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Taxation and policyholder behavior: the case of guaranteed minimum accumulation benefits

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Abstract
This paper considers variable annuity (VA) contracts embedded with guaranteed minimum accumulation benefit (GMAB) riders when policyholder's proceeds are taxed upon early surrender or maturity. These contracts promise the return of the premium paid by the policyholder, or a higher rolled-up value, at the end of the investment period. A partial differential equation valuation framework which exploits the numerical method of lines is used to determine fair fees that render the policyholder and insurer breakeven. Two taxation regimes are considered: one where capital gains are allowed to offset losses and a second where gains do not offset losses. Most insurance providers highlight the tax-deferred features of VA contracts. We show that the regime under which the insured is taxed significantly impacts prices. If losses are allowed to offset gains then this enhances the market, increasing the policyholder's willingness to participate in the market compared to the case when losses are not allowed to offset gains. With fair fees from the policyholder's perspective, we show that the net profit is generally positive for insurance companies offering the contract as a naked option without any hedge. We also show how investment policy, as reflected in the Sharpe ratio, impacts and interacts with policyholder persistency.

1. Introduction and motivation

Variable annuities (VAs) are notoriously popular in the US where the net asset value is approximately \$2.08 trillion as of June 2021 (Insured Retirement Institute, 2022). Conversely, there is a very thin market for VAs in Australia and Europe.[1] The VA market is relatively immature in Australia.[2] In Europe, the VAs' market was worth 188 billion in 2010 (EIOPA, 2011). However, after the Global Financial Crisis, their popularity decreased and various life insurers stopped offering such contracts.[3] VAs are among the

[1]We acknowledge that besides tax frictions there are unique factors in each market that might render supply of these products difficult. To name a few, in Germany, VA-type products are not popular due to a lack of fair value accounting (Russ and Kling, 2006). In Belgium, life insurance products with guarantees are liable to offer a minimum return guarantee, updated yearly by the Belgian Financial Market Authority, adding an additional layer of complexity and variability to any life insurance offering (FSMA, 2023).

[2]There are only a few notable players which includes AMP Financial Services, BT Financial Group, and MLC (Vassallo *et al*., 2016).

[3]For instance, Prudential's GMAB offering Pru Flexible retirement has been closed for business since 2018 (Prudential UK, 2018).

few assets which grow tax-deferred within the US and Australia.[4] Indeed, investors willing to save more than the guaranteed pension employer contributions can invest in a VA, gaining exposure to the equity markets, profiting from a tax-deferred investment to then annuitizing the account value upon retirement (Stanley, 2021).

VAs offer an opportunity to participate in the equity market while providing minimum guarantees in case of poor market performance. We focus on guaranteed minimum accumulation benefits (GMABs) which promise the return of the premium payment, or a higher rolled-up value at the end of the accumulation period of the contract.[5] The policyholder can surrender their contract anytime prior to maturity, incorporating often underestimated lapse risk. This is the risk that policyholders exercise their surrender options at a different rate than assumed at inception of the contract. Indeed, Moody's Investor Service (2013) highlights that underpricing lapse risk leads to significant write-downs and earnings charges for insurers.

Taxation levels are known to affect household financial behavior, yet few studies focus on the effect of institutional settings on the demand of insurance products including VAs. All proceeds for the policyholder, be it at maturity or surrender, are assumed to be taxed creating a valuation wedge between the insurer and policyholder. We study the effect of three taxation arrangements: no tax, losses offset (or not), and other capital gains on VAs. We aim to identify the extent to which taxation structures affect the demand for VAs and whether this might explain the lower popularity of such contracts. We find that allowing for losses to offset gains increases policyholder values and fees they are willing to pay for the contract, whereas the no offset case decreases the value and hence fee. We find low to no demand for some contract specifications. Yet, when fair fees from the policyholder's perspective are applied to the VA contracts, insurer's net average profit is positive in almost all taxation settings and median profit is positive in all taxation settings, and they are particularly affected by financial market parameters driving the dynamics of the underlying fund.

The bulk of existing literature has focused on risk-neutral valuation of VA contracts using a variety of techniques without considering income and wealth tax. Bauer *et al.* (2008), Bacinello *et al.* (2011), and Kélani and Quittard-Pinon (2017) provide universal pricing frameworks for various riders embedded in VA contracts when the underlying fund dynamics evolve under the influence of geometric Brownian motion (GBM) and Lévy markets, respectively. Incorporating a surrender option is a recent development that addresses the underpricing of lapse risk.[6] Bernard *et al*. (2014) note that it can always be optimal for the policyholder to surrender the contract anytime prior to maturity if the underlying fund value exceeds a certain threshold. As a means of disincentivizing early surrender, the authors consider an exponentially decaying surrender charge and use numerical integration techniques to determine optimal surrender boundaries. Such a penalty is needed as the possibility to lapse renders the product more profitable for the policyholder at the expense of the insurer (Piscopo and Rüede, 2018).[7] Various authors have since extended the framework in Bernard *et al.* (2014) to incorporate realistic market dynamics

[4]Both countries have a high share of private occupational or private pension savings to finance retirement. The Australian superannuation system, similar to 401(k) plans, is valued at $3 trillion as of December 2019 and is projected to increase to $3.5 trillion by 2020 (The Association of Superannuation Funds of Australia Limited, 2020). Maximizing their retirement savings is a high-stakes problem in both countries.

[5]There are various types of guarantees embedded in variable annuity contracts, and these can be classified into two broad categories, namely guaranteed minimum living benefits (GMLBs) and guaranteed minimum death benefits (GMDBs). GMLBs can be further divided into four subcategories as follows: guaranteed minimum accumulation benefits (GMABs), guaranteed minimum income benefits (GMIBs), and guaranteed minimum withdrawal benefits (GMWBs). A GMIB guarantees an income stream upon maturity of a GMAB for a given term if the policyholder chooses to annuitize. A GMWB guarantees a certain level of withdrawals during the life of the contract.

[6]Bauer *et al.* (2017) review the state of affairs with regard to the theoretical and empirical insights of policyholder behavior in variable annuities, including lapse risk.

[7]Others strive to avoid policyholder's surrender altogether by considering state-dependent fees, that is, fees that are paid when the account value is below a certain threshold (MacKay *et al*., 2017; Moenig and Zhu, 2018; Bernard and Moenig, 2019). Preliminary research indicates that for a 1-year maturity it might still make the product profitable, but the full effect of state-dependent fees on hedging must be investigated further in depth (Delong, 2014).

and computationally efficient methods.[8] These valuation frameworks determine fees which lie much higher than those observed in the market partly because taxes are not considered. Our general setting, considering two tax regimes, can be simplified to assess the classical case in the literature where taxes are not considered.

However, it is well known that taxes affect household financial behavior. Souleles (1999), Johnson *et al*. (2006), and Parker (1999) show that US households' consumption is significantly affected by income tax refunds as well as changes in social security taxes, covering old age survivor and disability insurance (OASDI) and health insurance (DI), respectively. These findings contradict classical life cycle theory as these tax-related cash flows are expected and considered in their optimal decision-making. Taxes also influence how to finance savings. Multiple studies show that taxes should affect portfolio allocation and asset holding in tax-deferred accounts.[9] However, as highlighted in Poterba (2002), little attention has been paid to the effect of institutional setting taxation on the demand of insurance products. The few studies focusing on this, such as Gruber and Poterba (1994), Gentry and Milano (1998), and Gentry and Rothschild (2010), note that tax incentives enhance the demand of health insurance for self-employed, VAs, and life annuities, respectively. Similarly, Horneff *et al*. (2015) show that purchasing VAs embedded with GMWB riders would increase when taxes are deferred, enhancing the welfare of retirees.

Taxation effects have been highlighted as possible explanation to the mismatch between theoretical and empirical values of VAs (Milevsky and Panyagometh, 2001; Brown and Poterba, 2006). Indeed, Moenig and Bauer (2015) resolve this partially by noting that incorporating taxation in the risk-neutral valuation of GMWB riders yields fees that closely match empirically observed values. In a subsequent paper, Bauer and Moenig (2023) find that providers can attach free death benefit riders to guaranteed minimum benefits as a strategy to disincentivize early surrender when income and capital gains taxation are considered. Ulm (2018) also highlight that, for the same taxation regime, the timing of tax affects VA policyholder's value, with taxation at maturity being more advantageous than taxation whenever proceeds are earned. In the same vein, this paper examines the impact of taxation on the optimal surrender boundaries for a GMAB when the policyholder behaves rationally with respect to the post-tax value of the contract and we find that the presence of taxation drives a substantial wedge between policyholder and insurer valuations.

These recent findings indicate that individuals might behave rationally with respect to their aftertax benefits. However, a fruitful strand of literature indicates that households do not behave rationally with respect to their financial planning and accumulation of retirement savings or retirement income product purchase, and that this may be due to lack of financial literacy (Lusardi and Mitchell, 2011, 2014; Bateman *et al*., 2018) or limited opportunities for the current generation to engage in social learning (Bernheim, 2002). However, the same literature on financial literacy indicates that high-income individuals and households score higher in financial literacy and numeracy measures, and this holds across most developed countries (Lusardi and Mitchell, 2011). This also translates to complex product ownership[10] and better financial decision-making (Agnew, 2006). Since VA ownership is more prevalent in high-income households (Brown and Poterba, 2006), we focus on high-income individuals marginal rate of taxation.

[8]Examples are Ignatieva *et al.* (2016) who provide a fast and efficient framework for valuing guaranteed minimum benefits using the Fourier space time-stepping algorithm and Kang and Ziveyi (2018) who incorporate stochastic volatility and stochastic interest rates and solve the pricing with surrender resulting free boundary problem using the method of lines.

[9]See Black (1980) and Tepper (1981) for their seminal work or Fischer and Gallmeyer (2016) for a recent review of the extensions to the Tepper–Black model. Chen *et al.* (2019) show that life insurance contracts with guarantees contracts lead to a higher expected utility level than traditional long positions in stocks when tax incentives are considered.

[10]Indeed, Poterba and Samwick (2003) indicate that households share of tax-advantaged assets increase with marginal income tax rate. Similarly, Inkmann *et al*. (2010) find that annuity ownership in the UK increases with financial wealth. These households are also more likely to seek financial advice (Finke et al., 2011; Hackethal *et al.,* 2012; Calcagno and Monticone, 2015) and hence benefit from tax management (see e.g., Hackethal *et al.,* 2012 for Germany and Cici *et al.,* 2017 for the US).

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The remainder of the paper is structured as follows: Section 2 presents the partial differential valuation problem to be solved with the aid of the method of lines algorithm. Section 3 analyzes the effect of the tax treatments (no tax, offset, and no offset) on insurer liabilities and policyholder contract values. A study of surrender is also performed. Sensitivities to the main financial market parameters are presented in Section 4. Finally, Section 5 analyzes the profit and loss statements of these products under the various tax regimes considered to assess the impact of the moneyness and tax. Concluding remarks are presented in Section 6.

2. Model and valuation approach

In this section, we provide the valuation framework for a VA contract embedded with a GMAB. We utilize a partial differential equation approach which is solved with the aid of a fast and accurate method of lines algorithm.

2.1. VA embedded with a GMAB

A GMAB rider discussed in this paper involves a policyholder entering into a VA contract by investing an initial amount x_0 into a mutual fund. Upon maturity of the contract, the policyholder is promised the greater of the minimum guarantee on the premium that is determined by a fixed continuously compounded guarantee growth rate, δ, $G(\delta) = x_0 e^{\delta T}$, and the fund value. The growth rate, commonly known as "roll-up," is typically applied to avoid diluting the value of the insurance feature as the years pass by. This is of special relevance for long maturity contracts such as VAs embedded with GMAB.[11] Due to no-arbitrage, we require that $\delta \leq r$. The existence of fair fees may impose an even stronger constraint on δ.

In order to finance management, guarantee, and transaction costs associated with providing the contract, we assume that the insurer charges a continuously compounded fee at rate q which is deducted as a percentage of the fund. We suppose that the underlying fund (S_ν)[12] follows a standard GBM[13] under the risk-neutral measure such as $dS_\nu = rS_\nu d\nu + \sigma S_\nu dW_\nu$. Here, r is the risk-free interest rate and σ is the volatility of the underlying fund. The investment component of the VA (x_ν) can be expressed as $x_\nu = e^{-q\nu} S_\nu$ where q is the continuously compounded management fee levied on the fund. Applying Ito's Lemma to the process x_ν yields the following dynamics:

$$dx_\nu = (r - q)x_\nu d\nu + \sigma x_\nu dW_\nu. \tag{2.1}$$

Upon maturity of the contract, the payoff of the policyholder can be represented as:

$$h(x_T, T) = [\max(G(\delta), x_T) - x_0 - C_0]_+ \tag{2.2}$$

$$= [\underbrace{[G(\delta) - x_T]_+}_{\text{capital gain on guarantee}} + \underbrace{x_T - x_0 - C_0}_{\text{capital gain on fund, net of all fees}}]_+ \tag{2.3}$$

where $[z]_+ = \max(z, 0)$ with x_0 being the initial value of the investment account. All other costs other than management fees for being invested in the VA contract are captured through C_0.[14] This cost is deductible for tax purposes only at maturity or surrender. This aligns with how tax is treated in Australia as all net

[11] Most papers on GMAB/GMMB in the literature do not model the roll-up. For instance, Delong (2014), Moenig and Bauer (2015), Moenig and Zhu (2018), MacKay et al. (2017), and Bernard and Moenig (2019) all study return-on-premium, that is $\delta = 0$.

[12] We reserve the use of t for the time to maturity and τ for the tax rate. Therefore, we use ν to denote the time elapsed since the inception of contract.

[13] We use GBM despite its pitfalls, such as the underestimation of the tails of the asset return distribution. However, an equivalent analysis would naturally follow with the use of sophisticated modeling frameworks.

[14] For instance, for the Australian MyNorth Investment product, these fees appear under the name "additional advice fees," see page 19 of AMP (2020).

losses can be carried forward to later income years (Australian Taxation Office, 2021). As income is only received upon surrender or maturity, it is reasonable to assume that C_0 is deducted at that moment.

We assume that the tax-deductible upfront costs C_0 do not include any commissions to third parties. That is, we assume that any commission paid to intermediaries is either borne by the insurer or not deductible for tax purposes if borne by the insured as it relates to separate services, such as the provision of financial advice. If commissions are considered on top of C_0, it would change the value to the policyholder but not the insurer liability. This is easily accommodated within our framework by solving for policyholder value equals

$$x_0 + C_0 + \text{commissions}$$

when determining fair fees.

We assume two tax treatments for losses. First, we assume that the taxable income cannot be negative in this case because capital losses incurred on the VA account cannot be offset against other income to reduce total taxes paid. This is in line with the approach in Moenig and Bauer (2015) in which there are no offsetting investments and capital losses are not incurred in the GMWB product. The case when losses offset gains is presented as an extension that reflects the tax treatment in Australia.

In addition to this, the GMAB contract permits the policyholder to surrender early. Policyholders are not eligible for the guarantee if they surrender early (Kang and Ziveyi, 2018). If the policyholder surrenders the contract at time ν from the inception of the contract, the insurer will pay $\gamma_\nu x_\nu$ from the investment account. Here, $(1-\gamma_\nu)$ is charged as a percentage of the current fund value. In the event of early surrender at time ν, the taxable income will thus be

$$[\gamma_\nu x_\nu - x_0 - C_0]_+. \tag{2.4}$$

In what follows, we will assume an exponentially decreasing surrender fee structure such that $\gamma_\nu = e^{-\kappa(T-\nu)}$. Let $u^p(x, \nu)$ be the value of the investment account to the policyholder where, as above, x represents the fund value and the time elapsed since the inception of the contract is ν. Therefore, the governing partial differential equation is the Black–Scholes equation that can be represented as:

$$\frac{1}{2}\sigma^2 x^2 u^p_{xx} + (r-q)\cdot x u^p_x - r u^p - u^p_t = 0. \tag{2.5}$$

Note that we have applied the transformation $t = T - \nu$ where t represents the time to maturity on the contract. We consider taxes on the boundary condition of the policyholder's valuation function, where the policyholder elects to surrender or receives the final payout from the GMAB contract. Detailed derivations have been relegated to Appendix A. In order to obtain the contract value from the policyholder's perspective, Equation (2.5) is solved subject to the following boundary conditions:

$$u^p(x, 0) = \max(x, G(\delta)) - \tau\left[\max(x, G(\delta)) - x_0 - C_0\right]_+, \tag{2.6}$$

$$u^p(s(t), t) = \gamma_{T-t} s(t) - \tau\left[(s(t)\gamma_{T-t} - x_0 - C_0\right]_+, \tag{2.7}$$

$$u^p(0, t) = (x_0 e^{\delta T} - \tau[x_0 e^{\delta T} - x_0 - C_0]_+)e^{-rt}, \tag{2.8}$$

$$u^p_x(s(t), t) = \gamma_{T-t} - \tau\gamma_{T-t}\mathbb{I}\left\{(s(t)\gamma_{T-t} - x_0 - C_0) > 0\right\}, \tag{2.9}$$

where γ_{T-t} is the proportion that the policyholder is allowed to keep subsequent to surrender, x_0 is the initial fund value (i.e., the "premium"), τ is the tax rate, $G(\delta)$ is the guarantee amount at maturity, and $s(t)$ is the minimum fund value to trigger surrender, given that there are still t years to maturity. The free boundary, $s(t)$, must be computed along with the valuation solution $u(x, t)$. The first two boundary conditions, Equations (2.6) and (2.7), represent the post-tax payoff at maturity or upon surrender, respectively, which occurs for rational agents when the fund value x exceeds $s(t)$. Equation (2.8) is the present value of the taxable income at maturity when the fund value is zero as given by Equation (2.2).

In that case, the guarantee is triggered and there is no incentive for the policyholder to leave the contract early.[15] Hence, in this case, the guarantee is paid with certainty and the payoff is deterministic. The final boundary condition, Equation (2.9), enforces the continuity of u_x at the boundary $x = s(t)$. If capital losses are allowed to offset gains, as is the case for nonqualified plans in the US (IRS, 2016) and Australian VAs, we replace $\left[\,\ldots\,\right]_+$ by $\left[\,\ldots\,\right]$ in the boundary conditions (2.6), (2.7), (2.8) and (2.9).

Insurer's perspective

As highlighted above, tax is a friction that distorts the valuation of the contract. This yields different results for the policyholder and insurer. The government receives a proportion of the payout, either at surrender or maturity, creating a gap between the value for the policyholder and the insurer's liabilities. To obtain the value of the contract from the insurer's perspective, henceforth to be referred to as the insurer's liabilities, the partial differential Equation (2.5) must be solved subject to boundary conditions which reflect the total before tax payments the insurer must make to the policyholder. The boundary conditions are equal to those presented in Equations (2.6)–(2.9) when $\tau = 0$. In this case, the initial net profit of the insurer is $x_0 + C_0 - u^i$, where x_0 is the initial premium paid by the policyholder and u^i is the value of the insurer's liabilitites.

Fair fee

In presence of taxation, the fee that renders the contract fair for the policyholder might differ from the insurer's fee. However, when $\tau = 0$, the fair fees obtained by solving the Partial Differential Equation (PDE) (2.5) subject to either the policyholder or insurer boundary conditions will be the same. We denote the policyholder fair fee q^p as:

$$q^p = \min\{q : x_0 + C_0 = u^p(x_0, T)\}. \tag{2.10}$$

This is the minimum fee rate such that the value of the contract at inception, when the time to maturity t is T, is equal to the initial premium paid by the policyholder. In other words, the net profit to the policyholder is zero. Similarly, the insurer perspective fair fee rate q^i can be determined implicitly as:

$$q^i = \min\{q : x_0 + C_0 = u^i(x_0, T)\}. \tag{2.11}$$

It is the smallest fee rate such that at inception of the contract when $t = T$, the liabilities of the insurer are equal to the initial amount they receive from the policyholder. This sets the net profit of the insurer to be 0.

2.2. Implementation and calibration

In order to solve Equation (2.5) subject to the initial and boundary conditions (2.6)–(2.9), we utilize the numerical method of lines algorithm. This is accomplished by truncating to the computational domain such that $\{(x, t) \in [0, X] \times [0, T]\}$.

It is well known that the method of lines is a fast, accurate, and efficient algorithm for solving such free boundary problems (Meyer and Van der Hoek, 1997; Chiarella *et al.*, 2009; Kang and Ziveyi, 2018). To obtain the contract values, Equation (2.5) is discretized in the t direction and continuity is maintained in x. Time is discretized uniformly starting at inception t_0 up to maturity T. Appendix B describes the step-by-step implementation of the method of lines algorithm used for the valuation of the contract. Since this algorithm provides contract values, we can find fair fees using the bisection method.

The first row in Table 1 shows the parameters used for the base case analysis in Section 3 where only the tax rate and regime are allowed to vary, removing any confounding effects. We further analyze the effect of the roll-up rate δ and maturity T. The parameters are calibrated using Australian market data. We select r based on the historical average of the cash rate in Australia, from 2009 to 2018, and σ based on ASX200 VIX index from 2009 to 2018. These values also coincide with Moenig and Zhu (2018) and Bernard and Moenig (2019). The marginal tax of τ is calculated based on the 0.45 marginal income

[15]This scenario is impossible in practice; however, the boundary condition is necessary for a well-posed problem.

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Table 1. *Financial base case parameters (first row) and sensitivity analysis (second row).*

	r	σ	τ	x_0	δ	T	κ
Base case	3%	20%	22.5%	1	0.75%	5	0.5%
Sensitivity	2.5%, 3.5%	15%, 25%	17.5%, 27.5%	–	[0%, 1.5%]	10, 15	0%,1%

Notes: the base case parameters (r, σ, τ) are calibrated using Australian market data and align with Moenig and Zhu (2018) and Bernard and Moenig (2019). The product specification (δ, T, κ) chosen aligns with the literature (see e.g., Shen *et al.*, 2016). Sensitivites to each parameter are shown in the second row. The initial premium of x_0 is chosen as 1 for convenience.

Table 2. *Contract values for the base case parameters in Table 1 and policyholder fair fee* $q = 0.0023$ *which illustrates numerical convergence.*

$\Delta t \backslash \Delta x$	10^{-1}	10^{-2}	10^{-3}	10^{-4}
10^{-1}	1.0748	1.0705	1.0706	1.0706
10^{-2}	0.9211	0.9891	1.0715	1.0705
10^{-3}	0.9210	0.9891	1.0749	1.0708

tax rate, multiplied by the discount of 0.50 for capital gains.[16] The value of x_0 is chosen to be unit as a convenient numerical value, since it is only the ratio $\frac{G(\delta)}{x_0} = e^{\delta T}$ which affects pricing. The maturity is assumed to be 5 years (with sensitivities at 10 and 15 years)[17] and the surrender penalty is chosen to be $\kappa = 0.5\%$ with sensitivities to 0% and 1% following Shen *et al*. (2016). The initial fee is chosen to be $C_0 = 7\%$[18]. Unless otherwise stated, these parameters will be used throughout the remainder of the paper. The second row of Table 1 shows the sensitivities that we will consider in Section 4. These allow us to further investigate the interaction between market conditions and tax treatments.

In addition, the following numerical parameters are used for the method of lines algorithm with the spacing in the x grid given by $\Delta x = 10^{-4}$ and the spacing in the t grid being $\Delta t = 10^{-3}$. The upper limits of the x grids are set to be four times the initial premium, that is, $X = 4 \cdot x_0$. We provide some justification for the choice of Δt and Δx in Table 2. As evident in Table 2, it is reasonable to assume that the solution converges to the third decimal places for the selected values of Δx and Δt.

3. The effect of the tax treatment

This section analyzes the effect of the tax treatment on the contract value of the policyholder and insurer, the fair fee, and surrender boundaries for three tax treatments. First, we study the case where capital losses can be used to offset capital gains from other investments. Second, we delve into the case where capital losses cannot offset gains from other investments, which is the case in previous literature (Moenig

[16]In Australia, there is no separate tax rate for capital gains. Instead, the capital gains are added to taxable income and taxed at the regular marginal income tax rate. Furthermore, we assume that the policyholder is a high net worth individual and therefore all investment earnings will fall under the highest tax bracket. As of 2018 in Australia, individuals in the highest tax bracket with income of \$180,001 and over pay \$54,097 tax plus 45c for each \$1 over \$180,000 (Australian Taxation Office, 2022).

[17]GMAB product is typically purchased during the accumulation phase with the bulk of existing literature assuming a typical policyholder aged between 50 and 60 purchasing the product for a unique premium base as a strategy of having a minimum guaranteed retirement amount. Depending on the age at inception, maturity will vary from 5 to 15 for a retirement age fixed at 65 years. Comparing our parametrization to the literature, we observe that there is no consensus on which value of T to consider. For instance, Delong (2014) uses $T = 1$, Moenig and Zhu (2018) and Bernard and Moenig (2019) use $T = 25$, whereas Moenig and Bauer (2015) and MacKay et al. (2017) use intermediate maturities of $T = 15$ and $T = 10, 20$ respectively. We believe that the choice of $T = 5$, 10, and 15 sufficiently presents the sensitivity to the maturity of the product.

[18]This corresponds to the policy acquisition expense rate in Moenig and Zhu (2018) and Bernard and Moenig (2019) and aligns with a front-load of MyNorth administration fees AMP (2020).

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and Bauer, 2015). More specifically, if the tax base exceeds the payoff of the asset, then the difference may not be claimed as a capital loss for taxation purposes. This implies that the value of the contract is always positive to the government. Finally, we assess the case where no tax is considered, which aligns with classical literature that abstracts from tax.

3.1. Insurer liabilities and policyholder contract values

In this subsection, we discuss the impact of increasing the level of taxation to the insurer liabilities and policyholder contract values across the three taxation treatments: tax-free, offset, and no offset. From an insurer's perspective, it is the value which applies for accounting and regulatory capital considerations. Indeed, insurers have to hold certain funds notwithstanding the marginal tax rate that policyholders have to pay to the government. Of course, the policyholder's value equals the insurer's liability whenever $\tau = 0\%$, which is the canonical modeling framework in the VA literature. The first glance shows that the presence of tax, compared to $\tau = 0\%$, creates a wedge between the policyholder and insurer value that reflects the increasing value of the contract to the government as the tax rate increases. Figure 1 presents the contract values from the policyholder and liability curves for the insurer as a function of fees charged for varying marginal rates of taxation and three maturities. Figure 2 shows the contract values for a fixed tax rate of 22.5% and varying levels of the roll-up rate δ.

The first obvious finding is that the insurer's liability crosses $x_0 + C_0$ at much higher rates than the policyholder contract value[19] in presence of taxation for the two treatments. In particular, for high maturities T, the fair fee, calculated as in (2.11), often does not even exist. It is natural that the insurer will have to follow the policyholder fair fee whenever taxes are considered. Indeed, it is the policyholder's behavior which detects the state of the contract at any given time. These findings suggest that the presence of taxes could substantially affect the supply of these products since the fair fee for the policyholder lies below the insurer's implied fair fee. This indicates that tax incentives need to be studied carefully as their presence and design can distort the market. This is of particular importance in a political environment that stimulates higher reliance of individuals on pension funds or private investment to sustain their retirement.

We note that the value of the policyholder decreases with increasing fee charges; indeed, higher fees reduce the level of the underlying fund and potential gains from the product. Similarly, the liability of the insurer decreases with fees charged as higher fees lead to higher income, lowering the liability toward the policyholder for the same fixed guarantee. For longer maturities and higher fees, we observe that liabilities drop suddenly as a consequence of immediate surrender. For instance in the offset case of Figure 1(a), the contract will not be viable for $T = 15$ as a rational policyholder will immediately surrender upon underwriting for fees higher than 1.8% when $\tau = 27.5\%$ and 2.6% when $\tau = 22.5\%$. In these extreme cases, the policyholder surrenders the contract immediately, and this results in a large drop in the insurer's liabilities due to the surrender penalties. When losses cannot offset gains, Figure 1(b), a similar behavior appears for $T = 15$ and $T = 10$, triggered by even smaller fees than the offset case.

Whether policyholders prefer higher or lower taxes depend on the tax treatment and fee rate. Generally, the value to the policyholder decreases with tax. Indeed, all gains are taxed both in the offset and no offset case, reducing the attractiveness of the product. However, we observe in Figure 1(a) that for fee rates considerably higher than q^p, the contract value increases with taxation for the offset case. This is because at fee rates much larger than q^p, for which the numerical values are shown in Table 3, the policyholder can expect to pay high fees. Therefore, if the tax rate decreases, the policyholder obtains less value from the tax deduction associated with having paid such fees. On the other hand, Figure 1(b), charging greater than the fair fee does not increase the policyholder value when the tax rate increases. Indeed, contrary to the offset case, higher fee payments yield lower gains with no reimbursement from the government. This results in the policyholder's value function converging to the same level for all tax

[19]The fair fee for the policyholder is the one that crosses $x_0 + C_0 = 1.07$ as well.

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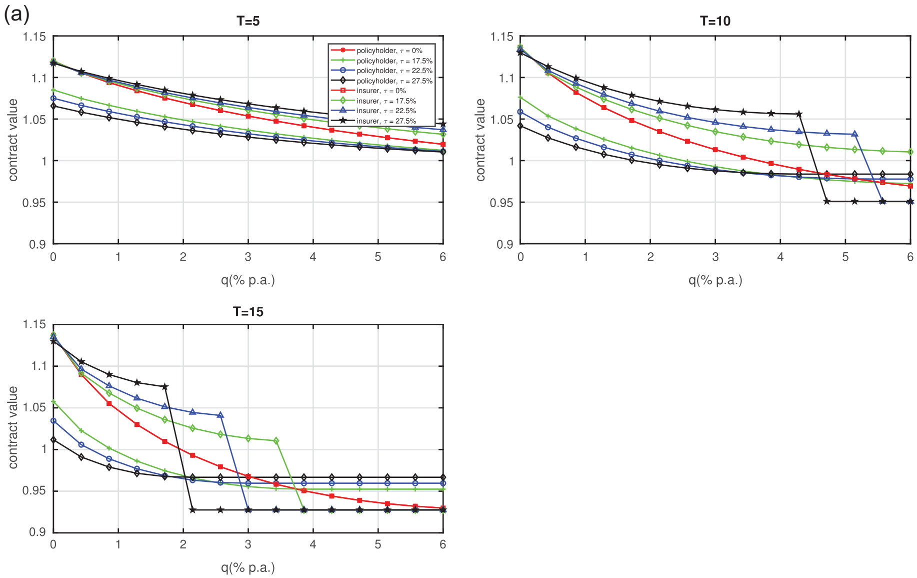
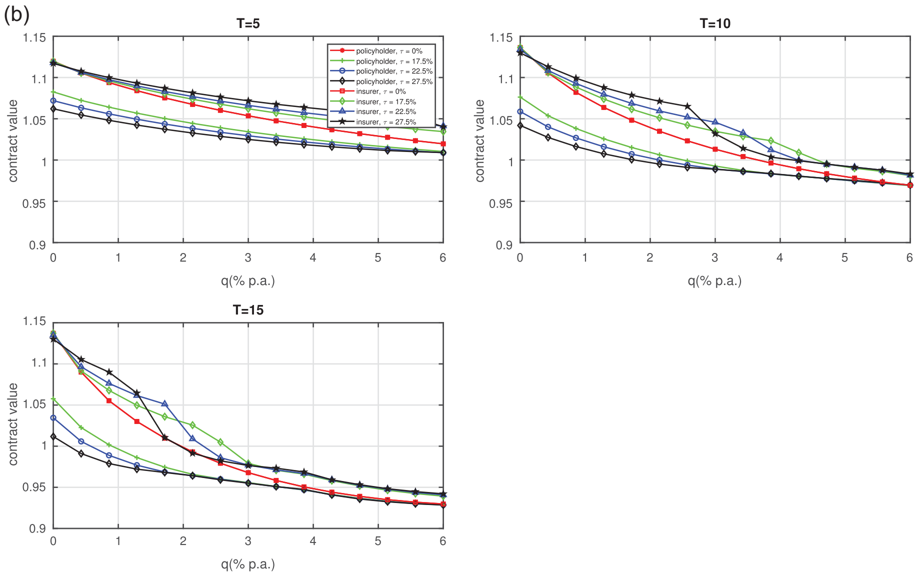


Offset: varying marginal rates of taxation τ



No offset: varying marginal rates of taxation τ

Figure 1. *Contract values from the policyholder and insurer perspective as a function of fees charged for varying marginal rates of taxation τ and maturity T. Financial parameters correspond to the base case of Table 1.*

rates considered, whereas in the offset case the value functions converge to a greater level for higher tax rates. This effect is more obvious when comparing to the tax-free case. Indeed, for high fees we observe that policyholder value in the offset case could be higher than when $\tau = 0\%$. However, for the no offset case, we observe that the policyholder value is always strictly lower than the tax-free case.

The insurer's liability increases with tax due to two reasons. First, fees decrease substantially in the presence of tax, lowering insurer's income for the same guarantee and boundary condition. Indeed, fees

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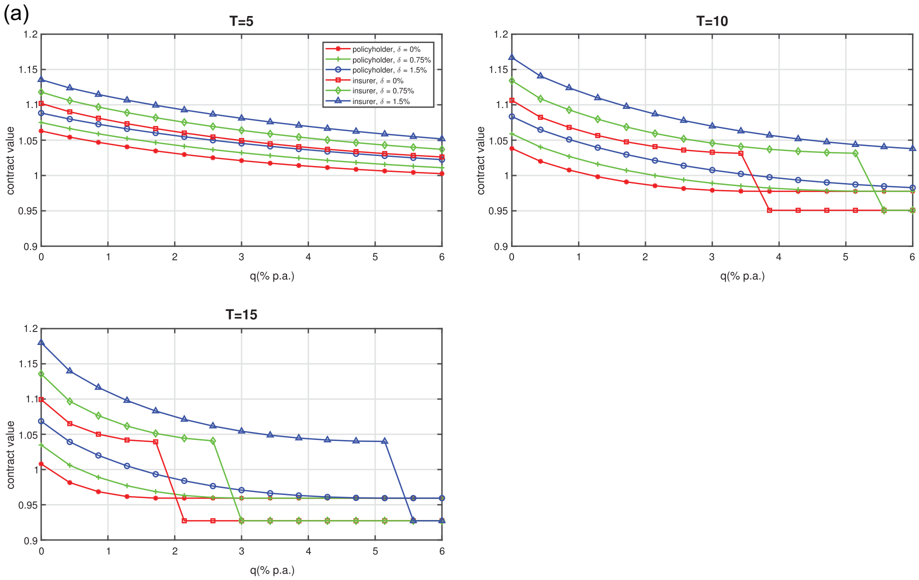
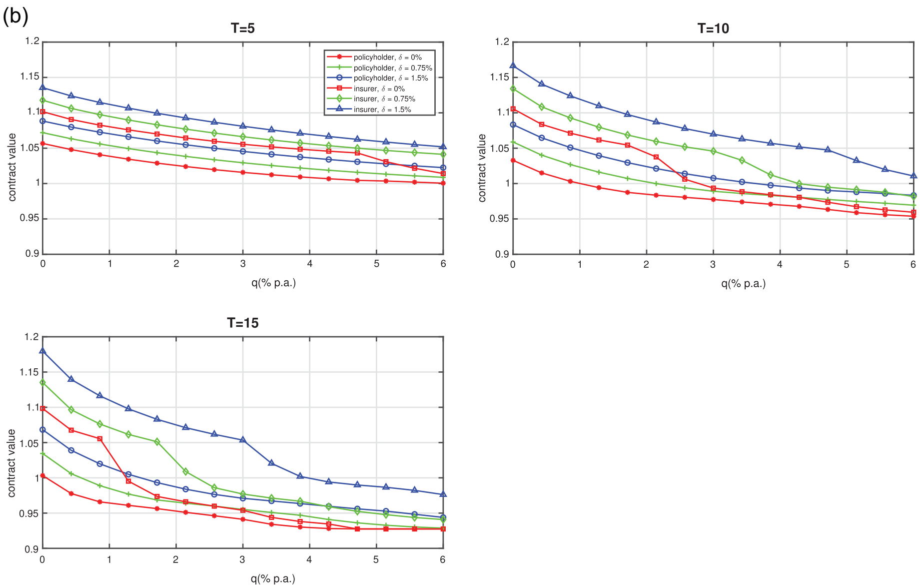


Offset: varying roll-up guarantee rates δ



No offset: varying roll-up guarantee rates δ

Figure 2. *Contract values from the policyholder and insurer perspective as a function of fees charged for varying roll-up rates δ and maturity T. Other parameters correspond to the base case of Table 1.*

decrease by at least two-thirds, greatly affecting the insurer's profitability. Secondly, surrender boundaries as presented in Subsection 3.2 increase substantially in presence of tax for the two treatments, increasing the likelihood of having to pay the guarantee. However, a clear difference arises when the no offset case is considered. We observe, Figure 1(a) versus (b), that the policyholder value functions and insurer liabilities converge to similar levels when high fees are charged. Of course, in that case taxable income would become zero and the level of taxation becomes irrelevant. The convergence for the insurer is erratic, as the impossibility to offset further acts as a friction to contract feasibility.

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Table 3. *Fair fees (% p.a.) for the policyholder (q^p) at various tax rates (τ%) and roll-up guarantees (δ%). Other parameters correspond to the base case of Table 1.*

a Offset allowed

	$T=5$							$T=10$							$T=15$						
	δ							δ							δ						
τ	0	0.25	0.5	0.75	1	1.25	1.5	0	0.25	0.5	0.75	1	1.25	1.5	0	0.25	0.5	0.75	1	1.25	1.5
0	1.16	1.41	1.69	2.00	2.34	2.73	3.17	0.76	0.97	1.22	1.50	1.83	2.21	2.66	0.27	0.37	0.49	0.64	0.85	1.10	1.41
17.5	0.07	0.23	0.43	0.66	0.92	1.21	1.54	n.a.	n.a.	0.06	0.25	0.48	0.75	1.06	n.a.	n.a.	n.a.	n.a.	n.a.	0.11	0.26
22.5	n.a.	n.a.	0.03	0.23	0.46	0.73	1.02	n.a.	n.a.	n.a.	n.a.	0.06	0.29	0.56	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
27.5	n.a.	n.a.	n.a.	n.a.	n.a.	0.21	0.47	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	0.06	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

b No offset allowed

	$T=5$							$T=10$							$T=15$						
	δ							δ							δ						
τ	0	0.25	0.5	0.75	1	1.25	1.5	0	0.25	0.5	0.75	1	1.25	1.5	0	0.25	0.5	0.75	1	1.25	1.5
0	1.16	1.41	1.69	2.00	2.34	2.73	3.17	0.76	0.97	1.22	1.50	1.83	2.21	2.66	0.27	0.37	0.49	0.64	0.85	1.10	1.41
17.5	n.a.	0.07	0.28	0.54	0.84	1.19	1.54	n.a.	n.a.	0.03	0.25	0.48	0.75	1.06	n.a.	n.a.	n.a.	n.a.	n.a.	0.11	0.26
22.5	n.a.	n.a.	n.a.	0.08	0.36	0.69	1.02	n.a.	n.a.	n.a.	n.a.	0.06	0.29	0.56	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
27.5	n.a.	n.a.	n.a.	n.a.	n.a.	0.17	0.47	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	0.06	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Notes: "n.a." implies that a fair fee does not exist. In other words, for all fee rates q, the value of the contract is less than $x_0 + C_0 = 1.07$ due to the interaction between financial parameters and guarantee level.

In summary, the presence of tax, compared to the tax-free case, decreases the value of the product in all tax treatments for reasonable, close to the fair fee, charges. However, if fees are substantially larger than the fair fee, we observe a greater value to the policyholder in the offset case as the tax credits are realized. If losses cannot offset capital gains, tax is only paid when there are investment gains. This will affect the behavior of the policyholder as they will try to avoid losses, that is, they will try to receive as much value of their contract while minimizing the fees paid. This will have a distortionary effect in the viability of such products in this taxation regime, especially for high marginal tax rates.

The contract values and insurer's liabilities are greatly affected by the roll-up rate δ. The higher the guarantee, the more valuable the contract will be and the higher fee the policyholder will be willing to pay. As a counterparty, the product becomes more expensive for the insurance company to offer, increasing liability payments accordingly. It is also interesting to note that for long maturities, the contract is also not viable for high fee levels and low δ. Indeed, when $T = 15$ the insurer liability jumps following immediate surrender whenever $\delta = 0\%$ and fees higher than 1.8%. However, for the same maturity we observe that the cutoff point for immediate surrender increases with δ. Overall, we can conclude that higher δ increases the viability of the product, despite the corresponding greater fee that is charged.

Table 3 summarizes the policyholder fair fees q^p (recall that q^i rarely exists in this setting as taxation distorts the offer of this product), that is, the fees that render the contract fair for the policyholder in three taxation treatments: tax-free, offset, and no offset. Firstly, we observe that fair fee increases for higher roll-up guarantee δ as suggested in Figure 2. Indeed, higher δ increases the minimum accumulation benefit making the product more attractive for the policyholder especially as the spread between the risk-free rate and δ decreases. We observe that the policyholder fee, q^p, decreases with tax rates. As earlier discussed, policyholders act so as to maximize post-tax contract value, and increasing tax rates reduce the potential gains for the market. Finally, we observe a negative effect of maturity. The higher the maturity, the lower the fee, to the extent it does not exist often for high τ and low δ. Even if the guarantee depends on T, the higher investment horizon makes it more likely for the underlying to outperform $G(\delta)$ rendering the contract less attractive.

We observe that the fair fee often does not exist, n.a. in Table 3, as the value function never crosses $x_0 + C_0$ for higher taxation rates. Indeed, the value function is always under $x_0 + C_0$, even for zero fees, and the product is not attractive from the policyholder's perspective. Note that the fair fee of n.a. reflects parameter combinations which make it infeasible for a rational policyholder to enter the contract. It can as well reflect that the policyholder is not willing to spend more than the upfront cost of C_0 in this product, implying that charging a zero fair fee while considering the C_0 could still render the contract profitable for the insurer, as there are other sources of income such as surrender fees in the event of the contract being surrendered early. We discuss this in Section 5.

Focusing on Table 3(a) and comparing it with Table 3(b), we note that for all cases associated with $T = 5$ and $\delta < 1.5\%$, the no offset case yields equal or lower fees than the offset regime. For instance, we find that in some cases the fee ceases to exist in the no offset case ($\delta = 0$ and $\tau = 17.5\%$) whereas they do in the offset case. The higher the δ, the lower the difference between the two taxation regimes. Indeed, from $\delta = 1.5\%$ this difference completely disappears. We hypothesize this might be due to policyholder having virtually only gains. In that case, whether losses can offset gains or not is of no relevance.

Another argument is based on the taxable income when $x_T < G(\delta)$ and the guarantee is triggered. In that case, the taxable income in Equations (2.6) and (2.8) solely depend on the relationship between δ and C_0. Indeed, the taxable income is given by:

$$[x_0 e^{\delta T} - x_0 - C_0],$$

which, for our chosen parameters of $x_0 = 1$ and $C_0 = 7\%$ simplifies to

$$1\left[e^{\delta T} - 1 - 7\%\right].$$

Clearly, when $T = 5$, we find that the taxable income is negative for $\delta < 1.25\%$ and positive otherwise. Whether or not losses can offset gains is less relevant when taxable income is *always* positive. The fees in the two taxation regimes hence coincide. A similar exercise for $T = 10$ yields positive taxable income

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from $\delta = 0.75\%$ and for $T = 15$ from $\delta = 0.50\%$. The fair fee analysis allows us to conclude that the impact of the taxation regime can be mitigated by a higher roll-up fee. However, in general, the offset case is beneficial to the insurer as the policyholder's higher willingness to pay is present. Fees are either higher or exist more often for the same guarantee level.

3.2. Optimal surrender behavior

The surrender boundary $s(\nu)$, as discussed in Subsection 2.1, is the minimum fund value required to trigger rational surrender, as a function of time since inception ν. Since no fees can be attached to the insurer valuation, we assume that the fee rate that is actually charged on the contract is q^p, which delivers zero profit to the policyholder. Surrender boundaries when no fair fee exist, such as in the base case for $\tau = 27.5\%$, are excluded from the analysis.

Figure 3 presents optimal surrender boundaries for various τ, δ, and T. First, we observe that the surrender boundary decreases with ν. Indeed, at $\nu = 5$, whenever the guarantee is maturing, the surrender boundary converges to the guarantee value. In all cases as the contract approaches maturity, the presence of taxation reduces the volatility in the final payoff, since the government absorbs a portion of both losses and gains. Furthermore, as time to maturity approaches zero, the surrender penalty approaches zero and hence the boundaries approach the guaranteed amount $G(\delta)$. Thus, the policyholder is more willing to remain invested at higher τ for smaller ν, which is indicated by the surrender boundary being shifted up, and this is consistent with findings in Bernard *et al.* (2014). If the policyholder has any amount in the fund exceeding the initial total payment, then they would prefer to surrender at Δt (and keep the fraction $e^{-\kappa\Delta t}$) before maturity rather than to pay fees in the time interval Δt for a guarantee which has a low probability of ending up in the money.

Second, Figure 3(a) and (b), show the boundaries for three tax treatments: offset, no offset, and tax-free. We observe that the boundary increases with tax. The surrender boundary increases as policyholders are less eager to surrender since they are paying lower fees. Complementary to this, reducing the post-tax value through higher taxes delays surrender as individuals are maximizing their post-tax value. Comparing Figure 3(b) with Figure 3(a), we observe that the surrender boundaries are higher in the no offset case. At $\nu = 0$, these differences can amount to 4% increase in the surrender boundary. Again, the fact that no losses can offset gains make policyholders stay longer in the contract, aiming to reach a certain post-tax value to compensate the loss of income through taxation. When the tax rate, τ, is set equal to zero, we reproduce results from the setting which has been extensively studied in the literature (Bernard *et al.,* 2014; Shen *et al*., 2016).

Higher δ corresponds to higher guarantee levels and hence higher fair fees, decreasing the surrender boundary accordingly as shown in Figure 3(c) and (d). At maturity, the convergence observed in the previous cases appear, but it happens at different levels corresponding to the varying $G(\delta)$. It is interesting to note that for low $\delta = 0.5\%$ which corresponds to a virtually free contract with a fair fee of 0.03% (Table 3), we have no surrender during the initial phases of the contract. However, after 3 year and 5 months, we observe that surrender is possible again. Upon approaching maturity, the underlying has had the possibility to increase more than the low guarantee, rendering surrender more likely.

Figure 3(e) and (f) use the base parameters from Table 1 with $\delta = 1.25\%$ and $\tau = 17.5\%$ instead of $\delta = 0.75\%$ and $\tau = 22.5\%$. This is because the base case scenario does not have fair fees for $T =$ 10 and $T = 15$ and hence no surrender boundaries to show. Akin to Bernard *et al.* (2014), we find that the surrender boundaries shift upward with maturity T. The higher the maturity, the higher the corresponding $G(\delta)$, but, contrary to δ sensitivity the higher guarantee comes with a lower fee since the insurer has a longer period to finance the guarantee. The low fees increase the boundary, indicating that the policyholder is willing to remain invested in the contract despite the higher probability to outperform the guarantee in the long term.

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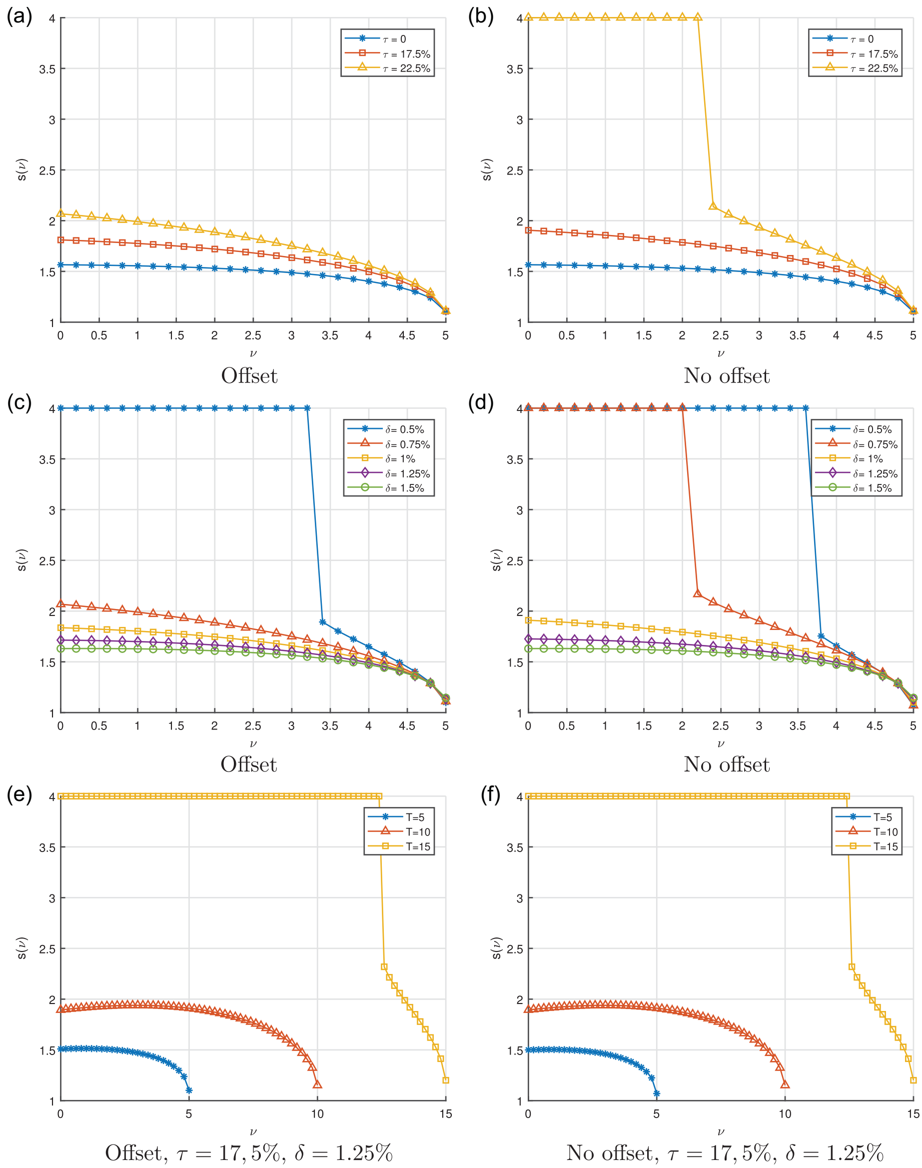

Figure 3. *Optimal surrender boundaries. Financial parameters correspond to the base case of Table 1. The first row presents sensitivity to τ, the second to δ, and the third to T.*
Notes: the sensitivity to T uses the base parameters from Table 1 with $\delta = 1.25\%$ and $\tau = 17.5\%$ instead of $\delta = 0.75\%$ and $\tau = 22.5\%$ since the base case scenario does not have fair fees for $T = 10$ and $T = 15$ and hence no surrender boundaries to show.

4. Sensitivity analysis

This section presents the sensitivity of the fair fees and surrender boundaries to alternative financial market and contract parameter specifications for three tax treatments: tax-free, offset, and no offset. We show the impact of varying the risk-free rate r, volatility σ, and surrender penalty κ on policyholder fair fees q^p. Unless otherwise stated, the rest of the parameters are given by the first row of Table 1. Globally, we observe that the no offset case, compared to the case where losses can offset gains, always yields

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Table 4. *Sensitivity analysis of fair fees (% p.a.): analysis of the impact of r, σ, and κ on policyholder (q^p) fair fees. The rest of the parameters are given in Table 1.*

a Financial market r and σ sensitivity

	Tax-free			Losses offset gains			Capital gains only		
$T=5$	$\sigma=0.15$	$\sigma=0.20$	$\sigma=0.25$	$\sigma=0.15$	$\sigma=0.20$	$\sigma=0.25$	$\sigma=0.15$	$\sigma=0.20$	$\sigma=0.25$
r=0.025	0.65	2.73	5.50	n.a.	1.01	3.24	n.a.	0.80	2.93
r=0.030	0.29	2.00	4.41	n.a.	0.23	2.10	n.a.	0.08	1.86
r=0.035	0.02	1.41	3.53	n.a.	n.a.	1.18	n.a.	n.a.	0.99

b Surrender penalty κ sensitivity

	Losses offset gains			Capital gains only		
	$\kappa=0$	$\kappa=0.005$	$\kappa=0.01$	$\kappa=0$	$\kappa=0.005$	$\kappa=0.01$
$\tau=0$	2.40	2.00	1.74	2.40	2.00	1.74
$\tau=0.175$	0.95	0.66	0.54	0.81	0.54	0.45
$\tau=0.225$	0.48	0.23	0.19	0.30	0.08	0.07
$\tau=0.275$	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Notes: "n.a." implies that a fair fee does not exist. In other words, for all fee rates q, the value of the contract is less than $x_0 + C_0 = 1.07$ due to the interaction between financial parameters and guarantee level.

slightly higher surrender boundaries. As discussed in Section 3, a combination of lower fees in the no offset case together with the fact that all gains are taxed makes the policyholder have a higher propensity to surrender and stay longer in the contract, trying to optimize their after-tax income.

In Table 4, a higher r is accompanied by a lower fee rate. Since policyholders can obtain a greater return in the risk-free market, they are willing to pay less to enter the contract for the same level of maturity guarantee. This is akin to put option prices where higher r implies cheaper put options. Hence as reflected in Figure 4(a) and (b), the surrender boundary is shifted up implying that the policyholder is still willing to remain invested in the contract in spite of the guarantee being worth less in relative terms. This holds since, the higher r, for the same volatility σ, the higher average value of the underlying and cutoff to surrender.

Furthermore, Table 4 shows that q^p increases with σ. This is because the guarantee is more attractive in a highly volatile, uncertain market. We show as well that greater market volatility corresponds to more savings for the policyholder when losses are allowed to offset gains. In contrast, when losses cannot be used to reduce tax payable, the policyholder fair fee is lower. Figure 4(c) and (d) show that the surrender boundary decreases with volatility. Higher uncertainty makes the guarantee more valuable, and as a consequence the surrender boundary needs to decrease such that the gain upon surrendering $(G(\delta) - s(\nu))$ compensates the higher option value. Similar to Figure 4(a) and (b), we observe that the boundaries are higher in the no offset case; however, the difference is smaller than in the sensitivity to r. We also note that fair fees do not exist when risk-free rates are very high or volatility is very low, indicating that the product is not interesting whenever the guarantee is too weak (high r) or has less added value (low σ).

In Table 4 as κ increases, q^p decreases because the insurer is also able to collect more from the larger surrender penalty they impose. We also show the impact on optimal surrender after increasing κ. As shown in Figure 4(e) and (f), a higher surrender penalty shifts the surrender boundary upward, since an increase in surrender penalty is accompanied by a decrease in the fair fee rate. From the insurer's perspective, the higher surrender penalty ensures the policyholder stays in the contract for a longer period of time.[20] Also, the surrender penalty is an exponentially decreasing structure with time-to-maturity.

[20]The boundary hitting 4 is a proxy for infinity as our numerical scheme is capped to $4 \cdot x_0$ with $x_0 = 1$ premium.

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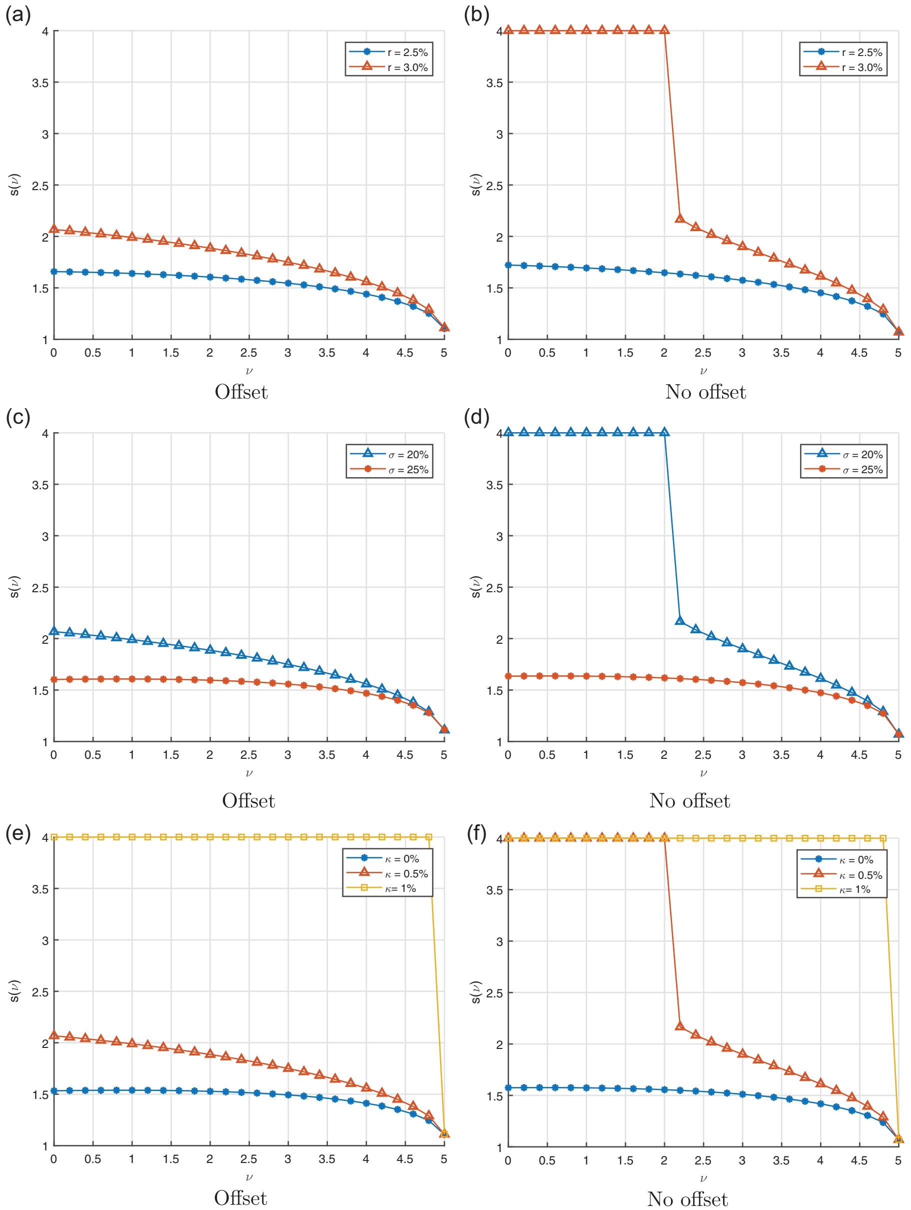

Figure 4. *Impact of interest rates (a,b), volatility (c,d), and surrender penalty (e,f) on optimal surrender for different times to maturity, with (a,c,e) and without (b,d,f) offset allowed. Other parameters correspond to the base case of Table 1.*

If the contract has a longer maturity and higher penalty fee, then it will not be economical for the policyholder to surrender early.

Based on the main and sensitivity analysis, we have identified that the fair fee increases substantially for higher guarantee value δ and volatility σ and it decreases the surrender boundary such that the gain upon surrendering $G(\delta) - s(\nu)$ compensates the higher option value. On the other hand, higher τ, r, κ, and T all decrease the fee rate and increase the surrender boundary accordingly.

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5. Profit and Loss analysis

To complement our analysis of the viability of the VA, we perform 1000 Monte Carlo simulations to analyze the profit and loss (P & L) profiles and surrender statistics for various parameter specifications akin to Moenig and Bauer (2015). Values provided in Tables 5 and 6 correspond to the real-world expectation at $t = 0$, discounted at the risk-free rate r. Since $x_0 = 1$, the results are expressed in terms of $\times 10^{-2}$ unless otherwise specified. We assume that the insurer sells the product at q^p following the policyholder bid price. Furthermore, the simulations of the underlying fund are done under the real-world measure assuming that the insurer writes a naked option without any hedge. The μ under the real-world measure is obtained as $\mu = r + SR \cdot \sigma$ where SR is the Sharpe ratio (SR) which represents risk-adjusted returns. We consider the following values for the SR 0.10, 0.25, and 0.45.[21] We abstract from any reinvestment of the fair fees in risky assets. We acknowledge that this is a stylized *worst case* analysis as it does not incorporate reinvestment risk or hedging.

The P & L tables provide an overview of the surrender fee that the insurer receives upon early surrender, the guarantee fees required to fund the insurance product, the initial fee C_0 as well as the cost of providing the guarantee. Surrender is triggered whenever the underlying excess the precomputed surrender boundary, which is the same for all SR. The surrender rate is then defined as the proportion of insurance contracts that are terminated prior to maturity and the time elapsed as the average time elapsed in the contract before surrender (if any).[22] Finally, the net profit is calculated as the guarantee fees, complemented by the surrender fee and upfront costs C_0 reduced by the guarantee cost. The net profit values on the tables should be interpreted as follows: a net average profit of 3.72×10^{-2}, base case with $SR = 0.10$ in the tax-free regime, means that the product with $x_0 = 1$ earns 3.72% average yield. Similarly, a net average profit of -1.04×10^{-2}, base case with $SR = 0.10$ in the offset case, means that a -1.04% average loss is incurred. The last rows also show the various net profit percentiles P_α for $\alpha = \{1, 25, 50, 75, 99\}$ in order to inform about their skewness. The results presented in the first to third set of results correspond to the case without taxation, with taxation and with the possibility to offset losses, and taxation without the option to offset, respectively.

First, let us delve into the effect of SR in the P&L dynamics for a given case and taxation regime. Early termination of the contract is more likely for higher SRs, that is, higher rewards-to-variability ratios. Recall that μ increases with SR, which implies a higher potential outside of the insurance, making it more likely to hit the surrender boundary. Intuitively, this also affects the guarantee cost. Indeed, it decreases with SR as it becomes less likely to trigger the guarantee at maturity. Of course, more lapses, for the same surrender boundary, are associated with a lower average time in the contract. Surrender and guarantee fees, for a given taxation regime, are not affected by SRs as much: the former slightly increase whereas the latter decrease with SR since surrender happens more often. However, the increase in surrender fees is insufficient to fully substitute the loss of regular guarantee fees. Globally, the net average and median profit increase with SR but primarily due to the decrease in guarantee cost. These general trends hold within each taxation regime considered.

The effect of tax is significant. Section 3 shows that fair fees decrease and surrender boundaries increase substantially when taxation is considered. The decrease (increase) is even greater whenever losses cannot offset gains. Indeed, having all proceeds taxed incentives the policyholder to stay longer as their aim is to maximize post-tax value. The stark decrease in fees translates in a similar decrease guarantee fee revenue. Despite the sizeable increase in surrender boundaries, Figure 3(a) and (b), the surrender rate increases only by 2–7%, depending on the scenario. Note that, although surrender rate is higher in the taxation regime with respect to the no tax regime, most of the surrenders are happening later on in the contract, increasing the average time elapsed accordingly. However, this increase does not increase the cost of the guarantee, on the contrary, the cost of guarantee decreases instead due to lower

[21] Sharpe ratios of 0.25 and 0.45 align with Moenig and Bauer (2015). We study the Sharpe ratio of 0.1 to study whether the product would still be profitable in a low performing economy.

[22] The average time elapsed is the average contract life of a given variable annuity. This corresponds to 5 years, the maturity of the contract, if there is never surrender. The lower the value, the less time policyholder spend in the contract, on average.

Table 5. *Profit and Loss profiles for various parameter specifications and Sharpe ratios.*

	Tax-free ($q^p = 2.00\%$)			Offset ($q^p = 0.23\%$)			No offset ($q^p = 0.08\%$)		
	SR			SR			SR		
Base case	0.10	0.25	0.45	0.10	0.25	0.45	0.10	0.25	0.45
Surrender fee $\times 10^{-2}$	0.70	0.94	1.28	0.54	0.76	1.04	0.46	0.64	0.93
Guarantee fees $\times 10^{-2}$	7.23	6.90	6.33	0.98	0.96	0.93	0.37	0.37	0.37
$C_0 \times 10^{-2}$	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Guarantee cost $\times 10^{-2}$	11.21	7.17	3.39	9.03	5.55	2.42	8.87	5.50	2.34
Net avg profit $\times 10^{-2}$	3.72	7.68	11.22	−0.52	3.17	6.55	−1.04	2.51	5.96
$P_{50} \times 10^{-2}$	13.26	13.83	13.95	8.21	8.46	8.85	7.48	7.68	8.09
$P_{25}; P_{75} \times 10^{-2}$	−6.7; 15.23	5.7; 15.43	12.6; 15.36	−8.4; 8.82	3.9; 9.22	8.2; 9.60	−8.5; 8.06	3.4; 8.39	7.5; 8.94
$P_1; P_{99} \times 10^{-2}$	−43.4; 17.61	−37.7; 17.60	−28.6; 17.42	−45.2; 10.81	−39.2; 10.90	−29.8; 11.03	−45.3; 10.10	−39.4; 10.34	−29.9; 10.58
Surrender Rate (%)	52	65	79	57	69	81	59	71	83
Avg time elapsed	4.01	3.68	3.21	4.33	4.06	3.73	4.44	4.24	3.92
	Tax-free ($q^p = 4.41\%$)			Offset ($q^p = 2.10\%$)			No offset ($q^p = 1.86\%$)		
	SR			SR			SR		
$\sigma = 0.25$	0.10	0.25	0.45	0.10	0.25	0.45	0.10	0.25	0.45
Surrender fee $\times 10^{-2}$	0.94	1.16	1.51	0.86	1.13	1.49	0.84	1.12	1.49
Guarantee fees $\times 10^{-2}$	13.26	12.72	11.47	7.05	6.71	6.16	6.35	6.04	5.54
$C_0 \times 10^{-2}$	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Guarantee cost $\times 10^{-2}$	16.69	11.32	5.91	14.14	9.09	4.51	13.86	8.94	4.38
Net avg profit $\times 10^{-2}$	4.51	9.56	14.08	0.77	5.75	10.13	0.33	5.22	9.65
$P_{50} \times 10^{-2}$	13.80	15.96	16.76	12.82	13.58	13.89	12.64	13.28	13.46
$P_{25}; P_{75} \times 10^{-2}$	−11.8; 20.17	2.3; 21.20	13.3; 21.25	−13.8; 15.05	1.2; 15.31	12.4; 15.34	−13.8; 14.44	0.8; 14.61	12.2; 14.60
$P_1; P_{99} \times 10^{-2}$	−49.5; 28.28	−44.0; 28.39	−35.1; 28.23	−52.1; 18.12	−46.5; 17.88	−37.4; 17.82	−52.4; 16.95	−46.8; 16.72	−37.6; 16.72
Surrender Rate (%)	50	62	76	52	65	78	54	66	79
Avg time elapsed	3.66	3.34	2.86	3.85	3.50	3.03	3.89	3.53	3.06

Table 5. *Continued.*

	Tax-free ($q^p = 2.34\%$)			Offset ($q^p = 0.46\%$)			No offset ($q^p = 0.36\%$)		
	SR			SR			SR		
$\delta = 1\%$	0.10	0.25	0.45	0.10	0.25	0.45	0.10	0.25	0.45
Surrender fee $\times 10^{-2}$	0.73	0.97	1.31	0.61	0.81	1.13	0.57	0.80	1.10
Guarantee fees $\times 10^{-2}$	8.29	7.87	7.21	1.86	1.82	1.74	1.49	1.45	1.39
$C_0 \times 10^{-2}$	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Guarantee cost $\times 10^{-2}$	12.18	7.89	3.82	9.73	6.08	2.72	9.62	6.00	2.67
Net avg profit $\times 10^{-2}$	3.84	7.95	11.69	−0.26	3.56	7.14	−0.56	3.25	6.83
$P_{50} \times 10^{-2}$	13.39	14.20	14.51	9.30	9.59	9.89	8.83	9.12	9.48
$P_{25}; P_{75} \times 10^{-2}$	−7.7; 16.12	4.7; 16.40	12.7; 16.43	−9.3; 9.87	3.2; 10.14	9.3; 10.40	−9.4; 9.39	2.8; 9.78	8.8; 10.08
$P_1; P_{99} \times 10^{-2}$	−44.2; 19.32	−38.5; 19.19	−29.4; 19.16	−46.1; 11.07	−40.1; 11.08	−30.7; 11.14	−46.2; 10.95	−40.2; 11.04	−30.8; 11.10
Surrender Rate (%)	51	64	77	56	67	81	57	69	81
Avg time elapsed	3.96	3.61	3.15	4.22	3.96	3.57	4.27	3.99	3.62

Notes: The first, second, and third block of results represent the base case, high volatility ($\sigma = 0.25$), and high guarantee value ($\delta = 1\%$) with the other parameters as depicted in Table 1. Net avg profit is calculated as Guarantee fees + Surrender fee $+C_0$ - Guarantee cost. P_x represents the xth percentile. Of course P_{25}, P_{75}, and P_{50} correspond to the quartiles and median, respectively. P_1 and P_{99} correspond to the extreme percentiles. This analysis includes C_0. To exclude it, $C_0 = 0.07$ should be substracted to the *Net avg profit* and percentile rows and analyze subsequently.

Table 6. *Profit and Loss profiles for varying r, σ, and κ for different Sharpe ratios.*

	Tax-free ($q^p = 2.73\%$)			Offset ($q^p = 1.01\%$)			No offset ($q^p = 0.80\%$)		
	SR			SR			SR		
$r = 2.5\%$	0.10	0.25	0.45	0.10	0.25	0.45	0.10	0.25	0.45
Surrender fee $\times 10^{-2}$	0.74	0.99	1.33	0.66	0.88	1.21	0.64	0.86	1.18
Guarantee fees $\times 10^{-2}$	9.48	8.98	8.15	3.84	3.72	3.47	3.12	3.02	2.85
$C_0 \times 10^{-2}$	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Guarantee cost $\times 10^{-2}$	13.29	8.69	4.35	10.87	6.88	3.25	10.60	6.71	3.13
Net avg profit $\times 10^{-2}$	3.94	8.27	12.12	0.63	4.72	8.44	0.16	4.17	7.90
$P_{50} \times 10^{-2}$	13.34	14.57	14.93	11.42	11.57	11.65	10.74	11.01	11.15
$P_{25}; P_{75} \times 10^{-2}$	−8.5; 17.02	3.7; 17.48	12.7; 17.39	−9.8; 11.86	3.0; 11.92	11.4; 11.93	−9.8; 11.23	2.5; 11.30	10.7; 11.33
$P_1; P_{99} \times 10^{-2}$	−44.9; 21.30	−39.2; 20.99	−30.3; 21.05	−46.7; 12.62	−40.8; 12.67	−31.5; 12.60	−46.9; 11.61	−41.0; 11.63	−31.6; 11.67
Surrender Rate (%)	49	63	77	54	66	79	56	68	81
Avg time elapsed	3.92	3.57	3.09	4.11	3.81	3.39	4.15	3.86	3.46
	Tax-free ($q^p = 2.00\%$)			Offset ($q^p = 0.66\%$)			No offset ($q^p = 0.54\%$)		
$\tau = 17.5\%$	SR			SR			SR		
	0.10	0.25	0.45	0.10	0.25	0.45	0.10	0.25	0.45
Surrender fee $\times 10^{-2}$	0.70	0.94	1.28	0.62	0.82	1.14	0.58	0.80	1.11
Guarantee fees $\times 10^{-2}$	7.23	6.90	6.33	2.63	2.57	2.44	2.18	2.13	2.04
$C_0 \times 10^{-2}$	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Guarantee cost $\times 10^{-2}$	11.21	7.17	3.39	9.54	5.94	2.66	9.40	5.85	2.59
Net avg profit $\times 10^{-2}$	3.72	7.68	11.22	0.70	4.46	7.92	0.36	4.08	7.56
$P_{50} \times 10^{-2}$	13.26	13.83	13.95	10.17	10.45	10.68	9.63	9.92	10.19
$P_{25}; P_{75} \times 10^{-2}$	−6.7; 15.23	5.7; 15.43	12.6; 15.36	−8.0; 10.73	4.5; 10.87	10.1; 11.01	−8.1; 10.19	4.1; 10.41	9.6; 10.64
$P_1; P_{99} \times 10^{-2}$	−43.4; 17.61	−37.7; 17.60	−28.6; 17.42	−44.8; 11.31	−38.8; 11.34	−29.4; 11.36	−44.9; 11.23	−39.0; 11.27	−29.5; 11.30
Surrender Rate (%)	52	65	79	56	68	81	58	70	83
Avg time elapsed	4.01	3.68	3.21	4.19	3.93	3.54	4.25	3.98	3.61

Table 6. *Continued.*

	Tax-free ($q^p = 1.74\%$)			Offset ($q^p = 0.19\%$)			No offset ($q^p = 0.07\%$)		
	SR			SR			SR		
$\kappa = 1\%$	0.10	0.25	0.45	0.10	0.25	0.45	0.10	0.25	0.45
Surrender fee $\times 10^{-2}$	1.05	1.48	2.07	0.01	0.02	0.07	0.01	0.02	0.07
Guarantee fees $\times 10^{-2}$	6.99	6.84	6.57	1.01	1.09	1.20	0.38	0.41	0.46
$C_0 \times 10^{-2}$	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
Guarantee cost $\times 10^{-2}$	11.08	7.09	3.38	9.03	5.61	2.42	8.88	5.50	2.35
Net avg profit $\times 10^{-2}$	3.97	8.23	12.27	−1.02	2.49	5.85	−1.48	1.94	5.18
$P_{50} \times 10^{-2}$	14.84	15.53	15.89	7.95	8.06	8.17	7.36	7.40	7.45
$P_{25}; P_{75} \times 10^{-2}$	−7.3; 16.07	4.7; 16.23	14.9; 16.31	−8.9; 8.16	3.0; 8.25	7.9; 8.40	−8.9; 7.44	2.9; 7.48	7.4; 7.53
$P_1; P_{99} \times 10^{-2}$	−43.7; 17.22	−37.9; 17.08	−28.7; 17.25	−45.2; 8.78	−39.3; 8.95	−29.8; 11.33	−45.4; 7.68	−39.4; 7.75	−29.9; 10.49
Surrender Rate (%)	51	63	77	55	67	80	58	70	83
Avg time elapsed	4.31	4.03	3.66	5.00	4.99	4.98	5.00	4.99	4.98

Notes: The first, second, and third block of results represent the high relative value of the guarantee case ($r = 2.5\%$), low tax ($\tau = 17, 5\%$), and high surrender penalty ($\kappa = 1\%$) with the other parameters as depicted in Table 1. The case $r = 3.5\%$ and $\tau = 27, 5\%$ are not presented as they yield no fee (Table 4). Net avg profit is calculated as Guarantee fees + Surrender fee $+C_0$ - Guarantee cost. Net extreme profit Ptiles represent the 1st and 99th percentile. Net profit Qtiles consider the 25th percentile and 75th percentile. This analysis includes C_0. To exclude it, $C_0 = 7$ should be substracted to the *Net avg profit* and percentile rows and analyze subsequently.

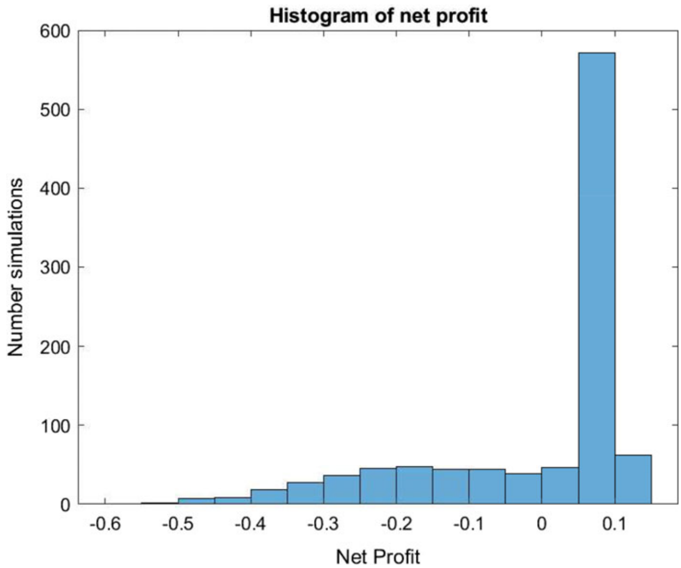

Figure 5. *Histogram of the net profit including C_0 over 1000 Monte Carlo simulations.*
Notes: the distribution depicted corresponds to the base case scenario ($r = 3\%$, $\sigma = 0.20$, $\delta = 0.75\%$, $\tau = 22.5\%$) with $SR = 0.10$.

fees facilitating a higher terminal account balance triggering the guarantee less often. This combined effect of tax skews the distribution further to the left and decreases the average profit and all percentiles. Of course, the more restrictive the tax treatment, as in the no offset case, the higher the impact.

Net average profit differs greatly from its percentiles. We observe that while some contracts with low *SR* under a taxation regime have negative or close to 0 *average* profit, with losses up to 1.48%[23] of the principal, they always have positive and much greater P_{50}, P_{75}, and P_{99}. As earlier stated, this is due to the P & L distribution, Figure 5 and P_1 in Tables 5 and 6, which has a low-frequency, high-severity negatively skewed distribution, affecting the average profit greatly when the guarantee kicks in. We observe indeed that the median profit is only slightly affected by increasing *SR*, whereas the net average profit is significantly affected through the sizeable decrease in guarantee cost. Even in the scenarios with low average profit, we observe that the product would be profitable with margins of at least 7.44% depending on the taxation regime with a real-world probability of 75%.

Tables 5 and 6 show the effect of increasing volatility σ[24], guarantee level δ[25], decreasing r and τ, while keeping all other parameters constant. Increasing σ, δ, and decreasing r and τ increases the fair fee, decreasing the account value. This triggers the guarantee more often, skewing the distribution further to the left. Yet, the increase in guarantee fees collected is sufficient to counter the increase in guarantee cost, yielding an increase in overall profitability, in average and median terms. The decrease in surrender boundaries, Figures 3 and 4, together with the lower underlying net of fees translates in slightly lower surrender and lower average time elapsed. Indeed, the changing shape of the boundary renders surrender slightly less common but makes it happen sooner in the contract.

Finally, the third panel of Table 6 shows the effect of increasing the surrender penalty κ. The slight decrease in fair fees, compared to the base case, lowers the guarantee fees accordingly and increase the

[23] Offset scenario in the $\kappa = 1\%$ from Table 6.

[24] Of course, μ increases with σ, yielding a higher potential return under the real-world measure. However, this increase does not translate in greater surrender given the mitigating effect of the greater fair fee.

[25] Recall that for δ approaching 1.5% we have the same fair fees in the two taxation regimes as there are virtually no losses anymore and hence the difference between them disappears.

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value of the underlying net of fees. This has a reduced impact in the guarantee cost. While surrender rate remains quite stable, the average time elapsed increases in the tax-free case by 3 months and in the taxed case to almost $T = 5$. Indeed, we know from Figure 3(e)–(f) that the surrender boundary attains the maximum level of 4, rendering surrender nearly impossible until just before maturity where the boundary drops. This virtual no surrender decimates the surrender penalty income in the taxed case, lowering the average profit. In the tax-free case, the greater penalty κ increases the surrender fee income in this case, having a slight positive effect on profitability on average terms.

6. Conclusions

Insurance providers benefit from the tax-deferred nature of VAs. However, the popularity of these products varies widely across countries. We show that the taxation regime, tax rate, and Sharpe ratio of the market are some of the key drivers of such demand. In this paper, we illustrate the impact of different taxation systems on policyholder behavior and the implications for insurers. In particular, we assess and compare the cases when losses are allowed to offset gains, and where losses are not allowed to offset gains. These two regimes reflect features of institutional arrangements in Australia, US, and most European countries.

Majority of existing literature on VA pricing abstract from the friction induced by taxation naturally leading to policyholder fair fees coinciding with insurer's expectations. However, upon introducing taxation, we show that wide gaps arise as a result of the interaction between the tax regimes and underlying policyholder behavior through surrender. We observe that individuals' threshold to surrender differs greatly from the no tax case at the beginning and end of the contract.

We formulate the valuation of a GMAB contract from the policyholder and insurer's perspective as a free boundary problem which is solved using the method of lines. The corresponding policyholder fair fee and insurer fair fee are computed. The numerical results show how the guarantee level δ, maturity T, risk-free rate r, volatility σ, and surrender penalty κ impact the pricing and optimal surrender behavior. This impact is determined by the fair fees and also on the particular taxation system. In particular, fair fees increase with δ and σ and decrease with r, κ, and T.

We show that allowing for losses to offset gains enhances the market, increasing the willingness to pay of the policyholder. However, fair fees and subsequently net profit still are still higher in the no tax case. However, the tax regime alone is not a sole driver of the attractiveness of the product. In financial markets with low volatility and high taxes, policyholders are only willing to enter the contract at very low fee levels. On the other hand, high volatility increases the attractiveness of the contract, as increasing guarantees do. For a particular financial market setting, we observe that product features such as the level of the guarantee can mitigate the effect of taxes. Intuitively, the greater δ compensates for the decreased post-tax income in a way that can still create demand for the product in adverse financial and institutional settings. Adjusting the volatility level to the extent that insurers can target a particular volatility of the fund is also a way of enhancing demand and profits. Despite the increase in fair fees in both cases, (rational) surrender is not affected as much, increasing guarantee revenue and decreasing the cost of actually providing the guarantee.

The profit and loss analysis shows that, despite charging the (low) demand fee, insurance providers' net median profit is always positive. This analysis is made under the assumption that the insurer writes a naked option without any hedge. The profit and loss distribution of the product is highly left-skewed, yielding positive returns on the product at high probability but with high losses at the tail. When losses offset gains, policyholders delay early surrender in order to receive the higher tax reductions reflecting their increased losses. However, if losses are not allowed to offset gains, then policyholders behave in such a way to maximize the post-tax value. Profitability of the insurer varies with the Sharpe ratio. For low Sharpe ratios, policyholders are more likely to hold their contracts until maturity and receive the benefit of the guarantee, diluting the insurer's profit. For high Sharpe ratios, the higher returns outside of the product incentivizes the policyholder to surrender since the guarantee offered can quickly become out of the money.

We identify at least two important directions for future research. Policyholder in our setting are always taxed upon surrender or upon reaching maturity. However, most taxation regimes offer incentives, either by reducing or removing tax altogether in order to cash out retirement income beyond a particular preservation age. Of interest will be disentangling the effect of this discontinuity in product pricing. Furthermore, we have identified hybrid products that combine the main underlying fund in most classical VA literature with a cash account that earns the risk-free rate. Combining the two allows for further tax optimization as cash flows can be transferred from one to the other delaying claim date and tax liability.

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A Appendices

A The Governing Partial Differential Equation

Applying Ito's Lemma in conjunction with the formula for dx in Equation (2.1) yields

$$\begin{aligned} du &= u_x dx + \frac{1}{2} u_{xx} (dx)^2 + u_\nu d\nu \\ &= u_x dx + \frac{1}{2} \sigma^2 x^2 u_{xx} d\nu + u_\nu d\nu. \end{aligned} \tag{A1}$$

Now consider a portfolio consisting of a long position in the GMAB contract (inclusive of both the cash account and investment account) and u_x units short in the underlying fund backing the investment account (without the cash account). The value of the portfolio can then be represented by $\Pi = u(x, \nu) + c(\nu) - u_x x$. Over a small time interval $d\nu$, the corresponding change in portfolio value, given that a continuously compounded management fee at rate qxu_x is paid out of the investment account, is given by:

$$d\Pi = du - u_x dx - qxu_x d\nu \tag{A2}$$

Substituting the known value of du from Equation (A1) into Equation (A2) implies that

$$d\Pi = \frac{1}{2} \sigma^2 x^2 \cdot u_{xx} d\nu + u_\nu d\nu - qx \cdot u_x d\nu.$$

Since this portfolio has no random component, that is, it does not have a dx term and must accumulate at the pre-tax risk-free rate. Substituting back $\Pi = u - u_x x$, we have

$$r(u - xu_x) = \frac{1}{2} \sigma^2 x^2 \cdot u_{xx} - qx \cdot u_x. \tag{A3}$$

In deriving the PDE (A3), we assume the existence of a complete, no-arbitrage market in which the participants (the policyholder and the insurer) can rebalance their portfolios without transaction costs. Instead, taxation is considered from an individual's perspective as it manifests at the boundary conditions of (A3), when the policyholder elects to surrender or receives the final payout from the GMAB contract.

Re-arranging Equation (A3) and applying the transformation $t = T - \nu$ where t represents the time to maturity on the contract, u will satisfy the PDE:

$$\frac{1}{2} \sigma^2 x^2 u_{xx} + (r - q) \cdot xu_x - ru - u_t = 0. \tag{A4}$$

B Method of Lines Implementation

In order to solve Equation (2.5), it is discretized in t and y directions and continuity is maintained in x. Let $0 = t_0 < t_1 < ... < t_n < ..., t_N = T$ be a uniformly space time grid and denote $u(x, t_{n'}) = u_{,n'}(x) = u_{n'}$. As highlighted in Meyer and Van der Hoek (1997) and Kang and Ziveyi (2018), the following finite

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difference approximations are used along the line $t = t_n$ (where we let $u_n = u$ to emphasize that u is presently being solved for as a function of x only):

$$u_t = \begin{cases} \dfrac{u - u_{n-1}}{\Delta t} & \text{if } n = 1, 2 \\ \dfrac{3}{2}\dfrac{u - u_{n-1}}{\Delta t} - \dfrac{1}{2}\dfrac{u_{n-1} - u_{n-2}}{\Delta t} & \text{if } n \geq 3. \end{cases} \tag{B1}$$

The method of lines as presented in Meyer (2015) can be used to solve the system of equations generated when Equations (B1) are used to approximate a solution for the partial differential Equation (2.5).

Substituting (B1) into (2.5) will give

$$\frac{1}{2}\sigma^2 x^2 u_{xx} + (r - q) \cdot x u_x - \tilde{c} u = \hat{f} \tag{B2}$$

where $\tilde{c} = r + \begin{cases} \frac{1}{\Delta t} & \text{if } n = 1, 2 \\ \frac{3}{2\Delta t} & \text{if } n \geq 3 \end{cases}$ and $\hat{f} = \begin{cases} -\frac{u_{n-1}}{\Delta t} & \text{if } n = 1, 2 \\ -\frac{4u_{n-1} - u_{n-2}}{2\Delta t} & \text{if } n \geq 3 \end{cases}$

Solving Equation (B2) requires the one-dimensional method of line solution, which is already discussed in great detail in Meyer (2015), which the following discussion is based on. We first rewrite (B2) as the two point boundary value problem:

$$u'(x) = v(x), \qquad u(0) = (x_0 e^{\delta T} - \tau[x_0 e^{\delta T} - x_0 - C_0]_+)e^{-rt_n} \tag{B3}$$

$$v'(x) = C(x)u + D(x)v + g(x), \qquad v(S) = \gamma_{T-t} - \tau\gamma_{T-t}\mathbb{I}\{[S\gamma_{T-t} - x_0 - C_0 > 0\} \tag{B4}$$

where $S = s(t_n)$ is the free boundary that needs to be computed along with the solution and

$$C(x) = \frac{2\hat{c}(x)}{\sigma^2 x^2}, \qquad D(x) = \frac{2(q - r)}{\sigma^2 x}, \qquad g(x) = \frac{2\hat{f}}{\sigma^2 x^2}.$$

The solution method of the system in (B3) and (B4) requires us to observe that the functions $u(x)$,$v(x)$ are related through the Riccati transformation $u(x) = R(x)v(x) + w(x)$. $R(x)$ and $w(x)$ are solutions to the initial value problems:

$$R' = 1 - D(x)R - C(x)R^2, \qquad R(0) = 0 \tag{B5}$$

$$w' = -C(x)R(x)w - R(x)g(x), \qquad w(0) = (x_0 e^{\delta T} - \tau[x_0 e^{\delta T} - x_0 - C_0]_+)e^{-rt_n} \tag{B6}$$

We first solve Equation (B5) using the implicit trapezoidal rule as detailed in Meyer (2015), although in principle any standard technique for first-order initial value problems can be employed. Equation (B5) depends only on the order of the difference schemes being used. Hence, in this case, there are actually only two possible solutions for $R(x)$ (depending whether n is greater than or less than 2). Thus, we solve for $R(x)$ outside the main loop and store the two separate solutions off-line. Once the values of $R(x)$ along the grid points are obtained, these known values can be used to solve Equation (B6). This is also done using the trapezoidal rule for Ordinary Differential Equations (ODEs) described in Chapter 3 of Meyer (2015).

Now we turn our attention to finding the exercise point $S = s(t_n)$. This is done by considering the function $\phi(x) = u(x) - R(x)w(x) - v(x)$ and noting that, by definition, it equals zero for $0 \leq x \leq S$. Thus, $\phi(S) = u(S) - R(S)w(S) - v(S) = 0$. Moreover, the boundary conditions of Equations (2.7) and (2.9) define what values $u(S)$ and $v(S)$ must take. In order to compute the appropriate S, we define the functions:

$$v_b(x) = \gamma_{T-t_n} - \tau\gamma_{T-t_n}\mathbb{I}\{x - x_0 - C_0 > 0\}$$

$$u_b(x) = \gamma_{T-t_n} x - \tau[\gamma_{T-t_n} x - x_0 - C_0]_+$$

and see that the value of S is the root of the equation $\tilde{\phi}(x) = u_b(x) - v_b(x)R(x) - w(x)$.

These values are known on the points along x, so we find S by identifying where a sign change occurs in function $\tilde{\phi}$. More specifically, one uses the fact that $\tilde{\phi}(x_s) \cdot \tilde{\phi}(x_{s+1}) < 0$ then S occurs in the interval $[x_s, x_{s+1}]$. We use linear interpolation to estimate S. If there are multiple sign changes, we refer to the root computed at the previous iteration and choose the one that is closest to it, as $s(t)$ must be continuous for this particular problem. From general financial reasoning, a small change in t or y should not produce a discontinuous jump in the surrender behavior for the GMAB.

Once S is found, the reverse sweep can proceed to solve for $v(x)$. Using the same linear implicit method used to find $w(x)$, the initial value problem in Equation (B4) can be solved. Since $x = S$ is not a point in the chosen grid, in order to perform the first backward step from $x = S$ to the nearest grid-point, we estimate the values of $C(S)$, $R(S)$, $g(S)$, and $D(S)$ using linear interpolation.

Since $v(x)$ is computed for $x < S$, we set the solution as:

$$u(x) = \begin{cases} R(x)v(x) + w(x) & \text{if } x < S \\ u_b(x) & \text{if } x \geq S \end{cases}$$

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