Macroeconomic Dynamics (2024), **28**, pp. 591–611
doi:10.1017/S1365100523000172

ARTICLE

The elastic origins of tail asymmetry

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Abstract
Based on a multisector general equilibrium framework, we show that the sectoral elasticity of substitution plays the key role in the evolution of asymmetric tails of macroeconomic fluctuations and the establishment of robustness against productivity shocks. A non-unitary elasticity of substitution renders a nonlinear Domar aggregation, where normal sectoral productivity shocks translate into non-normal aggregated shocks with variable expected output growth. We empirically estimate 100 sectoral elasticities of substitution, using the time-series linked input-output tables for Japan and find that the production economy is elastic overall, relative to a Cobb-Douglas economy with unitary elasticity. In addition to the previous assessment of an inelastic production economy for the USA, the contrasting tail asymmetry of the distribution of aggregated shocks between the USA and Japan is explained. Moreover, the robustness of an economy is assessed by expected output growth, the level of which is led by the sectoral elasticities of substitution under zero-mean productivity shocks.

Keywords: Productivity propagation; Structural transformation; Elasticity of substitution; Aggregate fluctuations; Robustness

JEL Classification: D57; E23; E32

1. Introduction

The subject of how microeconomic productivity shocks translate into aggregate macroeconomic fluctuations, in light of production networks, has been widely studied in the business cycle literature. Regarding production networks, the works of Long and Plosser (1983), Horvath (1998, 2000), and Dupor (1999) are concerned with input-output linkages, whereas Acemoglu et al. (2012, 2017), Acemoglu and Azar (2020) base their analysis on a multisectoral general equilibrium model under a unitary elasticity of substitution or Cobb-Douglas economy. In a Cobb-Douglas economy, Domar aggregation becomes linear with respect to sectoral productivity shocks, and because the Leontief inverse that plays the essential role in their aggregation is granular [Gabaix (2011)], some important dilation of volatility in aggregate fluctuations becomes explainable.

Moreover, peculiar aggregate fluctuations are evident from the statistical record. Figure 1 depicts the quantile-quantile (QQ) plots of the HP-detrended postwar quarterly log GDP using the Hodrick-Prescott (HP) filter against the standard normal for the USA (left) and Japan (right). These figures indicate that either the Cobb-Douglas or the normal shock assumption is questionable, since these assumptions together make the QQ plot a straight line. Acemoglu et al. (2017) explained the non-normal frequency of large economic downturns (in the USA), using non-normal (heavy-tailed) microeconomic productivity shocks. Baqaee and Farhi (2019), on the other hand, claim that a non-Cobb-Douglas economy (thus, with nonlinear Domar aggregation) can lead to such non-normality in macroeconomic fluctuations under normally distributed productivity shocks.

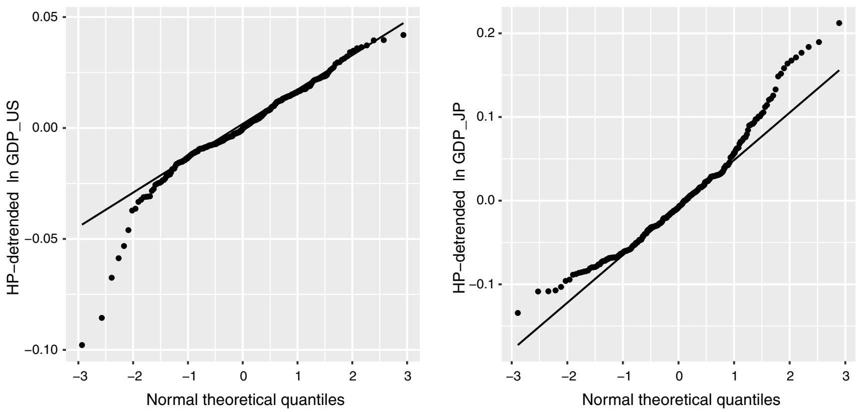

Figure 1. Quantile-quantile plots of postwar (USA:1947I–2021II; Japan:1955II–2020I) quarterly HP-detrended log GDP against the normal distribution for the USA (left) and Japan (right).
Source: Quarterly GDP data are taken from FRED (2021) and the Cabinet Office (2021).

Indeed, the asymmetric tails in the left panel of Figure 1 seem to coincide with the case in which the aggregated shocks are evaluated in a Leontief economy. We know this because a Cobb-Douglas economy with more alternative technologies can always yield a better solution (technology) than a Leontief economy with a single technology. Thus, if a Cobb-Douglas economy generates an aggregate output that corresponds to the straight line of the QQ plot, an unrobust Leontief economy that can generate less than a Cobb-Douglas economy must take the QQ plot below the straight line. This feature (of an inelastic economy) is also consistent with the analysis based on the general equilibrium model with intermediate production with a very low (almost Leontief) elasticity of substitution studied in Baqaee and Farhi (2019).

If the theory that the elasticity of substitution dictates the shape of the tails of the distribution of the aggregate macroeconomic shocks were to stand, Japan would have to have an elastic economy according to the right panel of Figure 1. Consequently, one basis of this study is to empirically evaluate the sectoral elasticity of substitution for the Japanese economy.[1] To do so, we utilize the time-series linked input-output tables, spanning 100 sectors for 22 years (1994–2015), available from the JIP (2019) database. We extract factor prices (as deflators) from the linked transaction tables available in both nominal and real values. We use the sectoral series of TFP that are also included in the database to instrument for the potentially endogenous explanatory variable (price) in our panel regression analysis. Note in advance that our sectoral average elasticity estimates ($\bar{\hat{\sigma}} = 1.54$) exceeded unity.

To ensure that our study is compatible with the production networks across sectors, we construct a multisector general equilibrium model with the estimated sector-specific CES elasticities. We assume constant returns to scale for all production so that we can work on the system of quantity-free unit cost functions to study the potential transformation of the production networks along with the propagation of productivity shocks in terms of price. Specifically, given the sectoral productivity shocks, the fixed point solution of the system of unit cost functions allows us to identify the equilibrium production network (i.e. input-output linkages) by the gradient of the mapping. By eliminating all other complications that can potentially affect the linearity of the Domar aggregation, we are able to single out the role of the substitution elasticity on the asymmetric tails of the aggregated shocks.

The remainder of this paper proceeds as follows. We present our benchmark model of a CES economy with sector-specific elasticities and then reduce the model to Leontief and Cobb-Douglas economies in Section 2. We also refer to the viability of the equilibrium structures with respect to the aforementioned economies and show that non-Cobb-Douglas economies are not necessarily prevented from exhibiting an unviable structure. In Section 3, we present our panel regression equation and estimate sectoral elasticities of substitution with respect to the consistency of the estimator. Our main results are presented in Section 4 where we show that our nonlinear (and recursive) Domar aggregators for non-Cobb-Douglas economies qualitatively replicate the asymmetric tails presented in this section. Section 5 concludes the paper. Replication data and Stata codes for this study are available at Nakano and Nishimura (2022).

2. The CES economy

2.1. Model

Below are a constant-returns-to-scale CES production function and the corresponding CES unit cost function for the jth sector (index omitted) of n sectors, with $i = 1, \cdots, n$ being an intermediate and a single primary factor of production labeled $i = 0$.

$$x = z\left(\sum_{i=0}^{n}(\alpha_i)^{\frac{1}{\sigma}}(x_i)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \qquad \pi = \frac{1}{z}\left(\sum_{i=0}^{n}\alpha_i(\pi_i)^{\gamma}\right)^{1/\gamma}$$

Here, $\sigma = 1 - \gamma$ denotes the elasticity of substitution, while α_i denotes the share parameter with $\sum_{i=0}^{n}\alpha_i = 1$. Quantities and prices are denoted by x and π, respectively. Note that the output price equals the unit cost due to the constancy of returns to scale. The Hicks-neutral productivity level of the sector is denoted by z. The duality asserts zero profit in all sectors $j = 1, \cdots, n$, that is, $\pi_j x_j = \sum_{i=0}^{n}\pi_i x_{ij}$.

By applying Shephard's lemma to the unit cost function of the jth sector, we have:

$$s_{ij} = \alpha_{ij}\left(\frac{z_j\pi_j}{\pi_i}\right)^{-\gamma_j} \tag{1}$$

where s_{ij} denotes the ith factor cost share of the jth sector. For later convenience, let us calibrate the share parameter at the benchmark where price and productivity are standardized, that is, $\pi_0 = \pi_1 = \cdots = \pi_n = 1$ and $z_1 = \cdots = z_n = 1$. Since we know the benchmark cost-share structure from the input-output coefficients of the benchmark period a_{ij}, the benchmark-calibrated share parameter must therefore be $\alpha_{ij} = a_{ij}$. Taking this into account, the equilibrium price $\boldsymbol{\pi} = (\pi_1, \cdots, \pi_n)$ given $\boldsymbol{z} = (z_1, \cdots, z_n)$ must be the solution to the following system of n equations:

$$\begin{aligned}
\pi_1 &= (z_1)^{-1}\left(a_{01}(\pi_0)^{\gamma_1} + a_{11}(\pi_1)^{\gamma_1} + \cdots + a_{n1}(\pi_n)^{\gamma_1}\right)^{1/\gamma_1}\\
\pi_2 &= (z_2)^{-1}\left(a_{02}(\pi_0)^{\gamma_2} + a_{12}(\pi_1)^{\gamma_2} + \cdots + a_{n2}(\pi_n)^{\gamma_2}\right)^{1/\gamma_2}\\
&\vdots\\
\pi_n &= (z_n)^{-1}\left(a_{0n}(\pi_0)^{\gamma_n} + a_{1n}(\pi_1)^{\gamma_n} + \cdots + a_{nn}(\pi_n)^{\gamma_n}\right)^{1/\gamma_n}
\end{aligned}$$

where we can set the price of the primary factor π_0 as the numéraire.

Note that this system of equations is equivalent to Miranda-Pinto and Young (2022)'s specification of equilibrium price for a CES economy with sectoral distortions (Proposition 3), which reads as follows:

$$P^{1-\epsilon_Q} = a\cdot(Z\cdot\vartheta^{w})^{\cdot(\epsilon_Q-1)} + \left((1-a)\cdot(Z\cdot\vartheta^{m})^{\cdot(\epsilon_Q-1)}1'\right)\cdot\left(\Omega\cdot(P1')^{\cdot((1-\epsilon_Q)1')'}\right)'1$$

https://doi.org/10.1017/S1365100523000172 Published online by Cambridge University Press

The correspondence of symbols are $P = (\pi_1, \pi_2, \cdots, \pi_n)$, $(1 - \epsilon_Q) = (\gamma_1, \gamma_2, \cdots, \gamma_n)$, $Z = (z_1, z_2, \cdots, z_n)$, $a = (a_{01}, a_{02}, \cdots, a_{0n})$, and

$$(1 - a) \cdot \Omega' = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}.$$

The only difference is that the sectoral wedges $(\vartheta^m, \vartheta^w)$ are all equal to 1 in our model, since the firms are assumed to be unconstrained, that is, that there is no binding constraint other than the budget. Sectoral production choices are, therefore, undistorted in our model.[2]

For later convenience, we write the above system in a more concise form as follows:

$$\boldsymbol{\pi} = \langle \boldsymbol{z} \rangle^{-1} \boldsymbol{c} (\boldsymbol{\pi}; \pi_0) \tag{2}$$

Here, angled brackets indicate the diagonalization of a vector. Note that $\boldsymbol{c}$:$\mathbb{R}_+^n \to \mathbb{R}_+^n$ is strictly concave and $\boldsymbol{z} \in \mathbb{R}_+^n$. Consider a mapping $\mathcal{E}$ that nests the equilibrium solution (fixed point) $\boldsymbol{\pi}$ of equation (2) and maps the (exogenous) productivity $\boldsymbol{z}$ onto the equilibrium price $\boldsymbol{\pi}$, that is,

$$\boldsymbol{\pi} = \mathcal{E}(\boldsymbol{z}; \pi_0) \tag{3}$$

There is no closed-form solution to equation (3). However, one can be found for the case of uniform elasticity, that is, $\gamma_1 = \cdots = \gamma_n = \gamma$, which is as follows:

$$\boldsymbol{\pi} = \pi_0 \left(\boldsymbol{a}_0 \left[\langle \boldsymbol{z} \rangle^{\gamma} - \mathbf{A} \right]^{-1} \right)^{1/\gamma} \qquad \text{Uniform CES economy} \tag{4}$$

where the n row vector $\boldsymbol{a}_0 = (a_{01}, \cdots, a_{0n})$ is called the primary factor coefficient vector and the $n \times n$ matrix $\mathbf{A} = \{a_{ij}\}$ is called the input-output coefficient matrix. The case of a Leontief economy, where $1 - \gamma = 0$, equation (4), can be reduced straightforwardly as follows:

$$\boldsymbol{\pi} = \pi_0 \boldsymbol{a}_0 \left[\langle \boldsymbol{z} \rangle - \mathbf{A} \right]^{-1} \qquad \text{Leontief economy} \tag{5}$$

For the case of a Cobb-Douglas economy, where $\gamma = 0$, we first take the log and let $\gamma \to 0$ where L'Hôspital's rule is applicable since $\sum_{i=0}^{n} a_{ij} = 1$ for $j = 1, \cdots, n$.

$$\ln \pi_j z_j = \frac{\ln \left(\sum_{i=0}^{n} a_{ij} (\pi_i)^{\gamma} \right)}{\gamma} \to \frac{\sum_{i=0}^{n} a_{ij} (\pi_i)^{\gamma} \ln \pi_i}{\sum_{i=0}^{n} a_{ij} (\pi_i)^{\gamma}} \to \sum_{i=0}^{n} a_{ij} \ln \pi_i$$

Thus, equation (4) can be reduced in the following manner:

$$\ln \boldsymbol{\pi} = (\boldsymbol{a}_0 \ln \pi_0 - \ln \boldsymbol{z}) \left[\mathbf{I} - \mathbf{A} \right]^{-1} \qquad \text{Cobb-Douglas economy} \tag{6}$$

It is notable that the growth of the equilibrium price $\text{dln}\, \boldsymbol{\pi} = (\text{dln}\, \pi_1, \cdots, \text{dln}\, \pi_n)$ is a linear combination of the growth of sectoral productivity $\text{dln}\, \boldsymbol{z} = (\text{dln}\, z_1, \cdots, \text{dln}\, z_n)$ in the case of a Cobb-Douglas economy.

Otherwise, the fixed point $\boldsymbol{\pi}$ given $\boldsymbol{z}$ can be searched for by using the simple recursive method applied to equation (2). Since the unit cost function $\pi_j = (z_j)^{-1} c_j(\boldsymbol{\pi}; \pi_0)$ is monotonically increasing and strictly concave in $\boldsymbol{\pi}$, we know by Krasnosel'skiˇı (1964) and Kennan (2001) that equation (2) is a contraction mapping that globally converges onto a unique fixed point, if it exists in $\mathbb{R}_+^n$. Note that if $\pi_0 = 1$ and $(z_1, \cdots, z_n) = (1, \cdots, 1)$, then $(\pi_1, \cdots, \pi_n) = (1, \cdots, 1)$ is an equilibrium, which must be unique. Moreover, note that obviously from equation (6), the existence of a

https://doi.org/10.1017/S1365100523000172 Published online by Cambridge University Press

positive fixed point $\boldsymbol{\pi} \in \mathbb{R}^n_+$ for any given $\boldsymbol{z} \in \mathbb{R}^n_+$ can be asserted for the case of a Cobb-Douglas economy. Specifically, it is possible to show from equation (6) that:

$$(\pi_1, \cdots, \pi_n) = \left(\prod_{i=1}^{n} \frac{e^{a_{01}\ell_{i1} \ln \pi_0}}{(z_i)^{\ell_{i1}}}, \cdots, \prod_{i=1}^{n} \frac{e^{a_{0n}\ell_{in} \ln \pi_0}}{(z_i)^{\ell_{in}}} \right) > (0, \cdots, 0)$$

where ℓ_{ij} denotes the *ij* element of the Leontief inverse $[\mathbf{I} - \mathbf{A}]^{-1}$. Conversely, $\boldsymbol{\pi}$ can have negative elements or may not even exist in $\mathbb{R}^n$ for non-Cobb-Douglas economies. One may see this by replacing $\boldsymbol{z}$ with small (but positive) elements in equations (4) and (5).

2.2. Viability of the equilibrium structure

From another perspective, $c_j(\boldsymbol{\pi}; \pi_0)$ is homogeneous of degree one in $(\pi_0, \cdots, \pi_n)$, so by Euler's homogeneous function theorem, it follows that:

$$\pi_j = \sum_{i=1}^{n} (z_j)^{-1} \frac{\partial c_j}{\partial \pi_i} \pi_i + (z_j)^{-1} \frac{\partial c_j}{\partial \pi_0} \pi_0 = \sum_{i=1}^{n} b_{ij}\pi_i + b_{0j}\pi_0$$

Here, by Shephard's lemma, b_{ij} denotes the equilibrium physical input-output coefficient. In matrix form, this is equivalent to:

$$\boldsymbol{\pi} = \boldsymbol{\pi} \nabla \boldsymbol{c} \langle \boldsymbol{z} \rangle^{-1} + \pi_0 \nabla_0 \boldsymbol{c} \langle \boldsymbol{z} \rangle^{-1} = \boldsymbol{\pi} \mathbf{B} + \pi_0 \boldsymbol{b}_0$$

Let us hereafter call $(\mathbf{B}, \boldsymbol{b}_0)$ the equilibrium structure (of an economy). Note that if $\boldsymbol{\pi}$ exists in $\mathbb{R}^n_+$, while $\boldsymbol{b}_0 \in \mathbb{R}^n_+$ (i.e. all sectors, upon production, physically utilize the primary factor), then $[\mathbf{I} - \mathbf{B}]$, where $\boldsymbol{\pi}[\mathbf{I} - \mathbf{B}] = \pi_0 \boldsymbol{b}_0$, is said to satisfy the Hawkins-Simon (HS) condition [Theorem 4.D.4, Takayama (1985); Hawkins and Simon (1949)].

The existence of a solution $\boldsymbol{y} \in \mathbb{R}^n_+$ for $[\mathbf{I} - \mathbf{B}]\boldsymbol{y} = \boldsymbol{d}$ given any $\boldsymbol{d} \in \mathbb{R}^n_+$ and the matrix $[\mathbf{I} - \mathbf{B}]$ satisfying the HS condition are two equivalent statements [Theorem 4.D.1, Takayama (1985)]. Thus, a structure that $[\mathbf{I} - \mathbf{B}]$ satisfies the HS condition is said to be *viable*. Conversely, for an *unviable* structure (that $[\mathbf{I} - \mathbf{B}]$ does not satisfy the HS condition), no positive production schedule $\boldsymbol{y} \in \mathbb{R}^n_+$ can be possible for fulfilling any positive final demand $\boldsymbol{d} \in \mathbb{R}^n_+$. For a Cobb-Douglas economy, we can assert that the equilibrium structure is always viable, since we know from equation (6) that it is always the case that $\boldsymbol{\pi} \in \mathbb{R}^n_+$. Otherwise, $\boldsymbol{\pi}$ may have negative elements, in which case the equilibrium structure must be unviable. An unviable equilibrium structure may never appear during the recursive process, however, if the equilibrium price search is such that it is installed in the recursive process of equation (2); instead, the recursive process will not be convergent since equation (2) maps into an open set $\mathbb{R}^n_+$.

Last, let us specify below the structural transformation (as the physical input-output coefficient matrix $\mathbf{B}$) and network transformation (as the cost-share structure or the monetary input-output coefficient matrix $\mathbf{S}$) given $\boldsymbol{z}$ in a uniform CES economy. Since an element of the gradient of the CES aggregator is:

$$\frac{\partial c_j}{\partial \pi_i} = a_{ij}(z_j)^{1-\gamma_j} \left(\frac{\pi_j}{\pi_i} \right)^{1-\gamma_j}$$

the gradient of the uniform CES aggregator can be written as follows:

$$\nabla \boldsymbol{c} = \langle \boldsymbol{\pi} \rangle^{\gamma-1} \mathbf{A} \langle \boldsymbol{\pi} \rangle^{1-\gamma} \langle \boldsymbol{z} \rangle^{1-\gamma}$$

Thus, below are the transformed structure and networks, where $\boldsymbol{\pi}$ is given by equation (4):

$$\mathbf{B} = \langle \boldsymbol{\pi} \rangle^{\gamma-1} \mathbf{A} \langle \boldsymbol{\pi} \rangle^{1-\gamma} \langle \boldsymbol{z} \rangle^{-\gamma}, \qquad \mathbf{S} = \langle \boldsymbol{\pi} \rangle^{\gamma} \mathbf{A} \langle \boldsymbol{\pi} \rangle^{-\gamma} \langle \boldsymbol{z} \rangle^{-\gamma}$$

Observe that $\mathbf{S} = \mathbf{A}$ in a Cobb-Douglas economy ($\gamma = 0$) and $\mathbf{B} = \mathbf{A}\langle 1/\boldsymbol{z} \rangle$ in a Leontief economy ($\gamma = 1$).

3. Estimation

Let us start by taking the log of equation (1) and indexing observations by $t = 1, \cdots, T$, while omitting the sectoral index (j). The cross-sectional dimension remains, that is, $i = 0, \cdots, n$. Here, we substitute p for π to emphasize that they are observed data and ζ for z to emphasize that they are parameters subject to estimation. For the response variable, we use the factor share a_{it} available as the input-output coefficient.

$$\ln a_{it} = \ln \alpha_i - \gamma \ln (\zeta_t p_t) + \gamma \ln p_{it} + \epsilon_{it} \tag{7}$$

Note that the error terms ϵ_{it} are assumed to be iid normally distributed with mean zero. The multifactor CES elasticity in which we are interested has been extensively studied in the Armington elasticity literature. Erkel-Rousse and Mirza (2002) and Saito (2004) apply between estimation, a typical strategy for the two-input case, to estimate the elasticity of substitution between products from different countries. Between estimation eliminates time-specific effects while saving the individual-specific effects such as the share parameter α_i. For a two-factor case, the share parameter is usually subject to estimation. However, for a multifactor case, the constraint that $\sum_{i=0}^{n} \alpha_i = 1$ can hardly be met. Moreover, we know in advance that $\alpha_i = a_i$ for the year that the model is standardized. Hence, we opt to apply within (FE) estimation in this study.

Below we restate equation (7) using time dummy variables such that γ and $\zeta_t p_t$ can be estimated from p_{it} and a_{it} via FE panel regression:

$$Y_{it} = \mu_1 + (\mu_2 - \mu_1) D_2 + \cdots + (\mu_T - \mu_1) D_T + \gamma X_{it} + \ln \alpha_i + \epsilon_{it} \tag{8}$$

where $Y_{it} = \ln a_{it}$, $X_{it} = \ln p_{it}$ and D_k for $k = 2, \cdots, T$ denotes a dummy variable that equals 1 if $k = t$ and 0 otherwise. For $t = 1$, $D_2 = \cdots = D_T = 0$ by definition, so we know that $\mu_t = -\gamma \ln (\zeta_t p_t)$ for $t = 1, \cdots, T$. The estimable coefficients for equation (8) via FE, therefore, indicate that:

$$\mu_t - \mu_1 = -\gamma \left(\ln \zeta_t p_t - \ln \zeta_1 p_1 \right) \qquad t = 2, \cdots, T$$

We may thus evaluate the productivity growth at t, based on $t = 1$, by the following formula:

$$\ln \zeta_t / \zeta_1 = -(\mu_t - \mu_1)/\gamma - \ln p_t / p_1$$

We face the concern that regression (7) suffers from an endogeneity problem. The response variable, that is, the demand for the ith factor of production by the jth sector, may well affect the price of the ith factor via the supply function. Because of such reverse causality, the explanatory variable, that is, the price of the ith commodity, becomes correlated with the error term that corresponds to the demand shock for the ith factor of production by the jth sector. To remedy this problem, we apply total factor productivity (TFP) to instrument prices. The JIP (2019) database provides sectoral TFP growth (in terms of the Törnqvist index) as well as the aggregated macro-TFP growth, for each year interval. It is generally assumed that TFP is unlikely to be correlated with the demand shock [Eslava et al. (2004); Foster et al. (2008)]. In our case, the ith sector's TFP to produce the ith commodity is unlikely to be correlated with the jth sector's demand shock for the ith commodity. Hence, TFP appears to be suitable as an instrument for our explanatory variable.

On the other hand, the price of the primary factor $i = 0$ can be nonresponsive to sectoral demand shocks. The primary factor consists of labor and capital services, while their prices, that is, wages and interest rates, are not purely dependent on the market mechanism but rather subject to government regulations and natural depreciations. Moreover, it is conceivable that the demand shock for the primary factor by one sector has little influence on the prices of its factors, labor, and capital, if not on their quantitative ratios demanded by the sector. Thus, we apply three exogenous variables as instruments for X_{0t}, namely (1) the macro-TFP, (2) the macro wage rate, and (3) the macro interest rate, which are available in time series in the JIP (2019) database. Specifically, we will be examining three instrumental variables in the FE IV regression of equation

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(8), namely, v_{it}^a, v_{it}^b, and v_{it}^c, all of which include the sectoral TFP at t, for $i = 1, \cdots, n$, and where v_{0t}^a = macro TFP at t, v_{0t}^b = macro wage rate at t, and v_{0t}^c = macro interest rate at t.

The results are summarized in Table 1. The first column (LS FE) reports the least squares fixed effects estimation results, without instrumenting for the explanatory variable. The second column (IV FE) reports the instrumental variable fixed effects estimation results, using the IVs reported in the last column. In all cases, overidentification tests are not rejected, so we are satisfied with the IVs we applied. Furthermore, first-stage F values are large enough that we are satisfied with the strength of the IVs we applied. Interestingly, the estimates for the elasticity of substitution $\hat{\sigma} = 1 - \hat{\gamma}$ are larger when IVs are applied. For later study of the aggregate fluctuations, we select from the elasticity estimates based on the endogeneity test results. Specifically, we use the LS FE estimates for sector ids 6, 12, 27, 52, 62, 70, 71, 81, 88, and, hence, IV FE estimates for the rest of the sectors. Finally, we note that simple mean of the estimated (accepted) elasticity of substitution is $\bar{\hat{\sigma}} = 1.54$.[3]

4. Aggregate fluctuations

4.1. Representative household

Let us now consider a representative household that maximizes the following CES utility:

$$H(\boldsymbol{h}) = \left((\mu_1)^{\frac{1}{1-\kappa}} (h_1)^{\frac{\kappa}{1-\kappa}} + \cdots + (\mu_n)^{\frac{1}{1-\kappa}} (h_n)^{\frac{\kappa}{1-\kappa}} \right)^{\frac{1-\kappa}{\kappa}}$$

The household determines the consumption schedule $\boldsymbol{h} = (h_1, \cdots, h_n)^\intercal$ given the budget constraint $W = \sum_{i=1}^n \pi_i h_i$ and prices of all goods $\boldsymbol{\pi} = (\pi_1, \cdots, \pi_n)$. The source of the budget is the renumeration for the household's supply of the primary factor to the production sectors, so we know that $W = \sum_{j=1}^n v_j$ (total value added, or GDP of the economy) is the representative household's (or national) income. The indirect utility of the household can then be specified as follows:

$$H(\boldsymbol{h}(\boldsymbol{\pi}, W)) = \frac{W}{(\mu_1(\pi_1)^\kappa + \cdots + \mu_n(\pi_n)^\kappa)^{1/\kappa}} = \frac{W}{\Pi(\boldsymbol{\pi})}$$

where Π, as defined as above, denotes the representative household's CES price index. Note that $H = W$ at the baseline $(z_1, \cdots, z_n) = (1, \cdots, 1)$ where $\Pi = 1$. Thus, H is the utility (in terms of money) that the representative household can obtain from its income W given the price change $\boldsymbol{\pi}$ (due to the productivity shock $\boldsymbol{z}$) while holding the primary input's price constant at $\pi_0 = 1$. In other words, H is the real GDP if W is the nominal GDP.

From another perspective, we note that the household's income W can also be affected by the productivity shock. When there is a productivity gain in a production process, this process can either increase its output while holding all its inputs fixed or reduce the inputs while holding the output fixed. In the former case, the national income W (nominal GDP) remains at the baseline level, which equals real GDP in the previous year $\overline{H}$, and GDP growth ($\Delta \ln H = \ln H - \ln \overline{H} = -\ln \Pi(\boldsymbol{\pi})$) is fully accounted for. In the latter case, however, the national income can be reduced by as much as $W = \overline{H}\Pi(\boldsymbol{\pi})$, in which case we have $\Delta \ln H = 0$ that is, no GDP growth will be accounted for.

Of course, the reality must be in between the two extreme cases. In this study, we conservatively evaluate national income (as nominal GDP) to the following extent:

$$W = \overline{H} \left(\mu_1(1/z_1)^\kappa + \cdots + \mu_n(1/z_n)^\kappa \right)^{1/\kappa} = \overline{H}\Pi(1/\boldsymbol{z})$$

The real GDP under the equilibrium price, which equals the household's expenditure, can then be evaluated as follows:

$$\ln H = \ln W - \ln \Pi(\boldsymbol{\pi}) = \ln \overline{H} - \ln \Pi(\boldsymbol{\pi}) + \ln \Pi(1/\boldsymbol{z}) \quad (9)$$

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Table 1. Estimation of the elasticity of substitution for all 100 sectors

	LS FE		IV FE					
id	$\hat{\sigma}$	s.e.	$\hat{\sigma}$	s.e.	1st F*1	Overid.*2	Endog.*3	IVs*4
1	1.119	0.082	3.540	0.317	106	0.11 (0.735)	92.07 (0.000)	la, c
2	1.422	0.093	2.739	0.273	145	0.44 (0.507)	29.21 (0.000)	a, c
3	0.994	0.065	2.850	0.262	94	0.79 (0.373)	76.80 (0.000)	lc, c
4	1.178	0.065	1.947	0.154	228	1.42 (0.234)	32.66 (0.000)	lb, c
5	0.782	0.054	2.104	0.150	197	0.99 (0.319)	121.19 (0.000)	la, c
6	1.167	0.071	1.288	0.202	139	0.72 (0.397)	0.41 (0.523)	la, c
7	0.936	0.062	1.698	0.192	124	0.18 (0.668)	19.12 (0.000)	la, c
8	0.724	0.062	1.118	0.137	260	0.02 (0.884)	10.63 (0.001)	la, c
9	0.553	0.070	1.384	0.183	187	0.15 (0.696)	26.18 (0.000)	la, c
10	0.962	0.076	1.519	0.275	86	0.10 (0.752)	4.56 (0.033)	a, c
11	0.671	0.076	2.145	0.261	111	0.10 (0.752)	42.06 (0.000)	la, c
12	1.052	0.093	1.312	0.264	139	0.10 (0.754)	1.11 (0.292)	la, c
13	0.389	0.064	1.190	0.177	163	0.87 (0.351)	25.61 (0.000)	la, c
14	0.805	0.048	1.241	0.115	217	1.45 (0.229)	18.33 (0.000)	a, c
15	0.608	0.048	1.402	0.114	250	0.65 (0.421)	68.15 (0.000)	la, c
16	0.153	0.051	0.898	0.116	179	3.60 (0.166)	58.22 (0.000)	la, lc
17	0.840	0.059	2.723	0.184	183	1.84 (0.174)	186.29 (0.000)	la, lc
18	0.723	0.047	1.515	0.133	160	0.69 (0.407)	46.70 (0.000)	la, lc
19	0.479	0.049	2.605	0.233	94	1.55 (0.214)	176.49 (0.000)	la, c
20	0.880	0.047	1.888	0.136	176	0.00 (0.998)	79.31 (0.000)	la, c
21	0.637	0.047	0.978	0.098	303	0.93 (0.335)	16.09 (0.000)	la, c
22	0.533	0.049	0.781	0.102	307	0.00 (0.997)	7.90 (0.005)	la, c
23	−0.111	0.047	1.946	0.243	78	1.49 (0.223)	150.55 (0.000)	la, c
24	0.502	0.048	1.765	0.211	74	0.16 (0.687)	51.81 (0.000)	la, c
25	0.605	0.038	0.908	0.069	455	1.73 (0.188)	28.83 (0.000)	la, c
26	0.855	0.046	1.644	0.105	285	1.83 (0.176)	82.84 (0.000)	la, c
27	0.158	0.050	0.200	0.104	302	0.04 (0.844)	0.21 (0.643)	la, c
28	0.585	0.042	1.446	0.105	241	0.13 (0.716)	101.26 (0.000)	la, c
29	0.624	0.054	3.700	0.645	19	0.02 (0.901)	60.56 (0.000)	b, lc
30	0.510	0.052	1.295	0.192	88	0.00 (0.961)	20.24 (0.000)	b, lc
31	0.641	0.044	2.076	0.185	94	0.33 (0.565)	101.52 (0.000)	b, lc
32	0.890	0.041	1.209	0.106	185	1.96 (0.162)	10.96 (0.001)	b, lc
33	0.414	0.047	1.215	0.140	148	0.03 (0.854)	43.16 (0.000)	b, a
34	0.325	0.044	0.978	0.102	260	0.66 (0.417)	57.24 (0.000)	b, a
35	0.500	0.046	1.095	0.115	203	0.92 (0.337)	34.54 (0.000)	b, a
36	0.615	0.047	1.054	0.113	223	0.06 (0.808)	19.04 (0.000)	b, a
37	0.434	0.046	0.787	0.084	450	0.96 (0.327)	26.22 (0.000)	b, la
38	0.410	0.049	0.907	0.106	286	1.05 (0.306)	29.97 (0.000)	lc, c
39	0.656	0.047	1.480	0.108	272	0.12 (0.728)	84.98 (0.000)	a, c
40	0.410	0.045	0.876	0.087	382	0.65 (0.421)	41.80 (0.000)	a, c
41	0.490	0.041	0.975	0.066	698	1.13 (0.289)	98.99 (0.000)	a, c
42	0.460	0.045	1.022	0.099	285	1.48 (0.224)	44.20 (0.000)	a, c

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Table 1. Continued

	LS FE		IV FE					
id	$\hat{\sigma}$	s.e.	$\hat{\sigma}$	s.e.	1st F*[1]	Overid.*[2]	Endog.*[3]	IVs*[4]
43	0.746	0.039	1.400	0.085	318	0.02 (0.876)	89.63 (0.000)	la, c
44	0.199	0.045	0.689	0.094	318	0.23 (0.634)	37.50 (0.000)	a, c
45	0.777	0.043	1.482	0.081	473	0.14 (0.709)	124.74 (0.000)	la, c
46	0.725	0.044	1.192	0.083	414	0.03 (0.859)	47.79 (0.000)	la, c
47	0.438	0.044	1.050	0.084	432	0.54 (0.464)	83.15 (0.000)	a, c
48	0.199	0.042	0.938	0.090	343	0.00 (0.947)	103.70 (0.000)	a, c
49	0.698	0.045	1.182	0.111	205	0.48 (0.489)	24.00 (0.000)	a, c
50	0.467	0.048	0.807	0.109	244	1.24 (0.266)	12.39 (0.000)	a, c
51	0.471	0.039	1.057	0.092	252	0.10 (0.758)	56.61 (0.000)	a, c
52	0.574	0.061	0.682	0.142	223	0.65 (0.420)	0.71 (0.399)	a, c
53	0.786	0.062	1.685	0.154	213	0.04 (0.848)	45.58 (0.000)	a, c
54	0.797	0.051	1.535	0.139	175	0.05 (0.831)	36.42 (0.000)	a, c
55	0.794	0.048	1.410	0.135	158	0.25 (0.619)	25.89 (0.000)	a, c
56	0.749	0.052	1.126	0.152	137	0.18 (0.675)	7.17 (0.007)	a, c
57	0.829	0.066	1.628	0.172	185	0.03 (0.870)	27.69 (0.000)	a, c
58	0.122	0.054	0.338	0.114	281	0.66 (0.416)	4.60 (0.032)	a, c
59	0.319	0.041	0.704	0.087	300	1.54 (0.215)	26.23 (0.000)	a, c
60	0.134	0.057	2.522	0.246	111	2.67 (0.102)	196.47 (0.000)	la, c
61	0.160	0.047	2.548	0.218	118	0.30 (0.586)	308.57 (0.000)	la, c
62	1.114	0.070	1.011	0.218	113	0.13 (0.723)	0.25 (0.618)	a, c
63	1.222	0.067	3.412	0.249	120	0.13 (0.722)	135.59 (0.000)	lc, a
64	1.028	0.111	2.768	0.476	65	1.80 (0.180)	15.97 (0.000)	lc, a
65	0.863	0.053	1.696	0.130	233	0.21 (0.651)	57.10 (0.000)	a, c
66	1.004	0.050	0.758	0.121	210	0.87 (0.350)	5.06 (0.024)	lc, a
67	1.206	0.054	1.591	0.161	129	0.96 (0.326)	6.61 (0.010)	la, c
68	1.112	0.062	1.477	0.123	345	0.94 (0.334)	12.11 (0.001)	lc, a
69	1.164	0.078	2.641	0.160	383	0.86 (0.353)	137.82 (0.000)	lb, a
70	0.920	0.047	1.012	0.101	275	0.48 (0.488)	1.07 (0.302)	a, c
71	0.960	0.051	0.979	0.106	296	0.69 (0.407)	0.04 (0.838)	a, c
72	1.048	0.050	1.864	0.124	223	0.14 (0.713)	60.48 (0.000)	a, c
73	1.513	0.071	3.985	0.261	135	0.10 (0.758)	162.65 (0.000)	lc, c
74	0.890	0.061	1.556	0.145	231	1.03 (0.311)	27.46 (0.000)	la, c
75	0.703	0.060	1.375	0.134	260	0.99 (0.319)	33.47 (0.000)	lc, c
76	0.887	0.059	1.644	0.133	265	0.55 (0.460)	44.19 (0.000)	lb, c
77	0.904	0.049	1.448	0.111	257	0.97 (0.326)	32.11 (0.000)	lb, c
78	1.152	0.077	1.514	0.154	340	0.95 (0.331)	7.52 (0.006)	lb, c
79	0.647	0.058	0.926	0.133	239	0.27 (0.603)	5.49 (0.019)	b, c
80	0.617	0.056	1.223	0.125	270	0.14 (0.712)	31.68 (0.000)	b, c
81	0.601	0.052	0.649	0.110	285	0.30 (0.586)	0.24 (0.625)	lb, a
82	0.702	0.058	1.337	0.129	272	0.00 (0.995)	32.48 (0.000)	b, c
83	1.057	0.061	2.014	0.143	255	0.09 (0.763)	62.94 (0.000)	b, c

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Table 1. Continued

id	LS FE $\hat{\sigma}$	LS FE s.e.	IV FE $\hat{\sigma}$	IV FE s.e.	1st F*1	Overid.*2	Endog.*3	IVs*4
84	1.149	0.153	2.067	0.422	147	0.04 (0.836)	5.57 (0.018)	lb, a
85	1.147	0.059	1.989	0.130	285	0.73 (0.393)	59.29 (0.000)	lc, c
86	0.864	0.063	1.618	0.137	290	1.06 (0.304)	41.69 (0.000)	lb, a
87	1.136	0.073	1.891	0.167	249	1.04 (0.309)	26.77 (0.000)	la, c
88	0.667	0.049	0.755	0.095	365	0.70 (0.404)	1.15 (0.283)	a, c
89	0.738	0.056	1.578	0.126	282	1.18 (0.277)	63.45 (0.000)	a, c
90	0.990	0.050	1.719	0.138	170	1.02 (0.314)	36.10 (0.000)	a, c
91	0.831	0.056	1.578	0.129	264	0.00 (0.999)	46.26 (0.000)	a, c
92	1.241	0.054	2.084	0.134	227	0.56 (0.456)	54.39 (0.000)	la, c
93	1.073	0.076	1.616	0.192	191	0.17 (0.676)	9.78 (0.002)	a, c
94	0.970	0.053	1.495	0.111	311	0.28 (0.596)	30.94 (0.000)	la, c
95	0.253	0.076	1.757	0.223	122	0.06 (0.804)	68.04 (0.000)	la, c
96	0.571	0.054	1.542	0.122	290	0.38 (0.536)	94.91 (0.000)	la, c
97	0.887	0.054	1.485	0.124	247	0.07 (0.799)	30.79 (0.000)	la, c
98	0.559	0.067	1.160	0.159	228	2.61 (0.106)	18.15 (0.000)	la, c
99	0.771	0.063	1.479	0.144	253	0.03 (0.865)	32.33 (0.000)	la, c
100	0.289	0.087	2.412	0.290	132	0.25 (0.616)	78.53 (0.000)	c, lc
101	0.997	0.043	1.198	0.091	389	0.56 (0.456)	21.20 (0.000)	a, a

Notes: For sector classifications (ids), see Table 2. The household sector is id = 101. Values in parentheses indicate p-values. *1 First-stage (Cragg-Donald Wald) F statistic for 2SLS FE estimation. The rule of thumb to reject the hypothesis that the explanatory variable is only weakly correlated with the instrument is for this to exceed 10.
*2 Overidentification test by Sargan statistic. Rejection of the null indicates that the instruments are correlated with the residuals.
*3 Endogeneity test by Davidson-MacKinnon F statistic. Rejection of the null indicates that the instrumental variables fixed effects estimator should be employed.
*4 Instrumental variables applied, where a, b, and c, indicate v^a, v^b, and v^c, respectively, and la, lb, and lc, indicate $\ln v^a$, $\ln v^b$, and $\ln v^c$, respectively.

If we assume Cobb-Douglas utility ($\kappa \to 0$) and normalize the initial real GDP ($\overline{H} = 1$), we have the following exposition:

$$\ln H = -\ln \Pi(\boldsymbol{\pi}) + \Pi(1/\boldsymbol{z}) = -(\ln \boldsymbol{\pi} + \ln \boldsymbol{z})\, \boldsymbol{\mu}$$

where $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_n)^\intercal$ denotes the column vector of expenditure share parameters. The first identity indicates that the real GDP growth is the negative price index growth of the economy less the negative price index growth of a simple economy.[4] Moreover, if we assume a Cobb-Douglas economy ($\gamma_j \to 0$), we arrive at the following:

$$\ln H = -(\ln \boldsymbol{z}) \left([\mathbf{I} - \mathbf{A}]^{-1} + \mathbf{I} \right) \boldsymbol{\mu} = \sum_{j=1}^{n} \lambda_j \ln z_j$$

Note that λ_j is the Domar weight [Hulten (1978)] in this particular case.

The parameters of the utility function are also subject to estimation. By applying Roy's identity, that is, $h_i = -\frac{\partial H}{\partial p_i} / \frac{\partial H}{\partial W}$, we have the following expansion for the household's expenditure share on the ith commodity:

$$s_i = \frac{\pi_i h_i}{\sum_{i=1}^{n} \pi_i h_i} = \frac{\mu_i (\pi_i)^\kappa}{\sum_{i=1}^{n} \mu_i (\pi_i)^\kappa} = \mu_i \left(\frac{\Pi}{\pi_i} \right)^{-\kappa} \tag{10}$$

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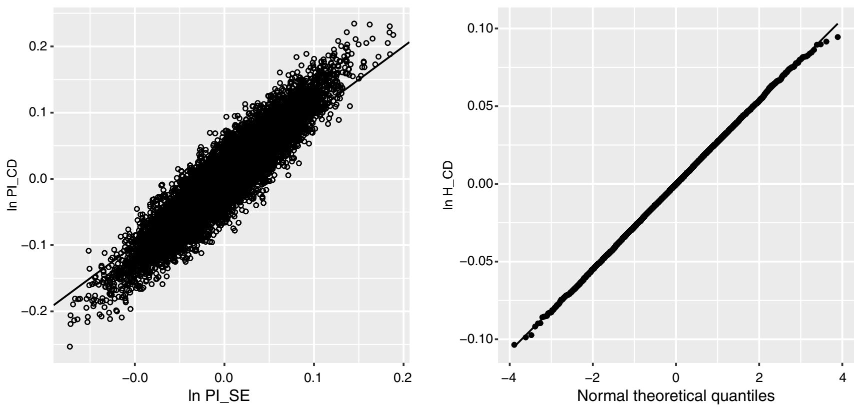

Figure 2. Left: Price index dispersion in Cobb-Douglas and Simple economies under Cobb-Douglas utility. Dispersion is larger in the Cobb-Douglas than in the Simple economy, due to the power-law granularity of the Leontief inverse. Right: QQ plot of the aggregate output fluctuations generated by the linear Domar aggregator for the Cobb-Douglas economy and utility.

where s_i denotes the expenditure share of the ith commodity for the representative household. By taking the log of equation (10) and indexing observations by $t = 1, \cdots, T$, we obtain the following regression equation where the parameter κ can be estimated via FE.

$$\ln m_{it} = \ln \mu_i - \kappa \ln (\Pi_t) + \kappa \ln p_{it} + \delta_{it} \tag{11}$$

As is typical, the error term δ_{it} is assumed to be iid normally distributed with mean zero. Here, we replace π with p to emphasize that they are observed data. For the response variable, we use the expenditure share m_{it} of the final demand available from the input-output tables. The cross-sectional dimension of the data for regression equation (11) is $i = 1, \cdots, n$, whereas it is $i = 0, 1, \cdots, n$ for equation (7). Thus, we apply sectoral TFP available for $t = 1, \cdots, T$ from the JIP (2019) database as instruments to fix the endogeneity of the explanatory variable. The estimation result for κ using time dummy variables as in equation (8) (such that we may retrieve the estimates for Π_t) is presented in Table 1 (id = 101).

4.2. Tail asymmetry and robustness

For a quantitative illustration, we study the distribution of aggregate output $\ln H$ when sectoral shocks $\ln \boldsymbol{z}$ are drawn from a normal distribution. Specifically, we use 10,000 $\ln z_j \sim \mathcal{N}(0, 0.2)$ iid samples for $j = 1, \cdots, n$, where the standard deviation (i.e. annual volatility of 20%) is chosen with reference to the annual volatility of the estimated sectoral productivity growth $\ln \zeta_j$ (see Appendix A). Let us first examine the granularity of our baseline production networks (i.e. 2011 input-output linkages). Below are both Cobb-Douglas price indices in terms of productivity shocks $\ln \boldsymbol{z}$ for the Cobb-Douglas and simple economies:

$$\ln \Pi_{\mathrm{CD}} = -(\ln \boldsymbol{z}) \left[\mathbf{I} - \mathbf{A}\right]^{-1} \boldsymbol{m}, \qquad \ln \Pi_{\mathrm{SE}} = -\left(\ln \boldsymbol{z}\right) \boldsymbol{m}$$

Here, we set the share parameter $\boldsymbol{\mu}$ at the standard expenditure share of the final demand $\boldsymbol{m} = (m_1, \cdots, m_n)^\intercal$ for the year 2011. Both indices must follow normal distributions because they are both linear with respect to the normal shocks $\ln \boldsymbol{z}$. The variances differ, however, and the left panel of Figure 2 depicts the difference. Observe the dilation of the variance in the Cobb-Douglas economy where the power-law granularity of the Leontief inverse causes the difference [Gabaix (2011); Acemoglu et al. (2012)].

Replacing the equilibrium price of equation (9) with the output of equation (3) yields the following Domar aggregator, where exogenous productivity shocks ($\ln \boldsymbol{z}$) are aggregated into the

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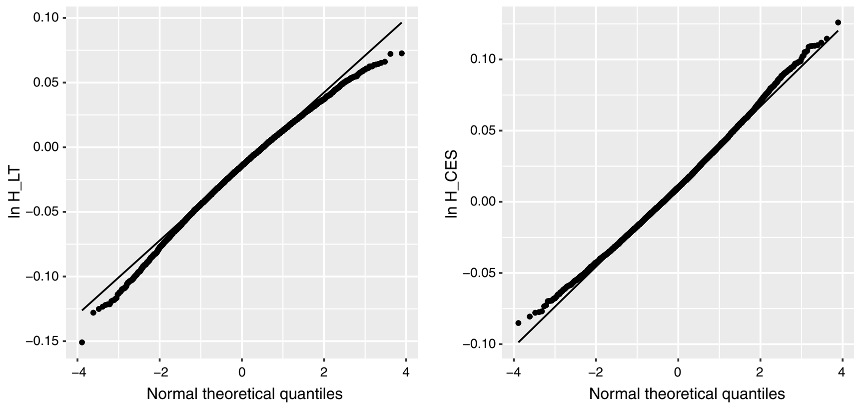

Figure 3. QQ plots of the aggregate output fluctuations generated by the Domar aggregators for Leontief (left) and elastic CES (right) economies under Cobb-Douglas utility.

growth of output ($\ln H$) for a CES economy.

$$\ln H = -\ln \Pi \left(\mathcal{E}\left(\boldsymbol{z};\pi_0 = 1\right)\right) + \ln \Pi\left(1/\boldsymbol{z}\right)$$

Note that this aggregator involves recursion in $\mathcal{E}$, as specified by equation (2), for a CES economy with non-uniform substitution elasticities. In this section, Cobb-Douglas utility is assumed for comparison with previous research, whence the Domar aggregator becomes:

$$\ln H = -\left(\ln \mathcal{E}\left(\boldsymbol{z}\right) + \ln \boldsymbol{z}\right) \boldsymbol{m} \qquad \text{CES economy} \qquad (12)$$

Again, we set the share parameter $\boldsymbol{\mu}$ at the standard expenditure share of the final demand. In the case of the Leontief economy, the closed form is available from equation (5) as follows:

$$\ln H = -\left(\ln\left(\boldsymbol{a}_0\left[\langle \boldsymbol{z}\rangle - \mathbf{A}\right]^{-1}\right) + \ln \boldsymbol{z}\right) \boldsymbol{m} \qquad \text{Leontief economy} \qquad (13)$$

The case for the Cobb-Douglas economy is also obtainable from equation (6) as follows:

$$\ln H = -(\ln \boldsymbol{z})\left(\left[\mathbf{I} - \mathbf{A}\right]^{-1} + \mathbf{I}\right) \boldsymbol{m} \qquad \text{Cobb-Douglas economy} \qquad (14)$$

As is obvious from the linearity of equation (14), the aggregate fluctuations must be normally distributed in the Cobb-Douglas economy. The resulting QQ plot is depicted in the right panel of Figure 2. Note further that $\mathbb{E}[\ln H] = 0$ because of the linearity of equation (14) and $\mathbb{E}[\ln \boldsymbol{z}] = (0, \cdots, 0)$; this is recognizable in the same figure. That is, the expected economic growth is zero against zero-mean turbulences. In other words, the unit elasticity of the Cobb-Douglas economy precisely absorbs the turbulences as if there were none to maintain zero expected growth. Such robustness is absent in a Leontief economy with zero elasticity of substitution. The left panel of Figure 3 illustrates the resulting QQ plot of the aggregate fluctuations generated by the same zero-mean normal shocks by way of equation (13).[5] In this case, we observe negative expected output growth, that is, $\overline{\ln H} = -1.57\%$, whose absolute value, in turn, can be interpreted as the robustness of the unit elasticity of substitution.[6] Moreover, we observe that normal shocks to the Leontief economy result in aggregate fluctuations with tail asymmetry similar to that depicted in the left panel of Figure 1.

In the CES economy with a sector-specific elasticity of substitution, we use the Domar aggregator of general type (12) with recursion. The right panel of Figure 3 depicts the resulting QQ plot of the aggregate fluctuations generated by zero-mean normal shocks. In this case, we observe positive expected output growth that is, $\overline{\ln H} = 1.10\%$. The value demonstrates the robustness of the elastic CES economy relative to the Cobb-Douglas economy. We also observe that normal shocks to the elastic CES economy result in aggregate fluctuations with tail asymmetry similar to that depicted

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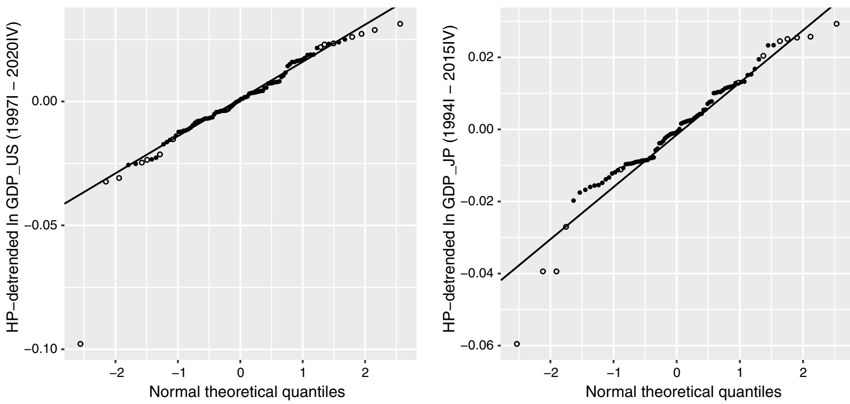

Figure 4. Quantile-quantile plots of recent (USA: 1997I–2020IV; Japan: 1994I–2015IV) quarterly HP-detrended log GDP against the normal distribution for the USA (left) and Japan (right). The time periods correspond to those when the sectoral elasticities are estimated. Open dots indicate 2007I–2009IV (GFC) and 2019IV– (COVID-19 pandemic) observations. Source: FRED (2021); Cabinet Office (2021).

in the right panel of Figure 1. For sake of credibility, we show in Figure 4 (right) the empirical aggregate output fluctuations focusing on the period 1994–2015 from which our sectoral elasticities are estimated. It is obvious that extreme observations belong to periods around the GFC (global financial crisis), which was a massive external (non-sectoral) shock to the Japanese economy. The plot otherwise appears rather positively skewed, as predicted by our empirical result ($\bar{\hat{\sigma}} = 1.54$).

Figure 4 (left) shows the aggregate output fluctuations focusing on the period 1997–2020 from which we estimated sectoral elasticities for the USA (see Appendix B for details). In this case, the extreme observations also belong to periods around the GFC and COVID-19 pandemic. The plot, however, seems to be in a straight line, which is consistent with our empirical result ($\bar{\hat{\sigma}} = 1.08$). Our elasticity estimates for the US economy also coincide with the estimates of the elasticity of substitution across intermediate inputs (ϵ_M all $= 1.05$) by Miranda-Pinto and Young (2022) based on 1997–2007 input-output accounts. These elasticity estimates, however, differ from those obtained by Atalay (2017) based on 1997–2013 input-output accounts that Baqaee and Farhi (2019) employed in their simulation ($\varepsilon = 0.001$). An inelastic economy as such is rather consistent with negatively skewed aggregate fluctuations spanning the postwar US economy as depicted in Figure 1 (left) than those of recent times depicted in Figure 4 (left).

5. Concluding remarks

Acemoglu et al. (2017)'s claim was that a heavy-tailed aggregate fluctuation can emerge from heavy-tailed microeconomic shocks because of the network heterogeneity of the input-output linkages (which will be fixed under Cobb-Douglas economy), even if the central limit theorem implies that the aggregate fluctuation must converge into a normal distribution. Our findings that the US economy in recent years has been essentially a Cobb-Douglas one, and that recent aggregate fluctuations exhibit a tail risk as captured in Figure 4 (left), therefore, indicate a negatively skewed distribution of microeconomic shocks. At the same time, our finding of Japan's elastic economy, together with the assumption of negatively skewed microeconomic shocks, provides a better understanding of the peculiar pattern of its recent aggregate fluctuations as captured in Figure 4 (right).

In the meanwhile, it is well documented that the Japanese have been more creative in discovering how to produce than in what to produce. The empirical results obtained in this study provide

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some evidence to believe that such a spirit is engraved in the nation's economy. Undeniably, the technologies embodied in a production function have been acquired over the long course of research and development. Japan must have developed its elastic economy through the grinding process of discovering more efficient and inexpensive ways to produce while overcoming the many external turbulences it confronted. Whatever the cause may be, an elastic economy equipped with many substitutable technologies must be favorable with respect to robustness against turbulence. Ultimately, human creativity expands the production function in two dimensions: productivity and substitutability, and the elasticity of substitution in particular, which brings synergism between the economic entities, must be worthy of further investigation.

Acknowledgements. The authors wish to thank the anonymous reviewers for their helpful comments and suggestions.

Financial support. JSPS Kakenhi Grant numbers: 19H04380, 20K22139.

Conflict of interest. The authors declare that they have no conflict of interest.

Notes

1 The role of elasticities in propagating shocks for multisector models with input-output linkages is also highlighted in Carvalho et al. (2020) with regard to Japan's supply chain.
2 This paragraph was included by request from the associate editor.
3 The uniform CES elasticity for the US production economy estimated by Atalay (2017) using military spending as an IV is reportedly approximately −0.1 with zero (Leontief) being unable to be rejected.
4 In a simple economy where there is no intermediate production, equation (2) is reduced as $\boldsymbol{\pi} = \boldsymbol{z}^{-1}$.
5 Note that several sample normal shocks (with annual volatility of 20%) made the equilibrium structure *unviable* in the Leontief economy. Such samples are excluded from the left panel of Figure 3.
6 Baqaee and Farhi (2019) also provide the foundation that an elastic (inelastic) economy implies positive (negative) expected output growth, based on the second-order approximation of nonlinear Domar aggregators.
7 We use the last 3 years' (2017, 2018, 2019) average of the accounts to instrument for the 2020 explanatory variable.

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Appendix A. GBM Property of sectoral productivity

A geometric Brownian motion (GBM) can be specified by the following stochastic differential equation (SDE):

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}B_t$$

where μ denotes the drift parameter, σ denotes the volatility, and $B_t \sim \mathcal{N}(0, t)$. Ito's Lemma implies that the above SDE is equivalent to the following:

$$\mathrm{d}\ln X_t = (\mu - \sigma^2/2)\mathrm{d}t + \sigma \mathrm{d}B_t$$

where this SDE is solvable by integration. The solution follows below:

$$\ln X_T = \ln X_0 + (\mu - \sigma^2/2)T + \sigma B_T$$

Since $B_T \sim \mathcal{N}(0, T)$, the first and the second moments for $\ln(X_T/X_0)$ can be evaluated as follows:

$$\mathbb{E}\left[\ln(X_T/X_0)\right] = (\mu - \sigma^2/2)T, \qquad \mathrm{var}\left[\ln(X_T/X_0)\right] = \sigma^2 T$$

There are several ways of estimating the volatility and the drift parameters of a GBM empirically from ℓ size of historical data $(X_1, \cdots, X_\ell)$. The obvious approach is the following, which is based on the sample moments:

$$\hat{\sigma} = \sqrt{\frac{1}{\ell-2}\sum_{k=1}^{\ell-1}\left(\Delta \ln X_k - \frac{\sum_{k=1}^{\ell-1}\Delta \ln X_k}{\ell-1}\right)^2}, \qquad \hat{\mu} = \frac{\sum_{k=1}^{\ell-1}\Delta \ln X_k}{\ell-1} + \frac{1}{2}\hat{\sigma}^2$$

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Table 2. GBM property of estimated productivity growth for all sectors

id	Sector	$\hat{\mu}$	$\hat{\sigma}$	Norm.
1	Agriculture	0.015	0.051	
2	Agricultural services	0.000	0.036	
3	Forestry	0.021	0.053	
4	Fisheries	0.001	0.065	
5	Mining	0.013	0.042	
6	Livestock products	0.010	0.118	
7	Seafood products	−0.002	0.060	
8	Flour and grain mill products	0.128	0.777	
9	Miscellaneous foods and related products	−0.016	0.073	
10	Beverages	−0.003	0.058	
11	Prepared animal foods and organic fertilizers	−0.023	0.113	
12	Tobacco	−0.072	0.222	
13	Textile products (except chemical fibers)	0.019	0.242	
14	Chemical fibers	−0.060	0.279	
15	Pulp, paper, and coated and glazed paper	−0.012	0.143	
16	Paper products	−0.071	0.576	No
17	Chemical fertilizers	−0.016	0.040	
18	Basic inorganic chemicals	−0.021	0.097	No
19	Basic organic chemicals	−0.043	0.146	No
20	Organic chemicals	−0.027	0.078	
21	Pharmaceutical products	0.192	1.362	
22	Miscellaneous chemical products	0.046	0.209	
23	Petroleum products	−0.057	0.300	No
24	Coal products	−0.063	0.259	No
25	Glass and its products	0.027	0.341	No
26	Cement and its products	−0.003	0.054	
27	Pottery	0.015	0.059	
28	Miscellaneous ceramic, stone, and clay products	0.001	0.067	
29	Pig iron and crude steel	−0.021	0.074	
30	Miscellaneous iron and steel	−0.049	0.292	
31	Smelting and refining of non-ferrous metals	−0.042	0.160	
32	Non-ferrous metal products	−0.113	0.272	
33	Fabricated constructional and architectural metal products	−0.053	0.220	
34	Miscellaneous fabricated metal products	0.405	1.638	
35	General-purpose machinery	−0.066	0.196	
36	Production machinery	−0.056	0.350	
37	Office and service industry machines	0.112	0.221	No
38	Miscellaneous business-oriented machinery	0.141	0.379	
39	Ordnance	0.024	0.107	
40	Semiconductor devices and integrated circuits	0.072	0.276	No
41	Miscellaneous electronic components and devices	0.310	1.641	No
42	Electrical devices and parts	0.032	1.509	

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Table 2. Continued

id	Sector	$\hat{\mu}$	$\hat{\sigma}$	Norm.
43	Household electric appliances	0.018	0.134	
44	Electronic equipment and electric measuring instruments	0.051	0.147	
45	Miscellaneous electrical machinery equipment	0.008	0.070	
46	Image and audio equipment	−0.136	0.523	No
47	Communication equipment	0.138	1.706	
48	Electronic data processing machines, etc∗1	0.127	0.980	
49	Motor vehicles (including motor vehicles bodies)	−0.028	0.305	
50	Motor vehicle parts and accessories	0.060	0.314	
51	Other transportation equipment	−0.059	0.584	
52	Printing	−0.005	0.078	
53	Lumber and wood products	0.015	0.073	
54	Furniture and fixtures	0.008	0.040	No
55	Plastic products	−0.011	0.097	
56	Rubber products	−0.053	0.206	
57	Leather and leather products	0.009	0.057	No
58	Watches and clocks	0.014	0.088	
59	Miscellaneous manufacturing industries	−0.003	0.091	
60	Electricity	0.000	0.053	
61	Gas, heat supply	−0.016	0.070	
62	Waterworks	−0.090	0.333	
63	Water supply for industrial use	−0.010	0.036	
64	Sewage disposal	−0.002	0.028	
65	Waste disposal	0.014	0.059	
66	Construction	−0.040	0.078	
67	Civil engineering	0.007	0.034	
68	Wholesale	0.008	0.099	
69	Retail	0.023	0.035	
70	Railway	0.107	0.488	
71	Road transportation	0.133	1.430	
72	Water transportation	−0.010	0.044	
73	Air transportation	−0.014	0.053	No
74	Other transportation and packing	0.016	0.071	
75	Mail	0.036	0.229	No
76	Hotels	0.018	0.061	
77	Eating and drinking services	0.014	0.076	
78	Communications	0.077	0.173	
79	Broadcasting	0.024	0.531	
80	Information services	−0.006	0.132	
81	Image information, etc∗2	−0.050	0.102	
82	Finance	0.032	0.193	
83	Insurance	0.014	0.093	
84	Housing	−0.108	0.284	No
85	Real estate	0.038	0.049	

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Table 2. Continued

id	Sector	$\hat{\mu}$	$\hat{\sigma}$	Norm.
86	Research	−0.013	0.076	
87	Advertising	0.015	0.047	
88	Rental of office equipment and goods	0.028	0.197	
89	Automobile maintenance services	0.000	0.066	
90	Other services for businesses	−0.006	0.058	No
91	Public administration	0.001	0.069	
92	Education	0.016	0.026	
93	Medical service, health, and hygiene	−0.001	0.072	
94	Social insurance and social welfare	0.031	0.061	
95	Nursing care	0.002	0.098	No
96	Entertainment	0.011	0.071	
97	Laundry, beauty, and bath services	0.013	0.143	No
98	Other services for individuals	0.050	0.123	
99	Membership organizations	0.030	0.118	
100	Activities not elsewhere classified	0.101	0.288	No

Note: The simple mean of all annual volatilities is 0.251. The normality of TFP growth rates is examined by the Shapiro-Wilk W test, where rejection of normality is indicated by the label “no,” and a blank is left otherwise.
*1 Electronic data processing machines, digital and analog computer equipment and accessories.
*2 Image information, sound information and character information production

Alternatively, Hurn et al. (2003) devised parameter estimates of the following based on the simulated maximum likelihood method.

$$\hat{\sigma} = \sqrt{\frac{1}{\ell-1}\sum_{k=1}^{\ell-1}\left(\frac{X_{k+1}}{X_k} - \frac{\sum_{k=1}^{\ell-1}\frac{X_{k+1}}{X_k}}{\ell-1}\right)^2}, \qquad \hat{\mu} = \frac{\sum_{k=1}^{\ell-1}\frac{X_{k+1}}{X_k}}{\ell-1} - 1 \tag{15}$$

The two methods produce very similar results for our data. Table 2 summarizes the estimated annual volatilities and drift parameters for all 100 sectors by formula (15) using the historical data of sectoral annual TFP from 1995 to 2015. Moreover, conforming to Marathe and Ryan (2005), we check the normality of the annual TFP growth rates using the Shapiro-Wilk W test. Normality was rejected in 19 out of the 100 sectors. The annual volatility, with a t-statistic over 2, ranges from 0.260 to 0.523, whereas the simple average concerning all 100 sectors is 0.251.

Appendix B. Sectoral elasticities of substitution for the us economy

This section is devoted to our estimation of sectoral substitution elasticities for the USA, in the same manner as we did for Japan. First, we create $n \times n$ input-output tables using the make and use tables of $n = 71$ industries in nominal terms for 24 years (1997–2020), available at BEA (2022). Next, we create tables in real terms by using price indices available as chain-type price indexes for gross output by industry. Note that the real value added of an industry is estimated by double deflation, so that price indices for value added can be derived from nominal and real value-added accounts. As for instruments, we utilize the integrated multifactor productivity (MFP), taken from the 1987–2019 Production Account Capital Table (BLS, 2022) of the BEA-BLS Integrated Industry-level Production Accounts, for n factor inputs.[7] To instrument primary factor prices, we apply three different instruments, namely, total factor productivity (i.e. aggregate TFP), a capital price deflator, and a labor price deflator, obtainable from the Annual total factor productivity and

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Table 3. Estimation of the elasticity of substitution for 71 US sectors

	LS FE		IV FE					
id	$\hat{\sigma}$	s.e.	$\hat{\sigma}$	s.e.	1st F*1	Overid.*2	Endog.*3	IVs*4
1	0.604	0.082	1.261	0.152	309	1.48 (0.223)	27.75 (0.000)	1, 2
2	0.532	0.101	1.348	0.187	302	0.10 (0.755)	32.42 (0.000)	l1, 2
3	0.533	0.087	1.124	0.182	197	1.14 (0.286)	15.67 (0.000)	1, l2
4	0.635	0.083	0.957	0.138	388	0.25 (0.616)	8.56 (0.003)	2, 1
5	1.143	0.085	1.549	0.161	298	0.03 (0.868)	11.76 (0.001)	l2, 1
6	0.812	0.106	0.749	0.190	313	0.03 (0.859)	0.16 (0.689)	1, 2
7	0.611	0.085	1.153	0.167	261	0.53 (0.469)	16.25 (0.000)	l1, 2
8	0.748	0.094	1.495	0.175	306	0.32 (0.571)	29.84 (0.000)	l1, 2
9	0.843	0.081	0.861	0.146	314	0.21 (0.649)	0.02 (0.878)	1, 2
10	1.037	0.079	0.746	0.151	262	0.68 (0.411)	3.82 (0.051)	1, l2
11	0.888	0.077	1.037	0.079	291	1.71 (0.190)	2.06 (0.151)	l1, 2
12	0.831	0.082	1.016	0.141	300	0.97 (0.324)	5.09 (0.024)	l1, 2
13	1.162	0.074	1.063	0.150	147	0.95 (0.330)	7.91 (0.005)	l1, 2
14	0.996	0.075	0.728	0.174	298	1.06 (0.304)	17.02 (0.000)	l1, 2
15	0.801	0.067	1.434	0.136	319	1.86 (0.173)	19.51 (0.000)	l1, 2
16	0.901	0.091	1.211	0.120	290	0.35 (0.554)	8.07 (0.005)	l1, 2
17	0.834	0.085	1.231	0.167	306	0.89 (0.346)	10.19 (0.001)	l1, 2
18	0.906	0.070	1.203	0.156	319	1.57 (0.211)	21.21 (0.000)	l1, 2
19	0.844	0.072	1.353	0.127	279	2.19 (0.139)	20.81 (0.000)	l1, 2
20	0.695	0.090	1.340	0.141	313	1.29 (0.256)	12.73 (0.000)	l1, 2
21	0.506	0.123	1.126	0.166	351	0.67 (0.413)	0.55 (0.458)	l1, 2
22	0.730	0.087	0.536	0.212	297	2.14 (0.144)	4.23 (0.040)	l1, 2
23	0.677	0.078	0.944	0.166	366	2.13 (0.145)	5.35 (0.021)	1, 2
24	0.768	0.095	0.930	0.135	167	2.01 (0.156)	5.44 (0.020)	l1, 2
25	0.530	0.088	0.288	0.226	268	0.18 (0.674)	12.00 (0.001)	l1, 2
26	0.703	0.087	0.954	0.174	302	1.45 (0.229)	19.79 (0.000)	l1, 2
27	0.740	0.062	1.253	0.165	307	1.77 (0.183)	34.09 (0.000)	l1, 2
28	0.833	0.081	1.237	0.112	324	0.02 (0.887)	8.88 (0.003)	l1, 2
29	0.764	0.066	1.147	0.144	279	0.04 (0.849)	8.70 (0.003)	l1, 2
30	0.956	0.102	1.028	0.125	315	0.07 (0.796)	13.84 (0.000)	l1, 2
31	0.721	0.064	1.461	0.186	320	1.16 (0.281)	17.47 (0.000)	l1, 2
32	0.279	0.146	1.067	0.114	276	0.16 (0.688)	18.20 (0.000)	l1, 2
33	0.720	0.074	1.214	0.271	279	1.97 (0.161)	24.69 (0.000)	l1, d2
34	0.765	0.093	1.255	0.140	91	0.51 (0.475)	2.98 (0.084)	d1, d2
35	0.584	0.114	1.168	0.257	289	0.16 (0.689)	24.71 (0.000)	l1, 2
36	0.860	0.116	1.416	0.215	273	0.78 (0.377)	10.90 (0.001)	l1, 2
37	0.083	0.139	1.377	0.216	256	1.24 (0.266)	1.70 (0.192)	l1, 2
38	0.621	0.104	0.217	0.258	262	0.07 (0.797)	20.22 (0.000)	l2, 3
39	0.608	0.106	1.348	0.208	318	0.57 (0.450)	7.61 (0.006)	l1, 2
40	0.911	0.096	0.996	0.188	304	0.00 (0.994)	44.01 (0.000)	l1, 1
41	0.993	0.089	1.862	0.183	308	0.18 (0.671)	2.48 (0.115)	1, l2

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Table 3. Continued

	LS FE		IV FE					
id	$\hat{\sigma}$	s.e.	$\hat{\sigma}$	s.e.	1st F*1	Overid.*2	Endog.*3	IVs*4
42	0.913	0.073	0.735	0.167	340	1.85 (0.174)	58.21 (0.000)	2, l3
43	0.773	0.094	1.675	0.132	330	0.09 (0.768)	5.19 (0.023)	l1, 2
44	0.762	0.084	1.072	0.171	293	0.11 (0.741)	38.11 (0.000)	l1, d2
45	1.120	0.101	1.514	0.153	219	1.58 (0.209)	31.97 (0.000)	l2, l3
46	0.804	0.126	2.055	0.203	333	1.49 (0.223)	4.13 (0.042)	l1, 2
47	0.691	0.116	1.154	0.223	235	1.32 (0.251)	28.22 (0.000)	l1, 2
48	1.226	0.269	1.664	0.233	326	0.00 (0.993)	0.06 (0.808)	1, l2
49	0.627	0.093	1.063	0.481	326	0.72 (0.397)	48.50 (0.000)	l1, 2
50	0.743	0.075	1.507	0.170	304	1.97 (0.160)	28.24 (0.000)	l1, 2
51	0.738	0.105	1.290	0.139	288	0.26 (0.613)	8.40 (0.004)	l1, 1
52	0.919	0.077	1.186	0.192	350	2.30 (0.130)	72.85 (0.000)	l1, 3
53	0.524	0.068	1.758	0.135	323	0.65 (0.419)	14.11 (0.000)	l1, 2
54	0.718	0.087	0.855	0.126	304	0.62 (0.431)	4.31 (0.038)	l1, 1
55	0.787	0.081	0.929	0.152	277	0.82 (0.365)	1.96 (0.161)	l1, l2
56	1.236	0.083	0.902	0.153	284	2.40 (0.121)	1.83 (0.176)	l1, 2
57	0.566	0.076	1.245	0.148	307	1.82 (0.178)	28.10 (0.000)	l1, 1
58	1.075	0.079	1.154	0.142	333	1.27 (0.260)	44.26 (0.000)	l1, 1
59	0.814	0.093	1.814	0.145	314	0.04 (0.849)	46.53 (0.000)	l1, 2
60	0.765	0.094	1.753	0.179	308	1.99 (0.159)	11.45 (0.001)	1, l2
61	0.683	0.084	1.235	0.177	298	0.52 (0.470)	17.88 (0.000)	1, l2
62	0.270	0.091	1.167	0.155	309	0.10 (0.758)	0.76 (0.383)	1, l2
63	0.667	0.076	0.364	0.170	325	0.83 (0.361)	2.11 (0.147)	1, l2
64	0.536	0.085	0.795	0.138	296	0.00 (0.965)	0.00 (0.965)	l1, 2
65	0.522	0.061	0.494	0.158	291	0.60 (0.438)	2.21 (0.138)	l2, 1
66	0.549	0.073	0.604	0.113	320	0.78 (0.378)	72.74 (0.000)	l1, 1
67	0.871	0.084	1.439	0.139	309	3.26 (0.071)	20.85 (0.000)	l1, 1
68	0.743	0.103	1.371	0.150	297	1.15 (0.284)	4.38 (0.036)	f1, 2
69	0.209	0.126	1.083	0.192	293	1.25 (0.264)	4.73 (0.030)	l1, 3
70	0.606	0.068	0.583	0.221	324	0.96 (0.327)	25.66 (0.000)	l1, 1
71	0.660	0.086	1.088	0.130	303	0.10 (0.752)	1.44 (0.230)	l1, 1

Notes: The id number corresponds to the numerical position of an industry of the input-output table of 71 industries BEA (2022). Values in parentheses indicate p-values.
*1 First-stage (Cragg-Donald Wald) F statistic for 2SLS FE estimation. The rule of thumb to reject the hypothesis that the explanatory variable is only weakly correlated with the instrument is for this to exceed 10.
*2 Overidentification test by Sargan statistic. Rejection of the null indicates that the instruments are correlated with the residuals.
*3 Endogeneity test by Davidson-MacKinnon F statistic. Rejection of the null indicates that the instrumental variables fixed effects estimator should be employed.
*4 Instrumental variables applied, where 1, 2, and 3 indicate v^a, v^b, and v^c, respectively, and l, f, and d indicate first lag, first forward, and first difference, respectively.

related measure for major sectors (BLS, 2022). Thus, our instrumental variables are v^a (sectoral MFP with aggregate TFP), v^b (sectoral MFP with capital price deflator), and v^c (sectoral MFP with labor price deflator), all of which are of $n+1$ dimension.

Estimation of sectoral elasticities of substitution was conducted according to the estimation framework presented in Section 3. The results are summarized in Table 3. The first column (LS FE) reports the least squares fixed effects estimation results, without instrumenting for the explanatory variable. The second column (IV FE) reports the instrumental variable fixed effects estimation results, using the IVs reported in the last column. In all cases, overidentification tests are not rejected. Moreover, first-stage F values are large enough. The estimates for the elasticity of substitution $\hat{\sigma}$ are larger when IVs are applied. According to the endogeneity test results, we accept the LS FE estimates for sector ids 6, 9, 10, 11, 21, 34, 37, 41, 48, 55, 56, 62, 63, 64, 65, and 71, instead of the IV FE estimates. The simple mean of the estimated (accepted) elasticities is $\bar{\hat{\sigma}} = 1.08$.

Cite this article: Nakano S and Nishimura K (2024). "The elastic origins of tail asymmetry." *Macroeconomic Dynamics* **28**, 591–611. https://doi.org/10.1017/S1365100523000172