Part III is devoted to the root-of-unity case. It covers both $U_q$ and $O_q$. When $q$ is a root of unity, these are polynomial identity algebras. Thus, PI-theory plays a central role in this part. The topics covered include Poisson structures on the centres, the Azumaya loci and Müller's theorem.

Both Parts II and III contain 10 chapters. The final chapter of each is devoted to open problems. There is a fine difference between the parts.

The final chapter of Part II is called *Problems and conjectures*. There a *problem* is usually a rather vague question, definitely important, but probably too general to be answered to everybody's satisfaction. On the other hand, a *conjecture* is a very precise question. *Conjectures* will provide work and important benchmarks for algebraists while *problems* will offer subjects for numerous tea (coffee, cacao, wine, vodka, sake, etc.)-time discussions across the globe.

The final chapter of Part III is called *Problems and perspectives*. With this slightly misleading title, it contains 11 *questions*, some of which are *problems*, while others are *conjectures* (in the above sense). It also contains a short exposition (a *perspective*?) of Lusztig's conjecture.

The book fills a gap in the quantum-groups literature that will be appreciated by many students and researchers in physics and mathematics. It can be used as a foundation to an advanced course on quantum groups. It may be also given to a postgraduate student for independent reading. Finally, the book is an important source of references and information for specialists.

D. RUMYNIN

DOI:10.1017/S0013091505224823

Ehrenpreis, L. *The universality of the radon transform* (Oxford University Press, 2003), 0 19 850978 2 (hardback), £80.

Let us start, as this 700-page monograph rather oddly does not, by saying what we mean by a Radon transform. The basic set-up is that we are interested in how an integral of a function restricted to a submanifold varies as we deform the submanifold. Consider then a family of smooth submanifolds $L_x$ ($x \in X$) of a smooth manifold $M$, parametrized by a smooth manifold $X$. Then for suitable (i.e. sufficiently smooth and decaying appropriately at infinity) functions $f$ on $M$ we define the *Radon transform* to be the function $Rf$ on $X$ defined by integration,

$$Rf(x) = \int_{L_x} f,$$

with respect to some suitable measure. Thus we have a map $R$ from functions on $M$ to functions on the parameter space $X$ and one can study this map (injectivity, smoothness properties, characterization of the range, etc.) and, where it is injective, try to find an explicit inverse. The name comes from Radon's work of 1917, where he showed that one can recover a suitably nice function on the plane if one knows its integral over every line.

The other thing that is missing is some explanation of the scope and aims of the book, whose title rather suggests that the intention is to survey the whole range of the Radon transform in modern mathematics. In fact, the book begins with 100 or so pages of introduction that I suppose are intended to illuminate the philosophy of the remainder but which I found disconnected, rather vague and hard to get to grips with. And the subject matter of the book is centred around the case of submanifolds of $\mathbb{R}^n$, mainly the case of families of affine subspaces, although there is some discussion of what the author calls 'nonlinear Radon transforms', which just means integrating over other sorts of submanifold.

Within these limitations, the reader will find a lot of interesting material, much of it pleasingly concrete, explicit and example-based and considerably more readable than the introduction. There is a lot on harmonic functions, on Cauchy problems and 'Watergate problems' (i.e. situations like specifying data for the wave equation on the time axis, so that an infinite number of data are required—the name comes from a rather contrived analogy with the Watergate scandal). There is a lot on situations such as integrating over lines in $\mathbb{R}^3$, where the Radon transform provides solutions of interesting differential equations. The horocycle transform for symmetric spaces gets an airing too, as do things as diverse as Poisson summation formulae, Hartogs–Lewy extension and Eisenstein series. There is an appendix too (by other authors) on tomography: the issues involved in implementing inverse transforms in practice in situations such as body scanners. Dipping into all this, I found a lot of illuminating discussion. Scattered among all this are numerous 'problems', many of which are interesting, but it is characteristic to my mind of a certain vagueness that pervades the book that there is no indication of their status: the graduate student will wonder whether they are supposed to be exercises or thesis projects.

So what is not in this book? The quick answer is that there is almost nothing on geometric developments, or developments only easily stated in a geometric context. To start at the beginning, as far as I know it was Funk in 1913 who first considered a 'Radon transform': integrating even functions on the sphere over great circles. There is nothing unusual of course in the 'wrong' name getting celebrated in this way, but it is symptomatic of a tendency in this area to avoid anything not based in $\mathbb{R}^n$.

The important work of Gelfand *et al.* [**1**], where it is shown that one can compactify the Radon transform for affine $k$-planes in $\mathbb{R}^n$ and regard it as a transform between sections of an appropriate line bundle over $\mathbb{R}P_n$ and sections of a line bundle over a Grassmanian, is cited in the bibliography but I cannot find it mentioned in the text*. It is from this geometric point of view that Funk's transform and Radon's become identical because even functions on the sphere are the same thing as functions on the projective plane, and then one sees Funk's transform as just a compactification of Radon's.

A whole area that is completely missing from this book is where one takes a Radon transform of something other than functions. To take just one example, consider integrating a 1-form on $\mathbb{R}P_n$ over projective lines. Michel [**2**] showed that a 1-form is in the kernel of this transform if and only if it is the exterior derivative of a function. He proved also a related result for symmetric 2-tensors which establishes infinitesimal Blaschke rigidity for projective space.

When the book does nod towards geometry, one senses the author to be on weaker ground. It seems very eccentric to me to refer to the group generated by orthogonal linear maps and translations on $\mathbb{R}^n$ as the 'affine group' (rather than the Euclidean group), although what most most of us call the affine group (i.e. that generated by $GL(n)$ and translations) is in fact the symmetry group of the Radon transform if one sets things up carefully, and I wonder whether enlarging the symmetry group in this way might clarify some arguments. A lot of the discussion of harmonic functions on $\mathbb{R}^4$, Bateman's representation and twistor theory (misspelt here as 'twister' by the way) seems rather opaque to me because the author does not use the geometry of Grassmannians and the Klein correspondence to elucidate what is going on.

Leaving these criticisms aside, this book contains a lot of good things, and workers in these areas of analysis will want to have it in their libraries. Given the small size of the appendix on tomography, I doubt whether one should really say (as the publishers do) that this book covers 'practical applications to X-ray and electrical impedance tomography', or indeed to give the potential readership as including researchers in 'physics, engineering and medical engineering'. This is a useful book, but it is primarily for analysts who work in $\mathbb{R}^n$.

## References

1. I. M. Gel'fand, S. G. Gindikin and M. I. Graev, Integral geometry in affine and projective spaces, *J. Sov. Math.* **18** (1982), 39–167.

* Why, by the way, has it not become standard practice to give a back-referencing list of page numbers with each reference in a book or paper—it would be perfectly easy to do and be very useful. Do we have to wait for e-books before we can do this sort of reverse look-up?

2. R. Michel, Sur quelques problèmes de géométrie globale des géodésiques, *Bol. Soc. Bras. Mat.* **9** (1978), 19–38.

T. N. BAILEY

DOI:10.1017/S001309150523482X

Magurn, B. A. *An algebraic introduction to K-theory* (Cambridge University Press, 2002), 0 521 80078 1 (hardback), £80.

Algebraic and topological $K$-theories originate in certain generalizations of the category, Vect, of vector spaces over a field. It is well known that any finite-dimensional vector space $V$ admits a basis, and any two bases are equivalent. Sending the isomorphism class $[V]$ of $V$ into $\dim V$ establishes a one-to-one correspondence between isomorphism classes, Iso(Vect), of objects in Vect and the set $\mathbb{N}$ of non-negative integers. As $\dim(V \oplus W) = \dim V + \dim W$, this is in fact a one-to-one correspondence of Abelian monoids which can be made via an obvious enlargement of both sides (i.e. adding formal differences $[V] - [W]$ to Iso(Vect) and negative integers to $\mathbb{N}$) into an equivalence of Abelian groups, $K_0(\text{Vect}) \simeq \mathbb{Z}$.

If one generalizes Vect to the category of finitely generated projective modules, $\text{Mod}_R$, over a ring $R$, then bases no longer exist (and even if they exist they may not be equivalent) so that dimension disappears. However, an analogue, $K_0(\text{Mod}_R)$, of the group $K_0(\text{Vect})$ survives! It is no longer isomorphic to $\mathbb{Z}$, in general, and measures the obstruction to existence of bases; another group, $K_1(\text{Mod}_R)$, describes their non-uniqueness. These are the first two floors of the tower of groups $K_n$ which are the main subject of study in algebraic $K$-theory.

If one generalizes Vect to the category of vector bundles over compact topological spaces, then one arrives at topological $K$-theory.

The book under review is an excellent introduction to the algebraic $K$-theory. It gives a nice overview of several deep problems solved by means of algebraic $K$-theory (such as the normal basis problem in number fields, the classification of normal subgroups of linear groups) and, rather surprisingly, assumes no prerequisite beyond standard undergraduate algebra. The book is very self-contained and can be recommended to graduate students.

Here is a list of contents:

**Part I. Groups of modules: $K_0$**

**Chapter 1.** Free modules (bases; matrix representations; absence of dimension)

**Chapter 2.** Projective modules (direct summands; summands of free modules)

**Chapter 3.** Grothendieck groups (semi-groups of isomorphism classes; semi-groups to groups; Grothendieck groups; resolutions)

**Chapter 4.** Stability for projective modules (Adding couples of $R$; stably free modules; when stably free modules are free; stable rank; dimension of a ring)

**Chapter 5.** Multiplying modules (semi-rings; Burnside rings; tensor products of modules)

**Chapter 6.** Change of rings ($K_0$ of related rings; $G_0$ of related rings; $K_0$ as a functor; the Jacobson radical; localization)

**Part II. Sources of $K_0$**

**Chapter 7.** Number theory (algebraic integers; Dedekind domains; ideal class groups; extensions and norms; $K_0$ and $G_0$ of Dedekind domains)

**Chapter 8.** Group representation theory (linear representations; representing finite groups over fields; semi-simple rings; characters)