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ALGEBRAIC DEFORMATIONS AND BICOHOMOLOGY

BY
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ABSTRACT. Recently we have introduced an enriched cohomology theory for categories that are tripleable (algebraic) over a category of modules. The cohomology admits a circle product, related to the obstruction problem for algebraic deformations, making the total complex a graded ring. We here offer similar constructions in two other situations – coalgebraic and bialgebraic categories. Examples include categories of bialgebras, sheaves of modules, and sheaves of algebras over a sheaf of rings.

Biocohomology theory was introduced by Van Osdol in order to provide a unified cohomology theory for bialgebras, Hopf algebras, and sheaves of algebras, as well as the usual algebraic and coalgebraic categories, [15]. All of these categories are bialgebraic over a category of modules. We have shown that the triple cohomology for an algebraic category may be enriched over the category of commutative coalgebras (**Coalg**), and that this gives rise to a circle product in cohomology, [8]. Here we extend those results to bialgebraic categories, and show how the resulting circle product arises in the deformation theory for these categories.

The homology of coalgebras was introduced by Eilenberg and Moore, using injective resolutions of comodules and the cotensor product, [5]. Cohomology was developed by Jonah using classical methods, [12], and reinterpreted in the context of triple cohomology by Van Osdol, [14]. In §1 we show that this theory may be enriched over **Coalg**, and that there is a circle product making $H^*(C,C)$ a graded algebra. In §2 we discuss bicohomology and the circle product there, and in §3 we show how this product appears as the obstruction to extending deformations of bialgebraic structures.

Throughout this paper **M** will denote an abelian monoidal-closed category with unit object k. **Coalg** is the category of cocommutative, coassociative, counitary coalgebras in **M**. It is cartesian-closed, and many categories of interest are enriched over it, [7]. Recall that a coalgebra C comes equipped with structure maps $\Delta : C \longrightarrow C \otimes C$ and $\xi : C \longrightarrow k$. A point (p) in C is an element satisfying $p\Delta = p \otimes p$ and $p\xi = 1$. A primitive over p is an element (d) satisfying $d\Delta = d \otimes p + p \otimes d$ and $d\xi = 0$. We use (M,N) to denote the cofree coalgebra over $\mathrm{Hom}(M,N)$ in **M**. The functor $(-,-) : \mathbf{M}^{op} \times \mathbf{M} \longrightarrow \mathbf{Coalg}$ defines the (trivial) enrichment of **M** over **Coalg**.

1. **Coalgebras and comodules.** Let k be a commutative ring and $\mathbf{M} = \mathbf{Mod\text{-}k}$. Let

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$\mathbf{C}$ denote the category of (non-cocommutative) coalgebras in $\mathbf{M}$. If C and D are in $\mathbf{C}$, there is an element of **Coalg**, denoted $\mathbf{C}(C,D)$, defining an enriched Hom functor $\mathbf{C}^{op} \times \mathbf{C} \longrightarrow \mathbf{Coalg}$. $\mathbf{C}(C,D)$ is the joint equalizer in **Coalg** of the following diagrams, [7]:

$$\begin{tikzcd}
 & (C,D)\otimes(C,D) \arrow[r, "\otimes"] & (C\otimes C, D\otimes D) \arrow[d, "{(\Delta,1)}"] \\
\mathbf{C}(C,D) \arrow[r, dashed] & (C,D) \arrow[u, "\Delta"] \arrow[r, "{(1,\Delta)}"] & (C, D\otimes D)
\end{tikzcd} \tag{1}$$

$$\begin{tikzcd}
\mathbf{C}(C,D) \arrow[r, dashed] & (C,D) \arrow[rr, "{(1,\xi)}"] \arrow[dr, "\xi"'] & & (C,k) \\
 & & k \arrow[ur, "\xi"'] &
\end{tikzcd} \tag{2}$$

From this it is easy to see that a point in $\mathbf{C}(C,D)$ represents a coalgebra map from C to D.

Van Osdol has shown that the category of D-D comodules is isomorphic to the category of abelian cogroup object under D. [14]. If $\psi : D \longrightarrow C$ is such an object, the classical comodule it represents is given by the cokernel. The cogroup counit is a map $u : C \longrightarrow D$. The coalgebra $D/\mathbf{C}(C,D)$ is defined as the equalizer in **Coalg** of the following diagram:

$$\begin{tikzcd}
D/\mathbf{C}(C,D) \arrow[r, dashed] & \mathbf{C}(C,D) \arrow[rr, "{(\psi,1)}"] \arrow[dr] & & \mathbf{C}(D,D) \\
 & & k \arrow[ur] &
\end{tikzcd} \tag{3}$$

Proposition 1. *If C is a cogroup object under D with counit $u : C \longrightarrow D$, then the primitives over u in $D/\mathbf{C}(C,D)$ represent the coderivations from* coker ψ *to D.*

Proof. That a primitive represents a coderivation follows from chasing around (1) and (2). That it maps from coker ψ follows from (3). That there are enough primitives follows from the cofreeness of (C,D) and the fact that $\mathbf{C}(C,D)$ is an equalizer.

In order to gain a great deal of generality, we will switch to the setting of triple cohomology. Let $U : \mathbf{C} \longrightarrow \mathbf{M}$ be a cotripleable functor with right adjoint $G : \mathbf{M} \longrightarrow \mathbf{C}$, yielding a triple (T,μ,η) on $\mathbf{C}$ and a cotriple (S,ϵ,δ) on $\mathbf{M}$. An object of $\mathbf{C}$ is an S-coalgebra, i.e. a map $b : B \longrightarrow BS$ in $\mathbf{M}$ satisfying well known conditions. This S-coalgebra we usually denote B^b, though we will omit the superscript when convenient. The triple T gives rise to a semisimplicial complex BT^* in $\mathbf{C}$. If C is an abelian cogroup object in $\mathbf{C}$, $\mathrm{Hom}_{\mathbf{C}}(C, BT^*)$ is a simplicial complex of abelian groups, and the usual triple cohomology groups $H^n(C, D)$ are defined to be the homology of the associated chain complex, whose boundary maps $d^n : \mathrm{Hom}_{\mathbf{C}}(C, BT^{n+1}) \longrightarrow \mathrm{Hom}_{\mathbf{C}}(C, BT^{n+2})$ are given by $xd^n = \sum_{i=1}^{n} xT^{i-1}\eta T^{n-i}$, [14].

The use of abelian cogroup objects may be avoided by using the enrichment of $\mathbf{C}$ over **Coalg**. The functor $S : \mathbf{M} \longrightarrow \mathbf{M}$ may be lifted to a natural map of coalgebras $(C, B) \longrightarrow (CS, BS)$, whether or not S is additive. The coalgebra $\mathbf{C}(C^c, B^b)$ is defined to be the equalizer in **Coalg** of the following diagram:

(4)
$$\begin{tikzcd}
\mathbf{C}(C^c, B^b) \arrow[r, dashed] & (C, B) \arrow[r, "S"] \arrow[dr, "{(1, b)}"'] & (CS, BS) \arrow[d, "{(c, 1)}"] \\
 & & (C, BS)
\end{tikzcd}$$

Proposition 2. *$\mathbf{C}(C^c, B^bT^{n+1})$ and (C, BS^n) are isomorphic.*

Proof. Dual to the algebraic case given in [7]. Let (A, BS^*) be the complex in **Coalg** given below:

(5)
$$0 \longrightarrow (A, B) \underset{a\cdot(\ )S}{\overset{(\ )\cdot b}{\rightrightarrows}} (A, BS) \underset{a\cdot(\ )S}{\overset{(\ )\cdot bS,\ (\ )\cdot\delta}{\rightrightarrows\!\!\!\to}} (A, BS^2) \ldots$$

This is the non-homogeneous complex for triple cohomology, dual to that given in [2]. It is easy to check that the adjunction $\mathbf{C}(A^a, B^bT^{*+1}) \longrightarrow (A, BS^*)$ preserves the boundaries of these complexes.

Proposition 3. *$\mathbf{C}(A^a, B^bT^{*+1})$ and (A, BS^*) are isomorphic as complexes of coalgebras.*

Proof. See [2]. We define $H^n(A, B)$ to be the homology of the associated chain complex in $\mathbf{M}$. The boundary map $d^n : (A, BS^n) \longrightarrow (A, BS^{n+1})$ is given by

$$xd^n = xbS^{n+1} + \sum_{i=1}^{n} (-1)^i xS^{i-1}\delta S^{n-i} + (-1)^{n+1} axS.$$

It is obvious from (5) that the points in $H^0(A, B)$ are precisely the coalgebra maps $A \longrightarrow B$. If $f \in H^0(A, B)$, the primitives over f form a subcomplex of (5), whose cocycles are denoted $C_f^n(A, B)$, coboundaries $B_f^n(A, B)$, etc. The cotriple version of Proposition 1 is:

Proposition 4. *If $f : A \longrightarrow B$ is the counit of the abelian cogroup object $B \longrightarrow A$, then $C_f^0(A, B) \simeq$ coder(A, B), while $H_f^0(A, B) \simeq$ coder(A, B)/inner-coder(A, B).*

The circle product may now be defined dually to that in the algebraic case, [8].

DEFINITION 5. *If* $x \in (A, BS^m)$ *and* $y \in (B, CS^n)$, *then* $x \circ y \in (A, CS^{m+n})$ *is defined by* $x \circ y = x \cdot yS^m$.

THEOREM 6. $(x \circ y)d = x \circ yd + (-1)^n xd \circ y$.

PROOF. Dual to the algebraic case, [8]. □

Thus the circle product lifts to a map of cohomology groups $\circ : H^m(A,B) \otimes H^n(B,C) \longrightarrow H^{n+m}(A,C)$, and $H^*(C,C)$ becomes a graded ring. Since $H^0(A,B) = \mathbf{C}(A,B)$ and the product $\circ : H^0(A,B) \otimes H^0(B,C) \longrightarrow H^0(A,C)$ is just composition, we have a graded category whose zero component is the category $\mathbf{C}$ with its enriched Hom into **Coalg**. If the cotriple is additive, for example the change of base from a category of comodules to another, Kleiner has shown that the circle product yields the Yoneda composition of long exact sequences, [11].

2. **Bicohomology.** Suppose that we are given a cotriple (S, ϵ, δ) and a triple (T, μ, η) on $\mathbf{M}$, and a mixed distributive law $\lambda : TS \longrightarrow ST$ ensuring harmony between the two, i.e. a natural transformation satisfying $\lambda \cdot T\delta = \delta T \cdot S\lambda \cdot \lambda S$, $S\mu \cdot \lambda = \lambda T \cdot T\lambda \cdot \mu S$, $S\eta \cdot \lambda = \eta S$, and $\lambda \cdot T\epsilon = \epsilon T$, [3]. A T-S-bialgebra A^a_α is an element of $\mathbf{M}$ equipped with maps $\alpha : AT \longrightarrow A$ and $a : A \longrightarrow AS$ making it a T-algebra and an S-coalgebra and satisfying $aT \cdot \lambda \cdot \alpha S = \alpha a$, [15]. A map between T-S bialgebras is a map in $\mathbf{M}$ that is both a T-algebra map and an S-coalgebra map.

The most obvious example of a bialgebraic category over **Mod-k** is, of course, the category of k-bialgebras, in the classical sense. Of much more interest is the following: Let X be a topological space and R a sheaf of commutative rings on X; let $\mathbf{M}$ be the category of ΠR_x-modules, S Godement's flabby sheaf cotriple on $\mathbf{M}$, and T the tensor algebra triple. Then the category of T-S-bialgebras is the category of sheaves of R-algebras on X, [15].

In order to clean up the notation needed below, we use (rather imprecisely) Λ to denote the obvious natural transformation $S^mT^n \longrightarrow T^mS^n$ obtained from repetitions of λ.

If A^a_α and B^b_β are T-S-bialgebras, the bicohomology groups $H^n(A,B)$ are obtained via a complex of complexes (see diagram next page).

The homology of the associated double complex in $\mathbf{M}$ is $H^n(A,B)$. We use ∂ and d to denote the horizontal and vertical boundaries respectively. Note that $aT^m\Lambda$ gives AT^m the structure of an S-coalgebra, and $\Lambda\beta S^n$ makes BS^n a T-algebra. Thus the rows and columns of the double complex are just those defining algebra and coalgebra cohomology respectively. The points in $H^0(A,B)$ again represent the T-S-bialgebra maps from A to B. Furthermore, the double subcomplex of primitives (over a given point (ψ) in $H^0(A,B)$) exactly gives biderivations. Hence, this double subcomplex defines the classical bicohomology groups $H^n_\psi(A,B)$. Obviously $H^n(A,B)$ and $H^n_\psi(A,B)$ are related by a long exact sequence involving the homology of the

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(6)

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
 & \vdots & \vdots & \vdots \\
0 \arrow[r] & (A, BS^2) \arrow[r, shift left] \arrow[r, shift right] & (AT, BS^2) \arrow[r, shift left=2] \arrow[r] \arrow[r, shift right=2, "(\ )T\Lambda\beta S^2"'] & (AT^2, BS^2) \ldots \\
0 \arrow[r] & (A, BS) \arrow[u, shift right=3, "(\ )bS"] \arrow[u, "(\ )\delta"] \arrow[u, shift left=3, "a(\ )S"'] \arrow[r, shift left, "\alpha(\ )"] \arrow[r, shift right, "(\ )T\lambda\beta S"'] & (AT, BS) \arrow[u, shift right=3, "(\ )bS"] \arrow[u, "(\ )\delta"] \arrow[u, shift left=3, "aT\lambda(\ )S"'] \arrow[r, shift left=2, "\alpha T(\ )"] \arrow[r, "\mu(\ )"] \arrow[r, shift right=2, "(\ )T\lambda\beta S"'] & (AT^2, BS) \ldots \arrow[u, shift right=3] \arrow[u] \arrow[u, shift left=3] \\
0 \arrow[r] & (A, B) \arrow[u, shift right, "(\ )b"] \arrow[u, shift left, "a(\ )S"'] \arrow[r, shift left, "\alpha(\ )"] \arrow[r, shift right, "(\ )T\beta"'] & (AT, B) \arrow[u, shift right, "(\ )b"] \arrow[u, shift left, "aT\lambda(\ )S"'] \arrow[r, shift left=2, "\alpha T(\ )"] \arrow[r, "\mu(\ )"] \arrow[r, shift right=2, "(\ )T\beta"'] & (AT^2, B) \ldots \arrow[u, shift right] \arrow[u, shift left] \\
 & 0 \arrow[u] & 0 \arrow[u] & 0 \arrow[u]
\end{tikzcd}
$$

quotient complex. Note that the boundary operator on the double complex is just $\partial + d$, [13].

Definition 7. *If* $x \in (AT^i, BS^j)$ *and* $y \in (BT^k, CS^m)$, *then* $x \circ y = xT^k \cdot \Lambda \cdot yS^j$.

Theorem 8. $x \circ y \in (AT^{i+k}, CS^{j+m})$ *and* $(x \circ y)(\partial + d) = x(\partial + (-1)^m d) \circ y + x \circ y((-1)^i \partial + d)$.

Proof. This follows from observing that $x \circ y = x \circ \Lambda\, y = x \Lambda \circ y$, the corresponding formulas for ∂ and d, and the following lemma.

Lemma 9. $(xT^i \Lambda)d = xdT^i \Lambda$ *and* $(\Lambda \cdot yS^m)\partial = \Lambda \cdot y\partial S^m$.

Proof. We will give the proof for $x \in (A, BS)$ and $i = 1$ and leave the general case to the reader.

$$
\begin{aligned}
(xT\lambda)d &= xT \cdot \lambda \cdot bTS \cdot \lambda S - xT \cdot \lambda \cdot T\delta + aT \cdot \lambda \cdot xTS \cdot \lambda S \\
&= xT \cdot bST \cdot S\lambda \cdot \lambda S - xT \cdot \delta T \cdot S\lambda \cdot \lambda S + aT \cdot xST \cdot S\lambda \cdot \lambda S = xdT \cdot \Lambda .
\end{aligned}
$$

□

Though the circle product is defined as a map $(AT^i, BS^j) \otimes (BT^k, CS^m) \longrightarrow (AT^{i+k}, CS^{j+m})$, this clearly may be extended to a circle product on the associated double

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complex, the dimensions working out just right. Theorem 8 then guarantees that we have a product at the level of cohomology, $\circ : H^m(A,B)\otimes H^n(B,C)\longrightarrow H^{m+n}(A,C)$. Hence, once again, the category of T-S-bialgebras becomes a graded category over **M**, $H^0(A,B)$ being the **Coalg**-valued Hom, and $H^*(A,A)$ is a graded ring object in **M**.

3. **Deformation theory.** Consider the following diagram in **Coalg**

$$\begin{tikzcd}
& (AT, A) \arrow[r, shift left, "\Delta\circ"] \arrow[r, shift right, "\mu\cdot(\ )"'] & (AT^2, A) \\
E \arrow[r, dashed] & (AT, A)\otimes(A, AS) \arrow[u, "\pi_1"] \arrow[d, "\pi_2"'] \arrow[r, shift left, "\tau\circ"] \arrow[r, shift right, "\cdot"'] & (AT, AS) \\
& (A, AS) \arrow[r, shift left, "\Delta\circ"] \arrow[r, shift right, "(\ )\cdot\delta"'] & (A, AS^2)
\end{tikzcd} \tag{7}$$

where E is the limit in **Coalg**. Note that τ is the "twist" isomorphism, while π_1 and π_2 are the projections in **Coalg**. If $\alpha\otimes a$ is a point in E, we find that $(\alpha\otimes a)\pi_1\Delta\circ = \alpha T\alpha=\mu\alpha=\mu\cdot(\alpha\otimes a)\pi_1$, $(\alpha\otimes a)\tau\circ = aT\lambda\alpha S=\alpha a$, and $aaS=a\delta$. Thus the points in E are simply pairs of maps making A a T-S-bialgebra.

A primitive $\alpha_1\otimes a+\alpha\otimes a_1$ over $\alpha\otimes a$ is given by a pair of primitives (α_1,a_1) over α and a respectively. If (α_1,a_1) is in E we have $a_1\circ a+a\circ a_1=a_1\delta$, $\alpha_1\circ\alpha+\alpha\circ\alpha_1=\mu\alpha_1$, and $\alpha_1\circ a+\alpha\circ a_1 = aT\lambda\alpha_1 S+a_1T\lambda\alpha S = a_1\circ\alpha+a\circ\alpha_1$. Hence, a primitive in E is precisely a primitive in $C^1(A,A)$, i.e. a classical 1-cocycle.

By a deformation of (α,a) we mean a sequence of divided powers c_* over $\alpha\otimes a$ in $(AT,A)\otimes(A,AS)$; c_* is determined by a double sequence $(\alpha_m,a_n)_{m,n\geqq 0}$ such that $\alpha_0=\alpha$,

$$a_0=a,\ \alpha_m\Delta=\sum_{i+j=m}\alpha_i\otimes\alpha_j,\ a_m\Delta=\sum_{i+j=m}a_i\otimes a_j,$$

$$c_m=\sum_{i+j=m}\alpha_i\otimes a_j,\ \text{ and } \alpha_m\xi=a_m\xi=c_m\xi=0 \text{ for } m>0.$$

The formal sum of a deformation in any coalgebra represents a formal point in that coalgebra. Hence, if c_* is contained in E, then (α_*,a_*) is a deformation of (α,a) towards another T-S-bialgebra structure on A. In that case we say (α_*,a_*) is a bialgebraic deformation of (α,a). This is the triple-theoretic version of the algebraic deformations introduced by Gerstenhaber, [9, 6]. If (α_*,a_*) is bialgebraic, from (7) we have $\sum \alpha_i \circ \alpha_j = \mu\alpha_m$, $\sum \alpha_i \circ a_j = \sum a_i \circ \alpha_j$, and $\sum a_i \circ a_j = a_m\delta$ $(i+j=m)$. These can be written

$$\begin{aligned} \alpha_m\partial &= -\sum \alpha_i \circ \alpha_j \\ a_m\partial + \alpha_m d &= \sum a_i \circ \alpha_j - \alpha_i \circ a_j \\ a_m d &= -\sum a_i \circ a_j \end{aligned} \tag{8}$$

In (8), and in what follows, the sum of the indices of each term is m, and no index is zero. If (α_*, a_*) is bialgebraic, the sums above form a 2-coboundary.

THEOREM 10. *If $(\alpha_n, a_n)_{n<m}$ is a truncated bialgebraic deformation of (α, a), then*

$$\left(-\sum \alpha_i \circ \alpha_j, \sum \alpha_i \circ a_j - \alpha_i \circ a_j, -\sum a_i \circ a_j\right) \tag{9}$$

is a 2-cocycle. This determines a primitive 2-cocycle whose vanishing in cohomology is necessary and sufficient for the existence of (α_m, a_m) extending the deformation.

PROOF. Temporarily write (9) as (X, Y, Z). We must show that $X\partial = 0$, $Zd = 0$, $Xd + Y\partial = 0$, and $Yd + Z\partial = 0$. The first is Lemma 3 in [6], and the second is dual to that. We verify that $-Xd = Y\partial$ and leave $Yd = -Z\partial$ to the reader.

$$\begin{aligned} \left(\sum \alpha_i \circ \alpha_j\right)d &= \sum \alpha_i \circ \alpha_j d + \alpha_i d \circ \alpha_j = \sum \alpha_i \circ a_j \circ \alpha_k - \alpha_i \circ \alpha_j \circ a_k \\ &\quad - \alpha_i \circ a_j\partial + a_i \circ \alpha_j \circ \alpha_k - \alpha_i \circ a_j \circ \alpha_k - a_i\partial \circ \alpha_j \\ &= \sum -\alpha_i\partial \circ a_j - \alpha_i \circ a_j\partial + a_i \circ \alpha_j\partial - a_i\partial \circ \alpha_j \\ &= \sum (a_i \circ \alpha_j)\partial - (\alpha_i \circ a_j)\partial . \end{aligned}$$

Thus (9) is an enriched cocycle obstructing (α_m, a_m); we denote it $\mathrm{obs}(\alpha_m, a_m)$. If c_m is any 1-cochain extending $(\alpha_n, a_n)_{n<m}$, then $c_m(\partial + d) - \mathrm{obs}(\alpha_m, a_m)$ is a primitive 2-cocycle whose class is independent of the choice of c_m, and which must be a boundary if an extension within E is to exist. □

COROLLARY 11. *If $H^2(A^a_\alpha, A^a_\alpha) = 0$, then every primitive 1-cocycle extends to a bialgebraic deformation of (α, a).*

This corresponds to the central theorem in algebraic deformation theory. We leave it to the reader to formulate the notions of formal bialgebraic automorphism and equivalent deformations. Imitating the algebraic case, it is then easy to show that the other results central to deformation theory carry over to the bialgebraic case. The use of bicohomology theory thus unifies, in more that just spirit, the classical deformation theory for sheaves of algebras and deformation theory for rings and algebras, (see [10]).

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