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Synthesis of Small and Large Scale Dynamos

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Abstract. Using a closure model for the evolution of magnetic correlations, we uncover an interesting plausible saturated state of the small-scale fluctuation dynamo (SSD) and a novel anology between quantum mechanical tunnelling and the generation of large-scale fields. Large scale fields develop via the α-effect, but as magnetic helicity can only change on a resistive timescale, the time it takes to organize the field into large scales increases with magnetic Reynolds number. This is very similar to the results which obtain from simulations using the full MHD equations.

1. Fluctuating field dynamics

The dynamics of the fluctuating magnetic field $\mathbf{B}$, is governed by the induction equation. The velocity is assumed to be the sum of a Gaussian random, delta-correlated in time $\mathbf{v}_T$, and an ambipolar diffusion type component $\mathbf{v}_D = a[(\nabla \times \mathbf{B}) \times \mathbf{B})]$. (Here $a = \tau/(4\pi\rho)$, τ is some response time, and ρ is the fluid density). Assuming that the magnetic field is also Gaussian random, Subramanian (1997, 1999; S99) derived closure equations for the longitudinal correlation function $M(r,t)$ and the correlation function for magnetic helicity density, $N(r,t)$. The random $\mathbf{v}_T$ has a longitudinal correlation function $T(r)$ and a correlation function for the kinetic helicity density, $C(r)$. Defining the operators $\tilde{D}(f) = (1/r^4)\,(\partial(r^4 f)/\partial r)$, and $D(f) = (\partial f/\partial r)$, we then have,

$$\dot{M} = 2\tilde{D}(\eta_{\mathrm{T}} DM) + 2GM + 4\alpha H; \quad \dot{N} = -2\eta_{\mathrm{T}} H + \alpha M, \tag{1}$$

where $H = -\tilde{D}DN$ is the correlation function of the current helicity and $G = -\tilde{D}DT$ is the effective induction. Also $\alpha = \alpha_0(r) + 4aH(0,t)$ and $\eta_{\mathrm{T}} = \eta + \eta_0(r) + 2aM(0,t)$ are functions resembling the usual α-effect and the total magnetic diffusivity. Here $\alpha_0(r) = -2[C(0) - C(r)]$ and $\eta_0(r) = T(0) - T(r)$, and η is the microscopic diffusivity. Note that at large scales $r \to \infty$, $\alpha \to \alpha_\infty = -2C(0) + 4aH(0,t)$ and $\eta_T \to \eta_\infty = \eta + T(0) + 2aM(0,t)$. The α-effect suppression is similar to the α-suppression formula first found by Pouquet et al. (1976); η_T is however enhanced by the growing field energy, as in ambipolar diffusion.

1.1. Small-scale dynamo saturation

The SSD problem, with $C(r) = 0$, $a = 0$ has solutions with $H(r,t) = 0$, and was first solved by Kazantsev (1968). A transformation of the form $\Psi(r)\exp(2\Gamma t) = r^2\sqrt{\eta_T} M(r,t)$, maps the problem of getting $\Gamma > 0$ modes into a bound state

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problem in time-independent quantum mechanics (QM). For turbulent motions on a scale L, with a velocity v, bound states obtain provided the magnetic Reynolds number (MRN) $R_m = vL/\eta > R_c \approx 60$, and imply M growing at rate $\sim v/L$. Further, the bound-state eigenfunction describes a field which is strongly concentrated within the diffusive scale, $r = r_d \approx L(R_m)^{-1/2}$, and curved on the scale L. For $a \neq 0$, η is simply replaced by an effective, time dependent diffusion $\eta_D = \eta + 2aM(0,t)$. So as the field (and M) grows, the effective MRN $R_D(t) = vL/\eta_D(t)$ is driven to the critical value R_c. The final saturated state is obtained when $R_D(t_s) = vL/(\eta + 2aM_L(0,t_s)) = R_c \sim 60$. So at saturation, the average energy density $E_B(t_s) = (3M_L(0,t_s)/8\pi) = (3/2)(\rho v^2/2)(L/v\tau)(1/R_c)$. For $\tau \sim L/v$, E_B is a small fraction $\sim R_c^{-1} \ll 1$, of the equipartition value. If we interpret the saturated field configuration in terms of flux ropes with peak field B_p, thickness r_d, and curved on scale L, then $E_B \sim (B_p^2/8\pi)Lr_d^2/L^3$. Using $r_d^2/L^2 \approx R_c^{-1}$, and $\tau \sim L/v$, we then have $B_p^2/8\pi \sim \rho v^2/2$, where, remarkably, the R_C^{-1} dependence has disappeared. So the SSD could saturate with the small-scale field of equipartition strength, being concentrated into flux ropes of thickness $LR_c^{-1/2}$, and curved on scale L.

1.2. Large scale dynamo as a tunnelling problem

For helical turbulence with $C(0) \neq 0$, new generation terms arise at $r \gg L$, due to the α-effect, in the form $\dot{M} = ... + 4\alpha_\infty H$ and $\dot{N} = ... + \alpha_\infty M$. These lead to the growth of large-scale correlations on a scale D, with a growth rate $\sim \alpha_\infty/D - \eta_\infty/D^2$, as in the large-scale α^2–dynamo. A special wavenumber, $k_{\rm p}(t) = \alpha_\infty(t)/\eta_\infty(t)$, is also picked out for any quasi-stationary state. For such states, which obtain when one neglects slow resistive evolution (see below), $(\partial N/\partial t) \approx 0$, implying $H \approx (\alpha/2\eta_T)M$. And if we define $\Psi = r^2\sqrt{\eta_T}M$, the equation $(\partial M/\partial t) \approx 0$, can again be mapped to a QM potential problem, for the zero-energy eigen-state. However the modified potential now tends to a negative definite constant value of $-\alpha_\infty^2/\eta_\infty$ at large r and so allows tunnelling (of the bound state) (see S99). In fact for $r \gg L$, one has an analytic solution $M(r) \propto r^{-3/2}J_{3/2}(k_{\rm p}r)$, exactly as one would get if the large scale field, $\mathbf{B}_0$, was random and force-free with $\nabla \times \mathbf{B}_0 = k_{\rm p}\mathbf{B}_0$.

2. Helicity constraint and resistively limited growth

The closure equations have also been solved numerically by Brandenburg and Subramanian (2000) (BS2K), adopting forms for $T(r)$ and $C(r)$ to match closely with the direct simulation of Brandenburg (2000, hereafter B2000) (Run 5). One sees an initial exponential growth of the magnetic field, which terminates when its energy becomes comparable to the kinetic energy. Note that our closure equations satisfy the helicity constraint $\dot{N}(0) = -2\eta H(0)$. The numerical solutions show that, after some time $t_{\rm s}$, the current helicity $\langle \mathbf{J} \cdot \mathbf{B} \rangle \propto H(0,t)$, is driven to a constant value which however is such that $|\alpha_\infty|$ remains finite. A constant $H(0,t)$ implies that the magnetic helicity $\langle \mathbf{A} \cdot \mathbf{B} \rangle \propto N(0,t)$ grows linearly at a rate proportional to η. During this phase the magnetic field correlations can extend to larger and larger scales. The corresponding magnetic energy spectra, $E_{\rm M}(k,t) = (1/\pi) \int_0^\infty M(r,t)\,(kr)^3\, j_1(kr)\, dk$ are shown in Figure 1.

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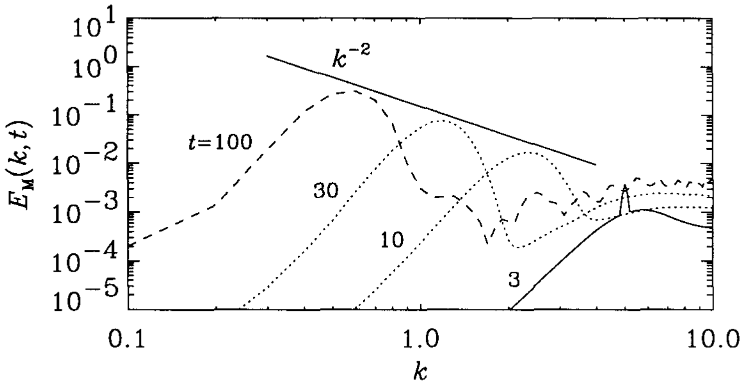

Figure 1. Evolution of magnetic energy spectra. Note the propagation of magnetic helicity and energy to progressively larger scales. The k^{-2} slope is given for orientation. (see BS2K)

The resulting magnetic field is strongly helical (cf. Section 1.2) and the magnetic helicity spectra (not shown) satisfy $|H_{\rm M}| \lesssim (2/k)E_{\rm M}$. The development of a helicity wave travelling towards smaller and smaller k, as seen in Figure 1, is in agreement with the closure model of Pouquet et al. (1976). We have also checked that to a very good approximation the wavenumber of the peak is given by $k_{\rm peak}(t) \approx k_p(t)$, as expected from Section 1.2, and it decreases with time because α_∞ tends to a finite limit and η_∞ increases. Further, since the large scale field is helical, and since most of the magnetic energy is by now (after $t = t_{\rm s}$) in the large scales, the magnetic energy is proportional to $\langle \mathbf{B}^2 \rangle \approx k_{\rm p} \langle \mathbf{A} \cdot \mathbf{B} \rangle$, and can therefore only continue to grow at a resistively limited rate. These results are analogous to the full MHD case (B2000); the helicity constraint is independent of the nature of the feedback! In conclusion, our closure model with ambipolar diffusion type non-linearity provides a useful model, enabling progress to be made in understanding nonlinear dynamos. One now needs to think of ways to relax the helicity constraint (cf. Blackman and Field (2000), Kleeorin et al. (2000)), so that large-scale magnetic fields can grow rapidly enough.

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