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On the Nonlinear Nature of the Turbulent α-Effect

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Abstract. Galactic magnetic fields are, typically, modelled by mean-field dynamos involving the α-effect. Here we consider, very briefly, some of the issues involving the nonlinear dependence of α on the mean field.

1. Introduction

Large-scale galactic magnetic fields are usually ascribed to be the result of some sort of hydromagnetic dynamo action (see, for example, the monograph by Ruzmaikin, Shukurov & Sokoloff 1988 and the review by Beck et al. 1996). Galactic dynamos have, almost without exception, been modelled by $\alpha - \omega$ mean field dynamos: in such a prescription, poloidal field is sheared by the differential rotation, ω, to produce a toroidal field, and, conversely, the reverse (toroidal to poloidal) step is achieved by the so-called 'α-effect' of mean field electrodynamics. A non-zero α-effect is attributable to a lack of reflexional symmetry in the underlying turbulence; in the galactic context such turbulence may be the consequence of exploding supernovae influenced by the galactic rotation.

Standard mean-field formalism is based essentially on a kinematic treatment in which the magnetic field has no influence on the velocity field. In reality, of course, dynamo-generated fields will eventually attain sufficient strength to react back on the driving flow. Consequently, one of the important points to address is the dependence of the α-effect on the mean (large-scale) magnetic field B_0 and also, crucially, its dependence on the magnetic Reynolds number Rm. This is a controversial issue which, in its simplest form, can be reduced to asking whether α satisfies a relation of the form

$$\alpha = \frac{\alpha_0}{1 + B_0^2/\langle u^2 \rangle}, \qquad (1)$$

where α_0 is the turbulent value and the magnetic field strength B_0 is measured in units of the Alfvén speed, or a relation of the form

$$\alpha = \frac{\alpha_0}{1 + Rm^\gamma B_0^2/\langle u^2\rangle}, \tag{2}$$

for some $O(1)$ value of γ.

Given that Rm in a galactic context is very large (estimates for Rm based purely on collisional processes are $O(10^{14})$, those based on ambipolar diffusion are $O(10^6)$), expressions (1) and (2) lead to very different conclusions; the former implies that the large-scale field can reach equipartition strength before α is suppressed, the latter that suppression occurs for a large-scale field smaller than equipartition by a factor of $Rm^{\gamma/2}$.

2. Physical Considerations

2.1. Two-dimensional Turbulent Diffusion

To illustrate some of the crucial ideas it is instructive to consider the simpler, but related, problem of the diffusion of a planar magnetic field due to a two-dimensional turbulent flow. Here dynamo action is impossible and decay of the field inevitable. The turbulent diffusion time t_T for a kinematic magnetic field over a scale L is given by the classical result

$$t_T \approx L^2/U\ell, \tag{3}$$

where ℓ is the scale of the energy-containing eddies. The dynamic decay time (i.e. taking into account the Lorentz force) can however be significantly enhanced (Vainshtein & Cattaneo 1992). If B_0 is the large-scale component of the field then, as shown by Zeldovich (1957), $\langle|\mathbf{B}|^2\rangle \approx RmB_0^2$; i.e. the fluctuating field is significantly ($O(Rm^{1/2})$) stronger than the large-scale field. Turbulent diffusion occurs through the generation of small-scale fields that eventually are annihilated by molecular processes. However, the generation of small-scale fields is inextricably linked with the generation of strong fields. If the fields so generated attain equipartition with the energy-containing eddies of the flow at scales larger than the diffusive scale then severe inhibition of the diffusive process occurs. Indeed, the dynamic timescale, t_D, for the decay of the magnetic field satisfies

$$t_D \approx \frac{L^2}{\eta}\left(\frac{1}{Rm} + \frac{1}{M^2+1}\right) \tag{4}$$

(Cattaneo & Vainshtein 1991), where M is the Alfvénic Mach number of the large-scale field ($M > 1$ (< 1) implying that the large-scale field is less than (greater than) equipartition strength). The most significant feature of equation (4) is that, in accordance with the issues discussed above, only very weak large-scale fields ($M \lesssim Rm^{1/2}$) are needed to influence strongly the diffusive process.

Cattaneo (1994) has reconsidered this problem to investigate the physical mechanism behind the suppression of diffusion. Taylor (1921) showed that for a purely passive scalar contaminant, the turbulent diffusivity D could be expressed in terms of the Lagrangian displacement of fluid particles ξ as

$$D = \frac{1}{4}\frac{d}{dt}\langle \xi^2 \rangle. \tag{5}$$

Cattaneo (1994) showed that the role of a (weak) large-scale magnetic field is to suppress the tendency of particles to undergo a random walk and hence to suppress the turbulent diffusivity of the field.

2.2. Three-dimensional Flows

For three-dimensional flows the situation is more complicated with the possibility (forbidden in 2D) of dynamo action. As shown by Moffatt (1974), at infinite Rm, α may be obtained formally from the Cauchy solution as $\alpha = -d\langle \xi \cdot \nabla \times \xi \rangle / dt$, showing clearly that α, like the turbulent diffusivity, is also a transport coefficient. It is then certainly conceivable that the generation of strong magnetic fields on small scales could inhibit α in an analogous manner to the suppression of turbulent diffusion discussed above. To determine whether this is indeed the case requires a combination of rigorous analysis and careful numerical simulations.

3. Numerical Results and Discussion

Cattaneo, Hughes & Thelen (2001) have considered the dependence of α on both B_0 and Rm, for the flow investigated by Cattaneo & Hughes (1996). Their results strongly support a nonlinear α-effect of the form of equation (2). Despite these compelling results, the subject, nonetheless, remains controversial and a number of papers have appeared arguing instead for expression (1) (e.g. Field, Blackman & Chou 1999). A full discussion of the issues involved, some of which are quite complex and beyond the scope of a brief communication such as this, will appear in our forthcoming papers.

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