

SUPER CATALAN NUMBERS AND FOURIER SUMMATIONS OVER FINITE FIELDS

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We find an algebraic interpretation of the super Catalan numbers through polynomial summation formulae over unit circles over finite fields of odd characteristic. While traditional Fourier analysis involves Riemann integration over the unit circle in the real number plane, we develop a purely algebraic integration theory without recourse to infinite procedures, and develop an algorithm for explicitly computing such Fourier sums for general monomials.

We consider three unit circles that arise from Euclidean geometry and two relativistic geometries, and demonstrate the strong relationship between the integration theory in each geometry. The algebraic integrals in the three geometries are called the Fourier summation functionals and take values in the same finite field.

The key results in this thesis are the existence and uniqueness of the Fourier summation functionals, as well as the explicit formulae for them in terms of the super Catalan numbers and their rational variants which we call the circular super Catalan numbers.

This investigation not only opens up new avenues in developing finite field harmonic analysis from a completely algebraic point of view, but also highlights many similarities to the traditional integration theory over the unit circle.

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