

work, and he has demonstrated his wide knowledge by placing it in a historical setting, finding, for example, that in some instances Ramanujan had been anticipated by classical writers, while in others—as for the Bell polynomials—he had obtained results later discovered by others.

The present volume is the first of three. It covers chapters 1–9 of the second notebook as well as the three quarterly reports submitted to the University of Madras under the terms of his scholarship. The various chapters, some of them in collaboration with other authors, have previously appeared separately in mathematical journals. They comprise a total of 759 different entries. With a very few exceptions, where the intent of the entry is not clear, proofs are given under appropriate conditions, or references are given to the literature, where the results are not new. Because of his lack of an orthodox mathematical background, Ramanujan, when he came to England, had little idea of what constituted a mathematical proof and, as Littlewood wrote, “if a significant piece of reasoning occurred somewhere, and the total mixture of evidence and intuition gave him certainty, he looked no further”.

Possibly because of some remarks made by Hardy, there has been an impression that much of Ramanujan’s work was not new and that the disappointment supposedly suffered by him on this account had an adverse effect upon his health. There has never been any strong evidence for this opinion and, as Professor Berndt has been the first to point out, it is truly remarkable how few of the entries are results already known at the time when they were written. It is, of course, true that many are outside the main stream of 20th century mathematics, and Ramanujan was perhaps the last great producer of mathematical formulae. If he had had an orthodox upbringing he might, as Hardy wrote, have discovered more that was new and of greater importance. On the other hand, as Professor Berndt points out, his output of results might have been severely curtailed, as he might have felt inhibited about stating results unrigorously obtained.

The chapters covered in the book deal with magic squares, inverse tangent formulae, series inversion, special functions and numbers such as Eulerian polynomials and numbers, Bernoulli numbers, Riemann zeta function, gamma function, divergent series, and the transformation and evaluation of infinite series. Infinite series abound throughout; some of them are divergent, but many can be considered as asymptotic series.

From such a plethora of results it is impossible to select representative samples. I content myself by selecting one entry, which Ramanujan knew to be false as he cancelled it, but, nevertheless, gives something of the flavour of his work:

“For $|x| < 1$,

$$\prod_{k=1}^{\infty} (1 - x^{p_k})^{-1} = 1 + \sum_{k=1}^{\infty} \frac{x^{p_1 + p_2 + \dots + p_k}}{(1-x)(1-x^2) \dots (1-x^k)},$$

where p_1, p_2, \dots denote the primes in ascending order.”

If c_n and d_n ($n \geq 0$) denote the coefficients of x^n on the left and right sides, respectively, then, quite amazingly, $c_n = d_n$ for $n \leq 20$, but $c_{21} = 30$ and $d_{21} = 31$.

The book, although expensive, is, as one would expect from the publishers, beautifully printed and produced.

R. A. RANKIN

UPMEIER, H. *Symmetric Banach manifolds and Jordan C*-algebras* (North-Holland Mathematics Studies 104, North-Holland, Amsterdam-New York-Oxford, 1985), 444 pp. Dfl. 150.

The theory of symmetric complex Banach manifolds and Jordan C*-algebras has been developing steadily over the last couple of decades. The advance has been guided by classical results for finite-dimensional manifolds and Lie groups and by parallel developments in Jordan C*-algebras closely linked to the theory of operator algebras. In this book the author studies the theory of symmetric manifolds over a Banach space and closely related areas. There are monographs on Jordan C*-algebras but not on their relationship with symmetric manifolds. This book fills an empty slot in the literature. The book is divided into two parts in a natural way. In the first part the theories of Banach manifolds over a Banach space and Banach Lie groups are

developed in a clear precise way from basic functional analytic beginnings. Not many examples are given but those that are are important and are studied in careful detail; the excellent discussion of the Grassmann manifold of a general Banach space is a good example of this. The material in the first half is treated as a well-organised unified subject in a manner not available elsewhere, even though most of the material here is well known and is available in books or monographs.

With the basic foundations well laid in the first half of the book, the main topics of symmetric Banach manifolds, symmetric Siegel domains, and their relationship with Jordan algebras and Jordan triple systems are studied in detail. Here the theory changes from the complex-analytic calculations of the manifolds and Lie groups to more algebraic operator-theoretic methods. This part of the book is a fascinating blend of algebra, Banach manifolds, and functional analysis mainly in the form of Jordan algebras of hermitian operators. The use of techniques from different areas of mathematics is a feature of the book. This book will be essential for those working in Jordan C^* -algebras, and is highly recommended for libraries as a clear readable account of the border between operator algebras and manifold theory.

A. M. SINCLAIR

HANYGA, A. *Mathematical theory of non-linear elasticity* (Ellis Horwood, 1985), 432 pp. £39.50.

During the last 15 years there has been a marked growth of interest in the existence and qualitative properties of solutions to both the static and dynamic equations of nonlinear elasticity. Far from being a sterile exercise in i-dotting, the study of these equations by applied analysts is leading to advances in the understanding of constitutive equations and of material instabilities such as those observed in phase transformations and fracture. A number of books are now appearing that expound the theory of nonlinear elasticity in the light of these recent developments.

The volume by Andrzej Hanyga under review is a scholarly presentation of mathematical elasticity. Chapter 1 consists firstly of mathematical preliminaries such as elements of differential geometry, measure and integration, functional analysis and Sobolev spaces, and secondly of a careful derivation of the fundamental theory. Chapter 2 concerns elastostatics and includes descriptions of monotone operator theory, direct methods based on the assumptions of quasiconvexity and polyconvexity of the stored-energy function, and remarks on regularity, constitutive assumptions and the complementary energy principle. Chapter 3, the longest in the book, is devoted to elastodynamics; some of the topics treated are simple and shock waves, the structure of solutions to hyperbolic systems of conservation laws via the study of functions of bounded variation, the Glimm difference scheme, entropy and viscosity admissibility criteria, uniqueness and stability results due to Dafermos and DiPerna, and finite time blow-up of solutions. Chapter 4, entitled "Geometric aspects of elasticity", deals with material symmetries and dislocations.

In his introduction the author expresses the hope that his book will be of use to engineers as well as mathematicians, but it is one of its less satisfactory aspects that there are few specific examples to link the theory to material behaviour. Further, even engineers of much greater mathematical sophistication than commonly encountered in the U.K. will find the text heavy going. A mathematically minded scientist or classical applied mathematician interested in learning some of the modern theory would probably do better to begin by reading the book by Marsden & Hughes [1]. Nevertheless, one of the strong points of Hanyga's book is the inclusion of much necessary and sometimes refined mathematical background complete with proofs. For mathematicians with a knowledge of applied analysis it has a useful selection of topics not duplicated elsewhere, and this, together with the careful and detailed exposition, makes it a significant and welcome addition to the literature.

J. M. BALL

REFERENCE

1. J. E. MARSDEN & T. J. R. HUGHES, *Mathematical foundations of elasticity*, Prentice-Hall, 1983.