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ABSTRACTS OF AUSTRALASIAN PHD THESES

SOME LATTICE POINT PROBLEMS

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This thesis concerns three lattice point problems, and the work on these problems is related by their common use of ideas from the geometry of numbers and their emphasis on 3-dimensional lattices.

The first part of this thesis, concerning subdivision of lattice cones, is related to a lattice point problem of Schinzel, see Schmidt [3], and includes and extends the work of my published paper [2]. A *lattice cone* is a cone with vertex at the origin and edges determined by lattice points, and a lattice cone is *basic* if its edges are determined by a basis of the lattice. It is proved that any lattice cone can be subdivided into a finite number of basic lattice cones, and various concepts of subdivision and associated bounds are also considered. The case of 2- and 3-dimensional cones is considered in detail. For example, it is proved that if the lattice points determining the edges of a 3-dimensional lattice cone form a basis for a sublattice of index m , then the subdivision into basic lattice cones can always be done with at most $2m - 1$ basic cones. In the course of the proof, we determine those 3-dimensional lattices which contain points at the vertices of a tetrahedron, one of which is the origin, but contain no other point inside or on the tetrahedron.

The second part of this thesis concerns a problem of Mahler on a quadratic analogue of the classical Farey sequence. Let K_n denote the

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set of all real quadratic or rational numbers which satisfy a polynomial equation

$$ax^2 + bx + c = 0$$

where a, b, c are integers, not all zero, satisfying

$$|a| \le n \ , \quad |b| \le n \ , \quad |c| \le n \ ,$$

and order the elements of K_n according to the natural order of the real numbers. It was conjectured by Brown and Mahler [1] on the basis of numerical evidence that the determinant of the coefficients of the primitive minimal polynomials of three consecutive numbers in F_n is always $0, 1$ or -1, except in the case when the middle number is rational. Although the calculations of A. Hesterman (private communication) show that this is not so, we will show that the result holds in certain subintervals determined by consecutive points of the classical Farey sequence, and holds for certain subsets of the quadratics in any such subinterval. We also show that the modulus of the determinant cannot exceed n (when the middle number is not rational).

The third part of this thesis deals with an index problem of Cassels. Let M be a 3-dimensional lattice in R^3 which has no points on the coordinate planes, and let X be a set of non-zero lattice points, one from each pair of opposite octants in R^3, each with minimal $F(\mathrm{x}) = |x_1| + |x_2| + |x_3|$ in its octant. Let $m \ge 1$ be the ratio $F(\mathbf{b})/F(\mathbf{a})$, where $\mathbf{a}, \mathbf{b}$ are two minimal points of X with least $F(\mathrm{x})$. Our main result is that if $l(X)$ denotes the sublattice of M generated by X, then

$$\big(M : l(X)\big) \le 4 \quad \text{if} \quad m \ge 11 \ .$$

References

[1] H. Brown and K. Mahler, "A generalization of Farey sequences: some exploration via the computer", *J. Number Theory* 3 (1971), 364-370.

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[2] L. Low, "A problem of Schinzel on lattice points", *Acta Arith.* 31 (1976), 385-388.

[3] Wolfgang M. Schmidt, "A problem of Schinzel on lattice points", *Acta Arith.* 15 (1969), 199-203.