BOOK REVIEWS - COMPTES RENDUS CRITIQUES

Ergodic properties of algebraic fields. By Yu. V. Linnik. Springer-Verlag, New York (1968). ix+192 pp. U.S. $11.

This book is mainly concerned with applications of ergodic concepts to solution of problems in number theory especially asymptotic results about solutions of diophantine equations. In §2 of the introduction the author makes quite clear his aim and the main ideas in the book. Intuitively speaking, the main theme is to construct "flows" of integral points i.e., a measure preserving transformation and the results of its iterates on integral points on spheres of increasing radii.

§1 of the first chapter is mainly statements of Birkhoff's individual ergodic theorem and Poincaré's recurrence theorem (this last theorem is a consequence of the first though not very well known). §2 is an application of the ergodic theorem to a problem in continued fractions.

Chapter 2 starts with a few facts about quaternions which will be found quite useful for subsequent reading though we must say we did not find the treatment adequate. In contrast to this, the treatment of the arithmetic of matrices in the next article is quite nice. Even though not all of the proofs are supplied the author succeeds in giving a good idea of what is going on.

Chapter 3 is quite technical. The main theorem gives a lower bond to the number of rotation of spheres which take primitive integral points into themselves.

Chapter 4 is the first place where ergodic concepts are used. The "ergodic theorem" of §2 is not the usual ergodic theorem; in simple terms this theorem gives the expected number of projections, in a region of the unit sphere, of a primitive integral point (on a sphere of radius m) and its images under the iterates of a rotation. Any reader familiar with probability will notice the probabilistic ideas reflected through this chapter.

Chapters 5 and 6 in the reviewer's opinion reflect the skill of the author. They are in the same vein as Chapter 4 though due to noncompactness the techniques are more difficult. To give a reasonably clear picture of the contents of these chapters would take at least a page.

On Chapter 7 the author discusses the asymptotic behaviour of solutions of systems of diophantine equations. This is of interest because of the possibility of representing algebraic fields by integral matrices, and such a representation automatically represents algebraic numbers by integral matrices.

Chapter 8 deals with the asymptotic distribution (number in regions of the group of unimodular matrices) of integral 3×3 matrices. It is rather technical; its aim is to generalize a lemma in Chapter 5. Definitive results are obtained here unlike Chapter 9 where only further generalization are examined briefly. Chapter 10, in author's own words, "stands alone". However, it is interesting to a probabilist. It should also please people interested in simulation of random numbers. This book is a valuable addition to the existing mathematical literature.

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An Introduction to Statistical Mathematics. BY A. M. MATHAI. S. Chand and Co., Delhi (1967). xvi+432 pp. U.S. $8.

Professor Mathai addresses his book to those attending a one-year course in Probability and Statistics and to those learning these subjects by themselves. The accent is firmly on the mathematical aspect of statistical procedures, a feature accurately reflected in the title.

The book can be divided into three sections, background mathematics leading up to probability and random variables, distribution theory, and inference. The generally fast pace seems appropriate in the middle section, in which few new concepts are introduced, but gives unsatisfactory results elsewhere. In particular it is arguable that most of the first two chapters should not have been included, since the treatment of set theory and linear algebra given there is far too compressed to help the reader previously unfamiliar with them, and unnecessary for others.

The treatment of inference is, in principle, general, but virtually all the detailed discussion relates to the normal and binomial distributions, for which most of the standard techniques are given. There is, however, little attempt to provide any coherent development of the theory, and the mechanics of statistical techniques are emphasized at the expense of their rationale.

On reading through the book one is left with the feeling that the subtlety of the material would have been more appropriately served by a more careful presentation. The abundance of misprints would seriously mislead anyone working alone, and incorrect or vague statements are hardly less in evidence. We learn, for example, that if, in successive trials, the probability of success is not constant the hypergeometric distribution is appropriate, also that the maximum likelihood ratio test usually gives a uniformly most powerful critical region for testing a simple or composite hypothesis. The short discussion of sampling from a finite population is rendered unconvincing by an erroneous definition of a simple random sample.

The author's pedagogical style involves repetitions of the sequence: theory, example (with solution) or proof, and comments. This is quite effective, particularly in the earlier chapters. Many exercises are provided, but occasionally these are badly linked with the text, and some cannot be answered without recourse to results and techniques not given in the book. For example, the reader is instructed