Star Clusters: From the Milky Way to the Early Universe
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A new method to simulate long-term evolution of stars near a massive Black Hole

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Abstract. N-body simulation is the necessary tool to investigate the evolution of star clusters. It is important to develop a time integration method which guarantees the appropriate accuracy and calculation cost.

Here we present a new simple method for long term simulation of stars around a massive black hole in stellar systems. Usually the time integration orbits of stars revolving a massive black hole requires much simulation time. We introduce a time transformation which is a kind of "inverse KS regularization" of time. Using our method, the integration of the long term evolution near a black hole (BH) becomes easier, especially applied to relatively large star clusters and the Galactic Center.

Keywords. methods: numerical, galaxies: evolution, galaxies: structure

1. Introduction

Following the long-term orbital evolution of stars near a supermassive black hole (SMBH) numerically is very difficult. It causes the slowing down of simulation of stellar systems around an SMBH (or SMBHs).

In Figures 1 and 2, the way to follow the orbits of stars (or dark matter particles) around a SMBH is shown schematically. The ticks correspond to time steps for each orbit.

Smaller the distance of a star from the SMBH is, larger its acceleration is. Therefore the required step size becomes smaller when the star approaches to the SMBH.

In order to integrate a stellar system containing a (SM)BH, algorithms using individual step sizes are adopted.

However, for cases of a galaxy with central SMBHs, traditional individual step size methods are not enough to carry out the simulation within a practical time. The difference in the time scales of evolution of the entire galaxy and the orbital period near the central SMBH is too large (Figure 3). If we adopt the individual timestep method in this case, the small steps part dominates the calculation. Therefore, a method using which we can avoid the emergence of too small stepsizes of particles near SMBH will be useful.

In the simulation of galaxies, what we pursue is not the exact orbital evolution of each star but evolution of statistical properties. From this point of view it will be useful if there is a method to simulate the secular orbital evolution of stars very close to BH, without integrating their orbits directly. One of the dominant evolution mechanism of orbits of stars very close to an SMBH is secular perturbation.

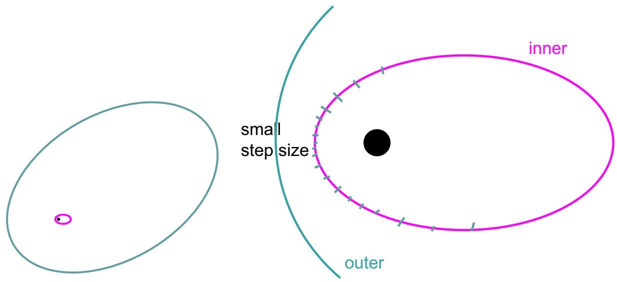

Figure 1. Step Sizes are schematically shown along orbits for a hierarchical system.

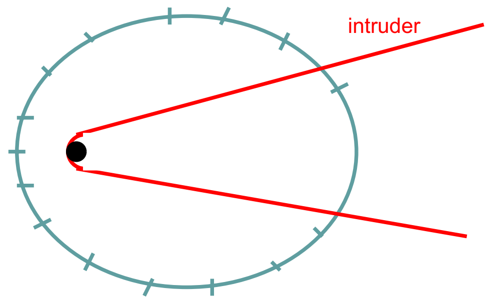

Figure 2. Step Sizes are schematically shown along orbits with an eccentric intruder.

Figure 3. If we adopt the individual temestep method, the small steps part dominates the calculation.

2. Principle

Strategy To slow down the intruders when they are close to BH.
Method Time transformation applied to each particle.

$$r dt = ds, \tag{2.1}$$

where, r is the distance from the BH and s is the "time" of each particle. Adopting the above transformation, the time integration of the orbit of each particle is done using s instead of t. The process is shown Figure 4.

3. Implementation

Since the times t are not the same for individual particles, the force calculation of each step is not appropriate.

If the purpose of the simulation is to investigate the evolution of a few body problem such as scattering experiments of 3-bodies, it will be better if the numerical integration of orbits of each particle is as exact as possible.

However, if the purpose of study is to investigate the evolution of stellar systems such as stars in the Galactic Center, to obtain exact orbits is not necessary.

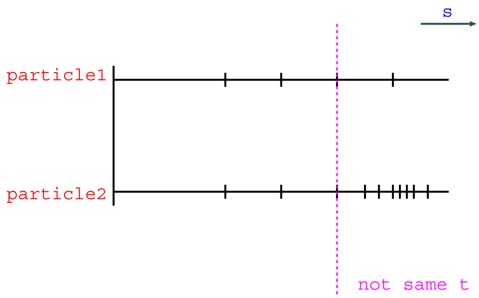

Figure 4. Integration using s.

In that case, it is enough that the long term evolution of distribution of orbital elements (semi-major axis a, eccentricity e, inclination i, argument of pericenter ω) of stars and the evolution of system structure (such as density profile, velocity profile...) are reproduced correctly in numerical simulations.

Through such a simulation, the evolution of orbit shall be consistent with that described by using the perturbation equation (e.g., Murray & Dermott 1999)

$$\frac{de}{ds} = n\Sigma_j \left(\frac{a_j}{a}\right)\left(\frac{m_j}{m_{SMBH}}\right) C_j e_j \sin(\omega - \omega_j), \tag{3.1}$$

$$\frac{d\omega}{ds} = n\Sigma_j \left(\frac{a_j}{a}\right)\left(\frac{m_j}{m_{SMBH}}\right) \left[2F_j - C_j \left(\frac{e_j}{e}\right) \cos(\omega - \omega_j)\right], \tag{3.2}$$

$$\frac{d\Omega}{ds} = n\Sigma_j \left(\frac{a_j}{a}\right)\left(\frac{m_j}{m_{SMBH}}\right) G_j. \tag{3.3}$$

Since the above equations expresses orbit averaged changes, they do not depend on the relative positions between particles at the exact time.

In order to obtain a result consistent with the evolution described using above equations, we introduce a weighted kick. Using it, the Leap-frog method becomes, for example,

$$\mathbf{v}_i = \mathbf{v}_i + 0.5 \cdot \Delta t \sum_j \boldsymbol{a}_{ij} \quad Normal \tag{3.4}$$

$$\mathbf{v}_i = \mathbf{v}_i + 0.5 \cdot \cdot \Delta t \cdot \sum_j w_j \boldsymbol{a}_{ij} \quad Weighted \tag{3.5}$$

$$w_j \propto 1/\dot{\phi}_j \tag{3.6}$$

where $\dot{\phi}_j$ is the angular velocity of each j particle which is the gravity source to i particle.

Test calculation

Here we show the preliminary result of the case of a stable hierarchical system with mass ratio $1 : 0.001 : 0.0001$. The comparison of long term orbital evolutions of the inner particle are shown in Figure 5.

Figure 5 shows our method works well.

4. Summary

Here we present a new simple method for long term simulation of stars around a massive black hole in stellar systems. Usually the time integration orbits of stars revolving a massive black hole requires much simulation time. We introduce a time transformation $rdt = ds$.

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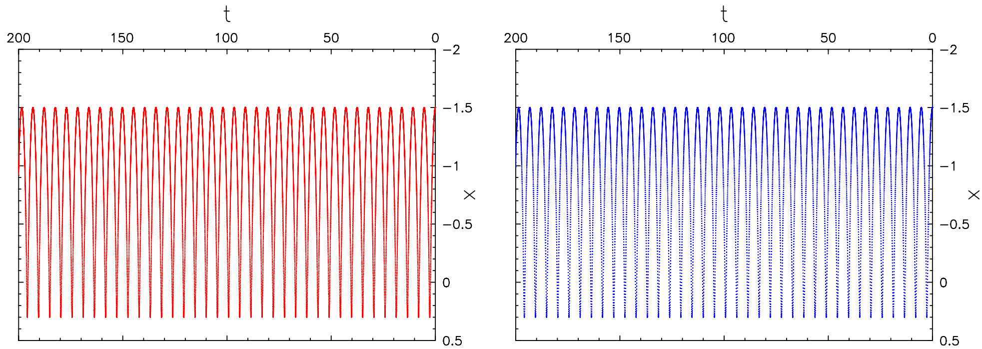

Figure 5. Comparison of results of normal Leap-frog (LF) calculation (left) and LF with weighted kick (right).

Using this method, the secular evolution of distribution of orbital elements can be simulated, in principle.

Further numerical test of the evolution of stellar system around an SMBH should be required.

Reference

Murray & Dermott 1999, *Solar System Dynamics*, (Cambridge University Press)

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