

## Boundary-artifact-free Observation of Magnetic Materials Using the Transport of Intensity Equation

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The transport of intensity equation (TIE) has been used to characterize magnetic materials, such as helical spin order [1] and two-dimensional skyrmions [2]. As described below the TIE can be converted to a Poisson equation [3], and in order to solve it we need to know the boundary condition. The commonly used FFT solver [4] implicitly assumes a periodic boundary condition, and often gives a huge slowly varying background when a set of the images to be used does not satisfy the presumed boundary condition. Therefore, in this report we use a selected area aperture placed at the image plane of the objective, and observe the propagated image wave that passes through the aperture by adjusting the intermediate lens (IL) as shown in Figure 1. In this way, we can obtain a boundary-artifact-free solution as explained below, which will facilitate the use of the TIE for quantitative magnetic characterization in the TEM. Using the IL to select the observation plane has another important advantage: the magnetic field around the sample will stay constant.

The transport of intensity equation (TIE) is a differential equation that describes the relation between the intensity distribution  $I(xy; z)$  and phase distribution  $\phi(xy; z)$  [3]:

$$\frac{\partial I(xy; z)}{\partial z} = -\frac{\lambda}{2\pi} \nabla_{xy} \cdot (I(xy; z) \nabla_{xy} \phi(xy; z)) \quad (1)$$

where  $\nabla_{xy}$  is a two-dimensional gradient operator. As shown by Teague [3] the TIE can be solved by introducing an auxiliary function  $\Phi(xy)$ , which satisfies  $\nabla_{xy} \Phi(xy) \equiv (I(xy; z) \nabla_{xy} \phi(xy; z))$ . Then the TIE (1) becomes

$$\frac{\partial I(xy; z)}{\partial z} = -\frac{\lambda}{2\pi} \nabla_{xy} \cdot (\nabla_{xy} \Phi(xy)) = -\frac{\lambda}{2\pi} \nabla_{xy}^2 \Phi(xy) \quad (2)$$

which is the two-dimensional Poisson equation for  $\Phi(xy)$ . Once  $\Phi(xy)$  is determined,  $\phi(x, y)$  can be obtained by solving another Poisson equation:

$$\nabla_{xy}^2 \phi(xy; z) = \nabla_{xy} \cdot \nabla_{xy} \phi(xy; z) = \nabla_{xy} \cdot (\nabla_{xy} \Phi(xy) / I(xy; z)) \quad (3)$$

In order to solve the Poisson equation we need to know the boundary condition. Paganin and Nugent proposed to solve the Poisson equation using Fast Fourier transform (FFT) [4]. In this case we assume the periodic boundary condition inherently imposed by the Fourier transform. Although this gives a quick and deterministic solution, it is problematic when the images do not satisfy the periodic boundary condition. Recently, in optics, the use of an aperture at the plane where we reconstruct the wave field is proposed, where the Poisson equation is solved using the discrete cosine transform (DCT) for Neumann boundary condition [5]. Along this line, we use the selected area (SA) aperture at the image plane of the objective as shown in Figure 1, and observe the wave propagation by controlling the intermediate lens (IL).

Figure 2 shows a set of three through-focus images of a 10-micron SA aperture obtained by adjusting the IL focus. Here, we used JEOL ARM-200F with CEOS double correctors operated at 200 kV. When the image wave propagates in vacuum, the size of the SA image varies as shown in (a) to (c), which indicates that the propagating wave is a diverging spherical wave. Figure 2d demonstrates that the

phase image retrieved by the DCT solver [6] shows a huge phase modulation. The line profile across the center of the retrieved phase exhibits a phase difference up to 1500 rad between the center and rim of the aperture, namely over 180 nm in the object space (see Figure 2e). This phase modulation is not an artifact, but shows the curvature of field of the image wave [7]. To the best of the authors' knowledge we don't know any reports in the field of TEM, including electron holography, about on direct observation of the curvature of field in TEM. This boundary-artifact-free technique will advance magnetic characterization using the TIE [8].

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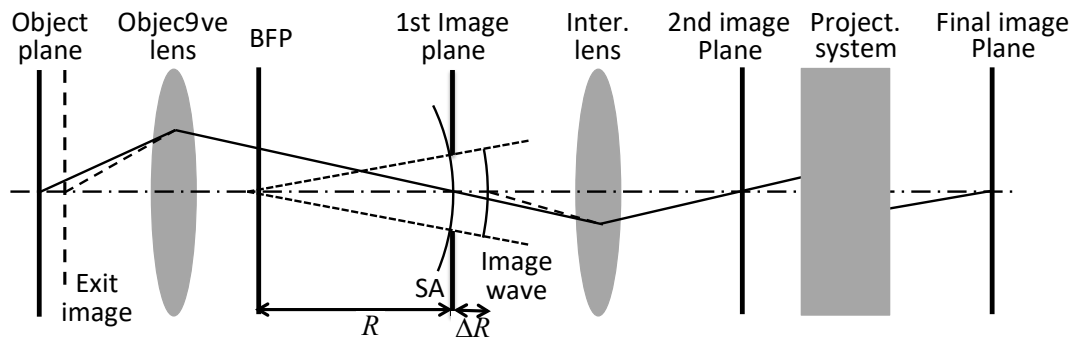
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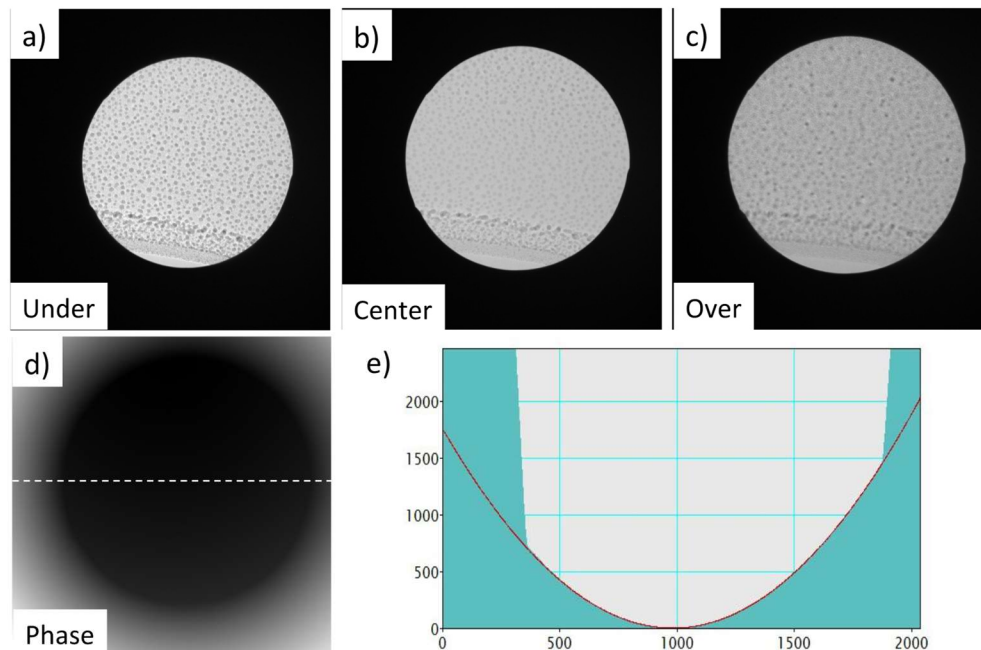
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**Figure 1.** Observation of image propagation. Here, we place the selected area (SA) aperture, and observe the wave field at the objective image plane by adjusting the intermediate lens (IL) focus.



**Figure 2.** (a) to (c): Experimental images of a selected area (SA) aperture. Retrieved phase distribution is shown in (d), and the parabola fitting to the retrieved phase in (e).