RELATIVISTIC AND PERSPECTIVE EFFECTS IN PROPER MOTIONS AND RADIAL VELOCITIES OF STARS

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ABSTRACT. The heliocentric motion of stars is investigated by taking into account all effects due to the geometry and due to the finite speed of light. It is shown that the proper motions as given in star catalogues contain the main light retardation term; an additional term of this type modifies Schlesinger's (1917) term but is less significant than a 3rd order perspective term neglected in conventional astrometry. Kapteyn's star serves as a numerical example. The ambiguous relativistic relation between Doppler shifts and radial velocities is discussed and demonstrated in a diagram. The ambiguity is solved with rigorous equations which allow to compute the inertial motion of a star from its proper motion, classical radial velocity, and distance. -

The new IAU concepts recommend the employment of rigorous methods in all branches of spherical astronomy. The various computational steps to be performed in "stellar reductions" (that is, in the reduction from observed apparent to mean catalogue positions, and vice versa) are described in the Astronomical Almanac 1984. All of these steps are treated rigorously, with only one exception: the perspective changes in the proper motions are still computed with Schlesinger's (1917) 2nd order term; terms of a higher order as well as light retardation effects are not considered at all.

The influence of Schlesinger's term on stellar motions has been discussed in the past by various astronomers (cf., for instance, van de Kamp, 1977). Eichhorn (1981) seems to be the first who employed vector relations and hence treated the geometrical part of the problem rigorously, but he also did not mention the retardation effect.

The only author who - to my knowledge - at least implicitly had considered the finite light-travel time was K.Schwarzschild (1894). He studied the so-called "secular aberration" and showed that the proper motion μ^*, as given in catalogues of mean star positions, is connected with the true inertial proper motion μ by the equation

$$\mu^* = \mu(1-\beta_r), \qquad (1)$$

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where

$$\beta_r = \text{radial velocity/speed of light} = v_r/c. \quad (2)$$

Schwarzschild's equation is the 1^{st} order approximation of

$$\mu^* = u/(1+\beta_r), \quad (3)$$

an equation which can be shown to be rigorously correct. It describes an effect which nowadays is used as one possible explanation for the so-called "super-luminal" velocities observed with the VLBI technique in the nuclei of active galaxies (cf. Kellermann and Pauliny-Toth, 1981). Ginzburg and Syrowatskii (1969) expressed their astonishment that effects of this type were not recognized for a long time. I found it fascinating that already Schwarzschild, while still being a student in Göttingen, knew about it. I think it is rather clear why his paper later was ignored: it concerned secular aberration, i.e. a rather uninteresting displacement of the heliocentric positions of the stars whose determination would either not at all or only with great difficulties and after very long times be possible, and hence could be ignored.

Today we are living in an era of ever-increasing measuring accuracy. If we consider the heliocentric (or barycentric) motion of stars, we should first investigate all effects involved before we decide which of them can be neglected. Recently I have carried out such an investigation (Stumpff, 1985; hereafter referred to as Paper I) and would like to take the opportunity of this conference to discuss the underlying problems and to demonstrate some of the results.

I am ignoring all the many problems with precession, nutation, parallax determination, solar system ephemerides, and the accuracy limitation of our present measuring techniques; in other words, I am assuming that all these problems have been ideally solved. One sees then easily that Earth-bound observations of apparent celestial positions and of apparent frequencies (of spectral lines, or of pulsars) can be rigorously reduced to the Sun (or the solar system barycenter). The results of such reductions are positions and frequencies which an observer located in the Sun (or in the barycenter) would observe as function of the light arrival time. From this information one can derive proper motion and radial velocity, and hence amount and direction of the total relative velocity, and so determine kinematical properties of a star and compute its position for other epochs.

I have investigated the following problem: An observer is located in the origin (Sun, or solar system barycenter) of an inertial reference frame. A star is moving along a linear orbit in this frame, i.e. with a constant velocity V_o in a constant direction. At an instant t_o it has a distance r_o from the observer and is emitting a light wave which the observer receives at the time $t_o^*=t_o+r_o/c$. Hence, at the instant t_o^*, the observer sees the star in the direction where it was at the instant t_o. This direction, which corresponds to the so-called "mean" heliocentric catalogue position at epoch t_o^*, is an apparent direction, i.e. affected by secular aberration. The direction of motion of the star at this instant is defined by the angle θ_o relative to the apparent line of

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sight, so that θ_o is an apparent angle; this determines the form (Eq.10) of the relativistic Doppler equation to be used in the computation of the radial velocity. The total vector of the relative velocity V_o has then the components $v_{r,o}=-V_o \cos\theta_o$ (radial) and $v_{t,o}=V_o \sin\theta_o$ (tangential). For an illustration see Fig.1 of this paper (where the subscript "o" is omitted). We repeat this consideration for another epoch, t, when the star has reached the distance r and is emitting another light wave which is received at the instant $t^*=t+r/c$ by the observer (see Fig.1 of Paper I). The observer finds that the star has moved along an arc ω across the sky in the time interval $\tau^*=t^*-t^*_o$.

Hence, if we want to understand what proper motion and perspective effects actually do mean, we have to compute the angle ω as function of the light-arrival time τ^*. Rigorous expressions for $\sin\omega$ and $\cos\omega$ are derived in Paper I (Eqs. 24) as function of τ^* and of

$$\left.\begin{aligned} \mu_o &= v_{t,o}/r_o = +(V_o \sin\theta_o)/r_o = \text{proper motion} \\ \rho_o &= v_{r,o}/r_o = -(V_o \cos\theta_o)/r_o = \text{normalized radial velocity} \\ \beta_{t,o} &= v_{t,o}/c, \qquad \beta_{r,o} = v_{r,o}/c, \qquad \beta_o = V_o/c. \end{aligned}\right\} \quad (4)$$

These quantities are "inertial", i.e. they refer to synchronized clocks in the reference frame. The observer, on the other hand, defines velocities with respect to the light-arrival time. Since he measures ω as function of τ^*, he can obtain the observational equivalent of the inertial proper motion by developing the observed $\omega(\tau^*)$ in a Taylor series:

$$\omega(\tau^*) = (\omega_o'\tau^*) + (\omega_o''\tau^{*2})/2 + (\omega_o'''\tau^{*3})/6 \quad \ldots \quad , \qquad (5)$$

where the prime indicates differentiation with respect to τ^*. Using the rigorous equations for $\sin\omega$ and $\cos\omega$, one obtains for the coefficients

$$\left.\begin{aligned} \omega_o' &= \tilde{\mu}_o , \\ \omega_o'' &= -2\tilde{\mu}_o\tilde{\rho}_o - \tilde{\beta}_{t,o}\tilde{\mu}_o^2 , \\ \omega_o''' &= 6\tilde{\mu}_o\tilde{\rho}_o^2 - 2\tilde{\mu}_o^3 + \Delta\omega_o''' , \end{aligned}\right\} \quad (6)$$

where

$$\tilde{\mu}_o = \mu_o/d_o, \quad \tilde{\rho}_o = \rho_o/d_o, \quad \tilde{\beta}_{t,o} = \beta_{t,o}/d_o , \qquad (7)$$

with

$$d_o = 1 + \beta_{r,o}. \qquad (8)$$

These expressions are completely rigorous. The term $\Delta\omega_o'''$ is of the 4^{th} order and can be ignored.

The main influence of the light-travel time is described by the fact that in the rigorous expressions the inertial velocity components

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are devided by the retardation factor d_o; its only other direct influence is caused by the second (3^{rd}order) term in ω_o''.

The proper motion μ_o^*, as given in a catalogue, is obviously identical with the first Taylor coefficient, and so we see that

$$\mu_o^* = \omega_o' = \mu_o/(1+\beta_{r,o}), \qquad \beta_{t,o}^* = \mu_o^* r_o/c, \tag{9}$$

as already Schwarzschild had shown. In other words, μ_o^* is the observational equivalent of the inertial parameter $\tilde{\mu}_o$, and hence the latter can be replaced by an observable in all rigorous equations.

Next we have to ask: What is the observational equivalent of the other important inertial parameter, viz. of the normalized radial velocity $\rho_o = v_{r,o}/r_o$? As I mentioned at the beginning, an apparent frequency measured on Earth can be reduced with any desired accuracy to the Sun. Let us denote this heliocentric frequency by f_o^*, because it is, in the same sense as μ_o^*, an "observable". Now if F is the rest frequency of the spectral line, then the relativistic Doppler equation is (since θ_o and hence $\beta_{r,o}$ are "apparent" quantities)

$$q_o = f_o^*/F = (1-\beta_o^2)^{1/2}/d_o, \qquad \beta_o^2 = \beta_{r,o}^2 + \beta_{t,o}^2. \tag{10}$$

The inertial radial velocity would follow from $v_{r,o} = c\beta_{r,o}$, but we do not obtain $\beta_{r,o}$ from the Doppler equation without knowing the tangential velocity! However, we can compute the 1^{st}order approximation (equivalent to the classical Doppler effect)

$$\beta_{r,o}^* = v_{r,o}^*/c = 1 - q_o, \qquad \rho_o^* = \beta_{r,o}^* \, c/r_o \, . \tag{11}$$

Introducing now

$$\delta_o = (1-\beta_o^2)^{1/2} - 1, \tag{12}$$

we get the following rigorous relations:

$$\begin{aligned} \mu_o &= d_o\mu_o^*, & \beta_{t,o} &= d_o\beta_{t,o}^*, \\ \rho_o &= d_o\rho_o^* + c\delta_o/r_o, & \beta_{r,o} &= d_o\beta_{r,o}^* + \delta_o \, . \end{aligned} \tag{13}$$

These equations can easily be solved by numerical iterations, starting with d_o=1 and δ_o=0. In this manner one obtains the inertial properties of the star's orbit, and they can then be used in the equations of the rigorous theory to predict heliocentric positions and velocity components for any instant τ^*.

As a numerical example I am considering Kapteyn's star. Its orbital elements are given by a parallax of 0".256, a proper motion of 8".89/year, and a radial velocity of +245 km/s. Table I gives an ephemeris, i.e. $\omega(\tau^*)$ in steps of 100 years, and the contribution of the perspective and light-travel time effects:

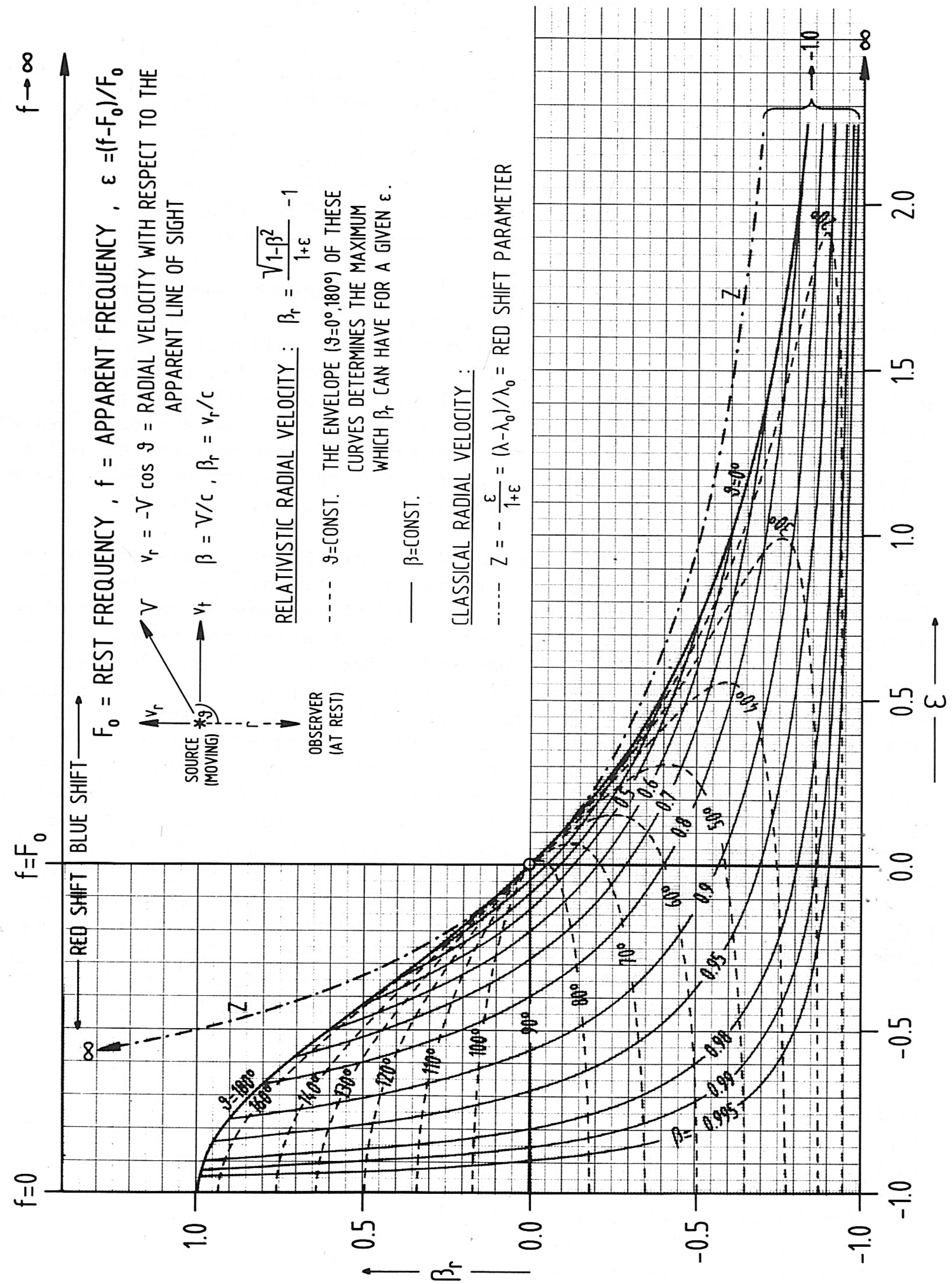

Figure 1. Radial velocity as function of Doppler shift

TABLE I. Ephemeris of Kapteyn's star

τ^*	ω	ω_{21}	ω_{22}	ω_3	$\Delta\omega$
100	883."3	-5."7	-0."001	+0."03	+0."03
200	1755.4	-22.8	-0.004	+0.25	+0.26
300	2616.5	-51.3	-0.010	+0.84	+0.85
400	3466.8	-91.2	-0.017	+1.99	+1.99
500	4306.3	-142.5	-0.026	+3.88	+3.86

ω_{21} is Schlesinger's term, ω_{22} is a 3^{rd}order light retardation effect, ω_3 is a 3^{rd}order perspective effect, and the $\Delta\omega$ in the last column gives the difference between the rigorous and the conventional method.

The additional terms contributed by a rigorous theory are very small, as this example shows, and probably not detectable at present. Hence my investigation may seem to be of a more or less academic nature, but this opinion may change in the future; VLBI observations of radio stars, or optical interferometry as proposed by Shapiro and Reasenberg during this symposium, may yield position errors far below 0."001.

Finally, I come back to the ambiguity of the relativistic conversion of apparent frequencies into radial velocities. The relativistic Doppler equations are very simple, and yet it is easy to make interpretation mistakes. I have constructed a diagram which may help to avoid such mistakes (Figure 1). The abszissa shows the apparent frequency of a spectral line, going from zero over the rest frequency F_o to infinity. Left from $f=F_o$ are the red shifts, right of it the blue shifts. Ordinate is the radial velocity (β_r) in units of c, covering the relativistic range -1 to +1. Plotted are the (dashed) curves θ=const and the (full) hyperbolae β=const. The curve (-·-·-·-) on top of all other curves gives the red-shift parameter Z which is identical with the classical radial velocity (in units of c) and belongs formally to the family of the hyperbolae β=const, with const=0. The horizontal line β_r=0 represents the transverse Doppler effect, and the vertical line $f=F_o$ (zero Doppler shift!) corresponds to all those cases where transverse and radial Doppler effects cancel each other. - The diagram illustrates very clearly that we can only assign a maximum value to the radial velocity if we have no other information than the Doppler shift.

REFERENCES.

Eichhorn,H.: 1981, Astron.J. 86, 915
Ginzburg,V.L., Syrowatskii,S.I.: 1969, Vistas in Astronomy 7, 375
Kellermann,K.I., Pauliny-Toth,I.I.K.: 1981, Ann.Rev.Astron.Astrophys. 19, 373
Schlesinger,F.: 1917, Astron.J. 30, 137
Schwarzschild,K.: 1894, Astron.Nachr. 136, 81
Stumpff,P.: 1985, Astron.Astrophys. 144, 232
van de Kamp,P.: 1977, Vistas in Astronomy 21, 289