

Phase Imaging: A Compressive Sensing Approach

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Since Wolfgang Pauli posed the question in 1933, whether the probability densities $|\Psi(\mathbf{r})|^2$ (real-space image) and $|\Psi(\mathbf{q})|^2$ (reciprocal space image) uniquely determine the wave function $\Psi(\mathbf{r})$ [1], the so called Pauli Problem sparked numerous methods in all fields of microscopy [2, 3]. Reconstructing the complete wave function $\Psi(\mathbf{r}) = a(\mathbf{r})e^{-i\phi(\mathbf{r})}$ with the amplitude $a(\mathbf{r})$ and the phase $\phi(\mathbf{r})$ from the recorded intensity, enables the possibility to directly study the electric and magnetic properties of the sample through the phase.

In transmission electron microscopy (TEM), electron holography is by far the most established method for phase reconstruction [4]. Requiring a high stability of the microscope, next to the installation of a biprism in the TEM, holography cannot be applied to any microscope straightforwardly. Recently, a phase retrieval approach was proposed using conventional TEM electron diffractive imaging (EDI). Using the SAD aperture as reciprocal-space constraint, a localized sample structure can be reconstructed from its diffraction pattern and a real-space image using a hybrid input-output algorithm [5].

We present an alternative approach using compressive phase-retrieval [6], which does not require a real-space image. The compressive sensing problem has the following formulation. First, we note that the complex-valued reciprocal-space wave-function is in approximation the Fourier transform of the (also complex-valued) real-space wave-function, $\Psi(\mathbf{q}) = \text{FT}[\Psi(\mathbf{r})]$, and subsequently the diffraction pattern image is given by $|\Psi(\mathbf{q})|^2 = |\text{FT}[\Psi(\mathbf{r})]|^2$. We want to find $\Psi(\mathbf{r})$ given a few differently coded diffraction pattern measurements $y_n = |\text{FT}[H_n\Psi(\mathbf{r})]|^2$, where the matrices H_n encode the mask structure of the aperture. This is a nonlinear inverse problem, but has been shown to be solvable even in the underdetermined case [6]. Since each diffraction pattern y_n contains diffraction information from selected regions of the same sample, the differences in each pattern contain local phase information, which can be combined to form a full estimate of the real-space wave-function.

To differently code the diffraction patterns, random complimentary pairs of checkerboard masks are cut into a 200 nm Pt foil covering a conventional TEM aperture (cf. Figure 1). Used as SAD aperture, subsequently diffraction patterns are recorded from the same sample area (cf. Figure 1). Hereby every mask blocks different parts of gold particles on a carbon support (cf. Figure 1 (Right)). First results of phase reconstruction of this novel approach are presented [7].

References:

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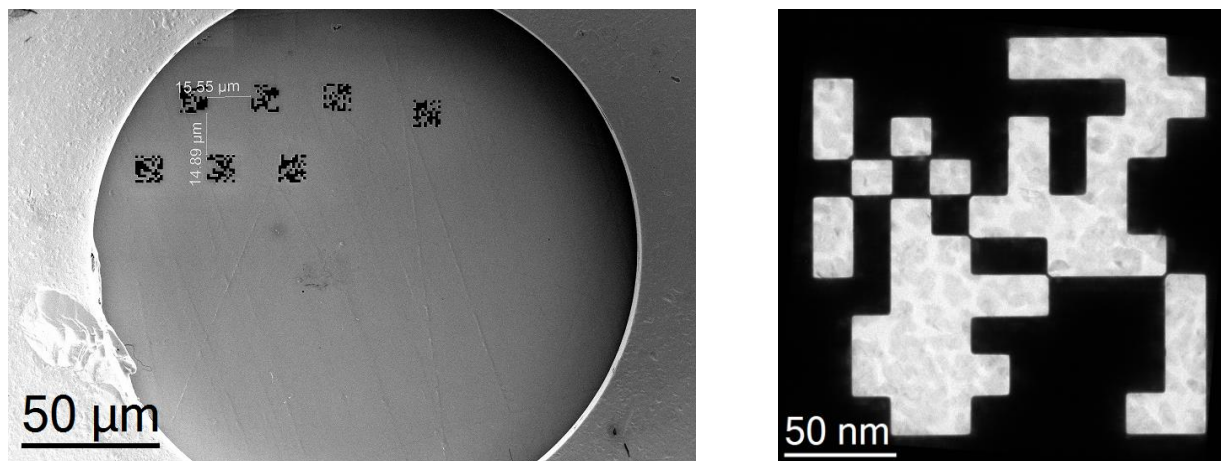


Figure 1. (Left) Conventional TEM aperture covered with 200 nm Pt foil. Random checkerboard masks are cut into the film using FIB. The masks have a size of 10 μm by 10 μm. (Right) Random mask in the SAD aperture holder blocking the image of gold nanoparticles on a carbon support. The diffraction pattern of this sample area is used for the phase reconstruction.

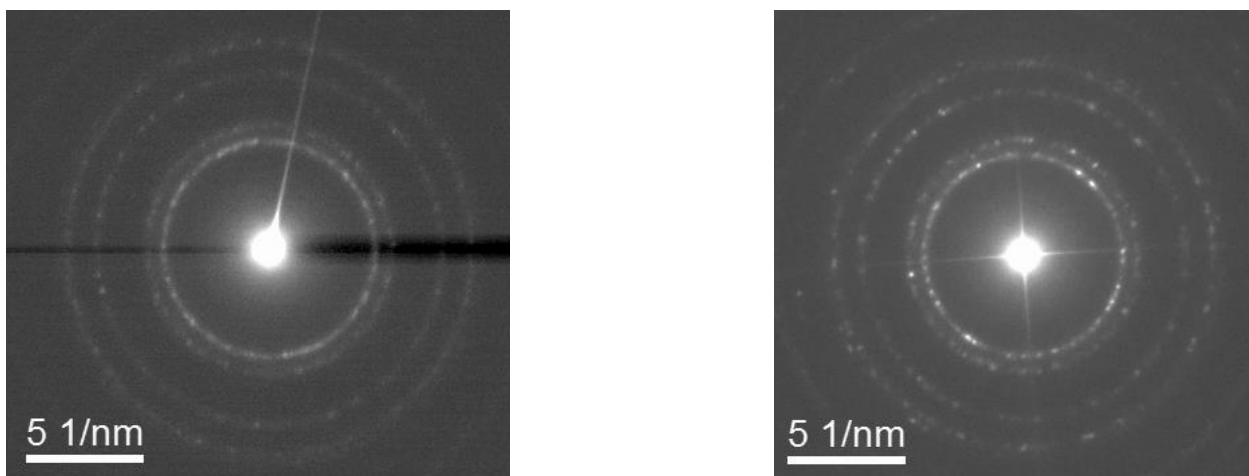


Figure 2. (Left) Diffraction pattern of gold nanoparticles acquired with no SAD aperture inserted. (Right) An example of a coded diffraction pattern using the SAD aperture in Figure 1.