of metals under random loads", *Nature* **177** (1956) 1176–1177.

[47] A. K. Head and N. P. Louat, "The distribution of dislocations in linear arrays", *Austral. J. of Physics* **8** (1955) 1–7.
[48] B. C. Hoskin, "Torsional rigidity of a plastic bar of circular section under combined tension and torsion", *Austral. J. Appl. Sci.* **12** (1961) 255.
[49] B. C. Hoskin and E. H. Lee, "The analysis of loaded flexible surfaces over sub-grades with visco-elastic material behaviour", *Amer. Soc. Civil Eng. J. Eng. Mech.*
[50] D. G. Hurley, "The effect of a small laminar separation bubble on the potential flow past an aerofoil", *A.R.L. Note A 148* (1955).
[51] D. G. Hurley, "The downstream effect of a local thickening of the laminar boundary layer", *J. Aero. Sci.* **22** (1956) 383–384.
[52] D. G. Hurley, "Note on the forces that act near the centre and the tip of a swept-back wing", *Aero. Quart.* **10** (1959) 127–144.
[53] D. G. Hurley, "Mass transfer cooling in a boundary layer", *Aero. Quart.* **12** (1961) 165–188.
[54] D. G. Hurley, "The use of boundary-layer control to establish free stream-line flows", in *Advances in Aeronautical Sciences 2* (ed. Th. von Kármán), (Pergamon, 1959) 662–708.
[55] D. G. Hurley, "A theoretical investigation of an aerofoil equipped with a split flap", *J. Austral. Math. Soc.* **2** (1961) 195–205.
[56] D. G. Hurley and G. F. Ward, "Experiments on the effect of air jets and surface roughness on the boundary layer near the nose of an NACA 64A006 Aerofoil", *A.R.L. Note A 128* (1953).
[57] L. V. King, "On the convection of heat from small cylinders in a stream of fluid. Determination of convection constants of small platinum wires with application to hot-wire anemometry", *Phil. Trans. Roy. Soc. London A* **214** (1914) 373–432.
[58] I. Langmuir, "Convection and conduction of heat in gases", *Phys. Rev.* **34** (1912) 401–422.
[59] H. C. Levey, "High speed flow of a gas past an approximately elliptic cylinder", *Proc. Camb. Phil. Soc.* **46** (1950) 479–491.
[60] H. C. Levey, "Exact solutions for transonic flow past cusped aerofoils", *A.R.L. Report A 87* (1954).
[61] H. C. Levey, "Two dimensional source flow of a viscous fluid", *Quart. Appl. Math.* **12** (1954) 25–48.
[62] H. C. Levey, "The effect of a small protuberance on the potential flow past an aerofoil", *A.R.L. Note A 134* (1954).
[63] H. C. Levey, "A shock-free airfoil", *J. Aero. Sci.* **23** (1956) 1129–1130.
[64] H. C. Levey, "Singular Perturbations: a model equation", *A.R.L. Note A 157* (1957).
[65] H. C. Levey, "The 'ground' interference of a carrier deck", *J. Roy. Aero. Soc.* **61** (1957).
[66] H. C. Levey, "The thickness of cylindrical shocks and the P.L.K. method", *Quart. Appl. Math.* **17** (1959) 77–93.
[67] H. C. Levey, "Heat transfer in slip flow at low Reynolds numbers", *J. Fluid Mech.* **6** (1959) 385–391.
[68] H. C. Levey, "The back effect of a wall on a jet", *Z. Ang. Math. Phys.* **11** (1960) 152–157.
[69] E. R. Love and J. P. O. Silberstein, "Elastic vibrations of a fan", *Australian Council for Aeronautics; Report ACA-15* (1945).
[70] E. R. Love and J. P. O. Silberstein, (with the assistance of J. R. M. Radok), "Vibration of stationary and rotating propellers", *Australian Council for Aeronautics; Report ACA-36* (1947).
[71] J. J. Mahony, "A critique of shock-expansion theory", *J. Aero. Sci.* **22** (1955) 673–680.
[72] J. J. Mahony, "Analytic treatment of two-dimensional supersonic flow, Part I Shock-free flow, Part II Flow with shock waves", *Phil. Trans. Roy. Soc. London A* **248** (1956) 467–515.
[73] J. J. Mahony, "Heat transfer at small Grashof numbers", *Proc. Roy. Soc. London A* **238** (1956) 412–423.

[74] J. J. Mahony, "The internal flow problem in axi-symmetric supersonic flow", *Phil. Trans. Roy. Soc. London A* **251** (1958) 1–21.

[75] J. J. Mahony, "The large deflection of thin cantilevered plates. Part I General theory, Part II Symmetrically loaded rectangular plate", *Quart. J. Mech. Appl. Math.* **14** (1961) 257–282.

[76] J. J. Mahony, "An expansion method for singular perturbation problems", *J. Austral. Math. Soc.* **2** (1962) 440–463.

[77] E. H. Mann, "Theory of the Borden tube", *J. Coun. Sci. Ind. Res. (Austral.)* **20**, (1946) 122.

[78] E. H. Mann, "An elastic theory of dislocation", *Proc. Roy. Soc. London A* **199** (1949) 376–394.

[79] M. A. Miner, *J. App. Mech.* **12** (1945) 159.

[80] C. S. Morawitz, "On the non-existence of continuous transonic flows past profiles", *Comm. Pure Appl. Math.* **9** (1956) 45–68.

[81] G. N. Patterson, "Ducted fans: design for high efficiency", *Australian Council for Aeronautics; Report ACA-7* (1944).

[82] G. N. Patterson, "Ducted fans: approximate methods of design for small slipstream rotation", *Australian Council for Aeronautics; Report ACA-8* (1944).

[83] G. N. Patterson, "Ducted fans: effect of the straightener on overall efficiency", *Australian Council for Aeronautics; Report ACA-9* (1944).

[84] G. N. Patterson, "Ducted fans: high efficiency with contra-rotation", *Australian Council for Aeronautics; Report ACA-10* (1944).

[85] A. F. Pillow, "A review of hydrodynamic stability and its bearing on transition to turbulent flow in the boundary layer", *A.R.L. Report A 35* (1945).

[86] A. F. Pillow, "The formation and growth of shock waves in the one-dimensional motion of a gas", *Proc. Camb. Phil. Soc.* **45** (1949) 558–586.

[87] A. F. Pillow, "The theory of heat regenerators in the unsteady state", *A.R.L. Report A 78* (1952).

[88] A. F. Pillow, "The free convection cell in two dimensions", *A.R.L. Report A 79* (1952).

[89] J. R. M. Radok, "The theory of aerofoils in unsteady motion", *Aero. Quart.* **3** (1952) 297–320.

[90] J. R. M. Radok, "General instability of simply supported rectangular plates", *J. Aero. Sci.* **21** (1954) 109–116.

[91] J. R. M. Radok, "The solution of eigenvalue problems of integral equations by power series", *J. of Math. and Appl. Mech.* **12** (1955) 413.

[92] J. R. M. Radok, "Problems of plane elasticity for reinforced boundaries", *J. Appl. Mech.* **22** (1955) 249.

[93] J. R. M. Radok, J. P. O. Silberstein and H. A. Wills, "A new theory for the strength of wooden box beams", *Australian Council for Aeronautics; Report ACA-40* (1948).

[94] J. R. M. Radok and L. F. Stiles, "The motion and deformation of aircraft in uniform and non-uniform atmospheric disturbances", *Australian Council for Aeronautics; Report ACA-41* (1948).

[95] P. G. Saffman and S. Tanveer, "Prandtl-Batchelor flow past a flat plate with a forward-facing flap", *J. Fluid Mech.* **143** (1984) 351–365.

[96] P. G. Saffman and S. Tanveer, "Vortex induced lift on two dimensional low speed wings", *Studies in Appl. Maths.* **71** (1984) 67–78.

[97] J. F. M. Scholes and G. N. Patterson, "Wind tunnel tests on ducted contra-rotating fans", *Australian Council for Aeronautics; Report ACA-14* (1945).

[98] J. F. M. Scholes, "Ducted fans: a monogram method of analysis", *Australian Council for Aeronautics; Report ACA-32* (1947).

[99] D. A. Secomb, "A wind tunnel investigation of Levey's shock-free aerofoil", *A.R.L. Note A 325* (1970).

[100] F. S. Shaw, "The torsion of solid and hollow prisms in the elastic and plastic range by relaxation methods", *Australian Council for Aeronautics; Report ACA-11* (1944).

[101] F. S. Shaw, *An introduction to relaxation methods* (Dover, New York, 1953).

[102] F. S. Shaw and J. P. O. Silberstein, "Stress analysis of an engine mount", *Australian Council for Aeronautics; Report ACA-17* (1945).

[103] J. P. O. Silberstein, "Plywood panels in end compresion. Curved panels with grain at various angles to generators", *Australian Council for Aeronautics; Report ACA-31* (1949).

[104] J. P. O. Silberstein, "On eigenvalues and inverse singular values of compact linear operators", *Proc. Camb. Phil. Soc.* **49** (1953) 201–212.

[105] R. C. T. Smith, "The buckling of flat plywood plates in compression", *Australian Council for Aeronautics; Report ACA-12* (1944).

[106] R. C. T. Smith, "The buckling of plywood plates in shear", *Australian Council for Aeronautics; Report ACA-29* (1946).

[107] R. C. T. Smith, "The approximate solution of equations in infinitely many unknowns", *Quart. J. Math.* **18** (1947) 25.

[108] W. J. Strang, "Transient source doublet and vortex solutions of the linearized equation of supersonic flow", *Proc. Roy. Soc. London A* **202** (1950) 40–53.

[109] W. J. Strang, "Transient lift of three-dimensional purely supersonic wings", *Proc. Roy. Soc. London A* **202** (1950) 54–80.

[110] J. J. Thompson and W. H. Wittrick, "The stress in certain cylindrical swept tubes under torsion and bending", *Australian Council for Aeronautics; Report ACA-43* (1949).

[111] R. W. Traill-Nash, "The symmetric vibrations of aircraft", *Aero. Quart.* **3** (1951) 1–22.

[112] R. W. Traill-Nash, "The anti-symmetric vibrations of aircraft", *Aero. Quart.* **3** (1951) 145–160.

[113] R. W. Traill-Nash, "On the excitation of pure natural modes in aircraft resonance testing", *J. Aero/Space Sci.* **25** (1958) 775.

[114] R. W. Traill-Nash and A. R. Collar, "The effects of shear flexibility and rotary inertia on the bending vibration of beams", *Quart. J. Mech. Appl. Math.* **6** (1953) 186–222.

[115] V. Volterra, *Ann. École Norm. Paris Ser. 3* **24** (1907) 401.

[116] R. A. Wallis, "The use of air-jets for boundary-layer control", *A.R.L. Note A 110* (1952).

[117] H. A. Wills, "Life of aircraft structures", in *Proc. 2nd. Intern. Aero. Conf. Inst. Aero. Sciences* (1949) 361–403.

[118] H. A. Wills, "Structural fatigue", *Aircraft* (1950) 12–14.

[119] W. H. Wittrick, "Stresses around reinforced elliptical holes, with application to pressure cabin windows", *Aero. Quart.* **10** (1959) 373–400.

[120] W. H. Wittrick, "Some simple transformation functions for square and triangular holes with rounded corners", *Aero. Quart.* **11** (1960) 195–199.

[121] W. H. Wittrick, "Preliminary analysis of a highly swept cylindrical tube under torsion and bending", *Australian Council for Aeronautics; Report ACA-39* (1948).

[122] W. H. Wittrick, "Torsion and bending of swept and tapered wings with rigid chordwise ribs", *Australian Council for Aeronautics; Report ACA-51* (1950).

[123] W. W. Wood, "Boundary layers whose streamlines are closed", *J. Fluid Mech.* **1** (1957) 77–87.

[124] W. W. Wood, "The asymptotic expansion at large Reynolds numbers for steady motion between non-coaxial rotating cylinders", *J. Fluid Mech.* **3** (1957) 159–175.

[125] W. W. Wood, "Properties of inviscid, recirculating flows", *J. Fluid Mech.* **22** (1964) 337–346.

[126] W. W. Wood, "Tunnel interference from slotted walls", *Quart. J. Mech. Appl. Math.* **17** (1964) 125–140.

[127] W. W. Wood, "Calculations for anemometry with fine hot wires", *J. Fluid Mech.* **32** (1968) 9–19.

[128] W. W. Wood, "Free and forced convection from hot wires", *J. Fluid Mech.* **55** (1972) 419–438.