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BOOK REVIEWS

KIRYAKOVA, V. *Generalized fractional calculus and applications* (Pitman Research Notes in Mathematics Vol. 301, Longman 1994), 388 pp., 0 582 21977 9, £39.

"Fractional Calculus" is the (less than ideal) name which has evolved for the study of operators involving derivatives or integrals whose orders are not necessarily positive integers. The subject is as old as Calculus itself. Away back in 1695, Leibniz and L'Hôpital were in correspondence about a possible meaning for $d^{1/2}y/dx^{1/2}$. In the ensuing 300 years, Fractional Calculus has been used to relate special functions to elementary functions and solve equations involving such special functions in a systematic fashion. Fractional integrals have also played a major role in harmonic analysis, notably in connection with weighted norm inequalities. Now there are even α-integrated semigroups of operators. However, it is with special functions that the present book is mainly concerned.

The book can be viewed in two ways. On the one hand, it may be regarded as the presentation of work by the author and co-workers which previously was only available in papers. From this angle, the contents might seem rather specialised and only a specialist would read every page. On the other hand, the book can serve as a mini-encyclopaedia of all that has happened in Fractional Calculus on the positive (or non-negative) real line. The list of references runs to 30 pages and contains no fewer than 519 items. There is also a citation index listing the references mentioned in individual chapters. This clearly illustrates the author's in-depth knowledge of the literature. The book serves a most valuable function in bringing together material from all over the world, including many items in Russian which may be largely unknown in the West, even to experts.

The casual reader can glean a good idea of the contents by reading the introduction. This shows how the operators to be discussed are related to products of Erdélyi–Kober operators. The latter operators, linked with the names of Arthur Erdélyi and Hermann Kober, have been studied extensively, both in theory and applications, over the last 50 years. It is possible to express products of such operators in terms of Meijer's G-function. This leads to the "generalized" fractional calculus of the title (not to be confused with fractional calculus for generalized functions!).

The first two chapters review and extend well-known results for Riemann–Liouville and Erdélyi–Kober operators. The first chapter is fairly elementary but the second chapter takes us into the realms of convolution, which is treated within a general framework developed by I. H. Dimovski. The work of Russian authors is featured prominently, notably the so-called Džrbashjan–Gelfond–Leontiev operators.

Chapter 3, running to over 100 pages, is the longest chapter. It deals with Hyper–Bessel operators, a typical example having the form

$$x^{\alpha_0}Dx^{\alpha_1}Dx^{\alpha_2}\dots x^{\alpha_{n-1}}Dx^{\alpha_n} \quad (D \equiv d/dx).$$

Connections with mathematical physics are discussed and the reader might be surprised to encounter a discussion of the Stokes phenomenon for the Airy equation. This is used to illustrate an extensive treatment of transmutation operators, of which the classical Poisson and Sonine integrals can be regarded as very special cases.

Chapter 4, devoted to generalized hypergeometric functions, begins with the quoting of a clarion call by Richard Askey exhorting us all to get educated in the usefulness of special

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functions. There follows a treatment of the function ${}_pF_q$ for $p \leqq q+1$. There are three cases: for $p<q$, ${}_pF_q$ can be regarded as of Bessel type, while $p=q$ and $p=q+1$ give Confluent and Gauss types respectively. Representations as Poisson integrals, formulae of Rodrigues type and representations in terms of Meijer's G-function are obtained.

In the final chapter, there are further extensions involving Fox's H-functions. Whereas most of the earlier theory is developed relative to spaces of continuous or differentiable functions, we now meet weighted L^p-spaces and the Mellin transform. The chapter concludes with brief references to a variety of topics including fractional finite differences and fractal dimension.

An appendix, running to 46 pages, contains all the properties of special functions necessary to make the book self-contained.

Although a lot of the material is technical and symbols, subscripts and superscripts abound, the author's presentation is admirable and the quality of English is exceptionally good. With so many formulae around, the author has devised a numbering system which, although largely standard, contains a few idiosyncrasies. For example, formulae (1.1.18) and (1.1.19) are squashed between (1.1.h) and (1.1.j). This makes the occasional cross-reference hard to find. The number of misprints is remarkably small and the overall appearance pleasing, thanks to the use of TEX.

As indicated earlier, the huge list of references almost justifies the book alone, but there is much else for the general reader and expert alike. The author is to be congratulated for her labours in producing a work of scholarship.

A. C. McBRIDE

NEUMANN, P. M., STOY, G. A. and THOMPSON, E. C. *Groups and geometry* (Oxford University Press, Oxford 1994), x + 254 pp., hardback 0 19 853452 3, £40; paper 0 19 853451 5, £19.50.

The reviewer of this text is immediately faced with an awkward and potentially embarrassing dilemma. The first two authors, Neumann and Stoy, explicitly affirm in the preface that the work may be idiosyncratic and, subsequently, they give a generous and touching eulogy to the deceased third author, Thompson. In this situation a reviewer is compelled to speculate on two questions, namely on whether to endorse their affirmation and whether, on the principle of the Latin tag *nil nisi bonum*, to refrain from criticism? An analysis of the text does allow both questions to be answered with some decorum.

The preface states that the work really consists of two books; of the 19 chapters, Chapters 1–10 on group theory and Chapter 19 on Rubik's magic cube have been written by Neumann and Stoy, who saw the book through the press, and Chapters 11–18 on geometry have been written by Thompson. Stylistically the two parts of the text cohere quite well although the later chapters have a tendency to be wordy in comparison with the earlier chapters. Some of the idiosyncrasy shows up in the unconventional titles of the chapters, titles such as "A menagerie of groups" and "A garden of G-spaces", and in the exotic choice of colours for the sides of Rubik's cube even if this choice is amusingly marred by a flaw in the proof-reading. Throughout the book there are interesting historical asides whose presence appears to reflect in particular the influence of Thompson.

The chapters on group theory are clearly written and emphasize aspects which will be relevant to the geometric applications. The group theory is itself developed from its very beginnings but throughout there is a presumption that the reader has some familiarity with the topics under discussion; in consequence, proofs are sometimes omitted or merely sketched. On the other hand the authors do not simply establish results but take pains to convey ideas; thus for example two different proofs of Sylow's Theorems are given so that the reader may better appreciate uses to which permutation groups may be put. Permutation groups do constitute a significant part of the group theory, concepts associated with groups acting on sets receiving an extensive treatment. Affine and linear groups are considered for their later applications in geometry. The counting

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