7. COMMISSION DE LA MECANIQUE CELESTE

PRÉSIDENT: M. CLEMENCE.

MEMBRES: MM. Armellini, Belorizky, Brouwer, Bucerius, Chazy, Chebotarev, Cox, Duboshin, Eckert, Fabre, Heinrich, Herget, Herrick, Milankovitch, Mineur, Moiseiev. Sadler, Slavenas, Stumpff, Subbotin, Sundman †, van Woerkom, von Zeipel.

Recommendation by V V Heinrich:

In the case of the planets our historical observations are not sufficiently extended to enable us to decide many questions of mechanical or cosmogonical character. So all theoretical workers should be advised to give up exact conclusions upon these matters, or else give them with all possible reserve. The case of satellites seems a little better. Very accurate observations have been available for about 50 years, and this corresponds to more than twenty-five centuries in the system of the inner planets. The suggestion would therefore be *to impress on observers the necessity of procuring the most careful observations of satellites using the most powerful American and especially Californian instruments.*

In connection with this recommendation G. M. Clemence wishes to express his conviction that the study of satellite motions is a fruitful field for investigation, which has been too much neglected in recent decades.

Recommendation by G. Armellini:

Io propongo che venga esaminato lo stato attuale delle tavole adoperate per il calcolo delle perturbazioni delle famiglie dei piccoli pianeti, secondo il metodo rapido di Bohlin dei gruppi di commensurabilita col moto medio di Giove, indicando quali siano i miglioramenti da introdurvi e di quali completamenti vi sia bisogno.

We report with regret the death of our esteemed member Karl Frithiof Sundman, Professor Emeritus of Astronomy and retired Director of the Observatory of the University of Helsingfors, on September 28, 1949. His obituary and bibliography of his mathematical and astronomical works, by G. Järnefelt, have appeared in *Acta Mathematica*, Vol. **83**.

On September 12, 1950, M. Clemence addressed a letter to every member of the Commission, requesting reports of recent activity in celestial mechanics, and recommendations for resolutions to be discussed. Reports of considerable length were received from all members except MM. Eckert, Mineur, and von Zeipel. The report which follows is largely a summary of these individual reports, which are too voluminous to be printed in full. But it fails to be a complete report of progress in celestial mechanics for two reasons: first, the individual members have reported chiefly on work in their own countries; and second, the subject-matter is purposely restricted to avoid as much as possible duplicating the reports of other Commissions, many of which fall to some extent in the field of celestial mechanics. This Report therefore should be supplemented by reference to the reports of Commissions 4, 17, 20, 22, 28 and 33. On the other hand, some material is included which might properly form a part of the report of some other Commission, owing to the danger of its being entirely overlooked. References are omitted, since they can readily be found under the author's name in the *Astronomischer Jahresbericht*.

Rotation of the Earth. H. Spencer Jones, in 1939, established the reality of fluctuations in the speed of rotation of the Earth, by showing that the observed fluctuations in the orbital longitudes of the Moon, the Sun, Mercury, and Venus are in fact proportional to their mean motions. This conclusion was later confirmed by Clemence's discussion of meridian observations of Mercury. More recently, Stoyko, Uhink, and Finch have independently established the existence of an annual variation in the speed of rotation, by studies of the performance of clocks. It is now recognized that, for many purposes, the mean solar second is not suitable as a fundamental unit of time, and the sidereal

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year has displaced it to some extent. Further remarks on this subject will be found in the report of the Sub-commission of Commission 4. Van den Dungen, Van Mieghen, and Cox have studied possible causes of the annual variation, and conclude that the explanation may be found in the annual variation of the momentum-transfer between the Earth and the atmosphere, resulting from friction. Assuming that complete compensation occurs at a uniform rate throughout the year, the calculated effects agree well with the observed ones. If complete compensation does not occur, a mechanism may be provided for irregular fluctuations in the speed of rotation.

Heinrich has studied the method of variation of arbitrary constants as applied to the rotation of a solid body around a fixed point.

Melchoir has studied the connection between the variations in the Chandlerian period of the polar motion and the rigidity of the Earth, and also the influence of partition of density in the interior of the Earth on luni-solar variations of gravity.

Mendes has studied the rotation of an ellipsoid using the functions of Lamé.

Cox and van den Dungen, extending the work of Hagen, Manucci, and Stein, have studied the deviations of falling bodies from the vertical under different conditions; they find mathematical expressions for the deviations that agree well with observation. Under their direction Brouet is studying the Coriolis effect experimentally, and he is also studying the case of uniformly decelerated falling bodies.

Bouny has published a paper on the mechanical proofs of the rotation of the Earth.

Problem of three bodies. Lahaye has studied the problem of three bodies with a view to solving it by iteration, and finds expressions for the co-ordinates that converge uniformly for all finite times even when simple collisions occur. He has also formulated existence conditions for periodic or stable solutions in the neighbourhood of the Lagrangian co-linear and equilateral configurations, and has established the existence of asymptotic solutions, expressible by means of convergent series.

Heinrich has deduced Hill's canonical elements with the true anomaly as independent variable by his own method, and has tried the application of his method to the canonical systems of Lagrange and Appel-Christoffel. He has also attempted to simplify the general Lagrangian method for the variation of arbitrary constants, and finds a rule-of-thumb (mechanical) method of carrying out the operations, which he applies to the problem of finding the canonical elements of a periodic solution. Using the same method he generalizes the elliptical elements of Levi Civita, with the eccentric anomaly as one of the angular elements, and analyses the Delaunay transformation. He also studies analytically the problem of two orbits linked together, considering collisions, and indicates a way of making the numerical calculations. He studies the non-canonical equations of Lagrange and Christoffel, with a view to their application to the same problem. As yet unpublished are two extensive papers, on certain classes of always stable orbits, and on a method for avoiding small divisors in celestial mechanics.

Chazy has reduced the equations for the relative motion of three bodies to three differential equations of the tenth order, dealing severally with simple collisions, hyperbolic motion where the three mutual distances become infinite with the time, and hyperbolic-parabolic motion where two distances are of the order 1 and the third of order 2/3. He has also investigated the propriety of reversing the motion in the problem of three bodies.

Jacques Levy has studied the minima of the mutual distances, and shows the existence of a solution where one distance has an infinite number of successive minima, which tend toward zero as the time increases indefinitely.

Uno, Bélorizky, Sémirot, and Dramba have studied the problem of collisions, both double and triple, and find series for the co-ordinates proceeding, in the different cases, according to different simple fractional powers of the time.

Georges Meyer has studied periodic solutions in the problem of n bodies in the neighbourhood of relative equilibrium, which generalize the solutions of the problem of three bodies given by Euler and Lagrange, and the solutions asymptotic to these periodic solutions.

Mendes has studied the problem of n bodies with variable masses.

Lahaye, in a series of papers on collisions in the problem of n bodies, studies simultaneous double collisions, triple collisions, and quadruple collisions, and obtains convergent series proceeding according to rational powers of the time in the first case, and according to rational and irrational powers of the time in the other two cases. He also obtains a regularizing function which permits the definition of the co-ordinates and the time by convergent integral iterations, converging much more rapidly than the series of Sundman.

Belorizky has studied the convergence of the entire series by which Sundman has integrated the problem of three bodies, with a view to discovering whether these series can be applied to the motions of the planets, the Moon, and comets. He has made some trials under conditions more favourable than those actually existing, but the resulting series are so slowly convergent as to be less useful than the classical series heretofore employed.

Savchenko has studied the theory of the motion of bodies with variable masses.

Moiseev has studied the theory of characteristics of the contacts of trajectories with the curves of a given auxiliary family, the theory of the orbital stability of trajectories, and the theory of the stability of motion with finite deviations. His associates have published a number of articles devoted to concrete applications of these methods to particular cases of the three-body problem and other problems.

Lemaitre has published a paper on the canonical variations in Keplerian motion.

Armellini has studied asymptotic solutions of the general equations of motion.

Levi-Civita has studied the regularization of the three-body problem by means of a system of variables that remain finite at the time of a collision.

Planetary Theory. Nechvile has treated a new form of differential equations for the resonance case of planetary motion, where the two finite bodies move in Keplerian ellipses, and the motion of the disturbed body is in the same plane. With the true anomaly as independent variable, he transforms the system to differential equations of Fuch's type, of the second order, exactly integrable with the assistance of power series.

Brouwer in 1937 investigated the accumulation of errors in methods of numerical integration. He found that the accumulation of error is independent of the method used; it consists of a part in the mean longitude having a mean value proportional to the 3/2 power of the number of steps, plus contributions by errors in the other elements that increase proportionally to the 1/2 power of the number of steps.

Merton has devised a modification of the perturbation-of-elements method, which should prove advantageous for the rigorous numerical integration of the special perturbations of periodic comets.

Herrick has devised another modification of the same method, especially adapted for large eccentricities. In connection with the accumulation of error by this method, he says in a letter: 'I am now satisfied that the procedure. . does not avoid the accumulation of error of a double summation, as I believed and implied in writing my first article. Accordingly it will be preferable, I believe, to adopt Merton's suggestion for direct integration of the mean anomaly. This method is thus equivalent to all others considered by Brouwer, and its advantage (so far as the accumulation of error is concerned) will lie only in longer intervals and fewer steps, when these are possible.'

Brouwer has published a method for the integration of the equations of general planetary theory in rectangular coordinates, using canonical variables. The method resembles Hansen's in some respects, but is more laborious than Hansen's if only perturbations of the first order of the masses are considered. For perturbations of higher order the new method appears to have decided advantages. A numerical application, not yet published, has been made by M. S. Davis, who shows that the first-order perturbations obtained by this method and by Hansen's are identical.

Moiseev has studied the theory of secular and long-period perturbations, and has developed a method for calculating them separately from the terms of short period, showing that the methods of Gauss, Gauss-Fatou, and Hill-Delaunay may all be regarded as the result of a certain averaging of the disturbing function. He has pointed out a number of new averaging schemes which may be used for secular and long-period perturbations, as well as for the construction of intermediary orbits. Rein pointed out a variant of the limited single averaged elliptical three-body problem.

Whipple's work on the identity of the Taurid meteors with Encke's comet was followed by Brouwer's new analysis of the secular variations of the elements of this comet. With van Woerkom he has made a new solution of the secular variations of the elements of the principal planets, which represents an improvement on previous solutions in two respects: more precise values of the planetary masses were used, and the effects of the second-order terms due to Jupiter and Saturn were included, it has not yet been possible to include Pluto in the solution. The results have been applied to the secular variations of the minor planets, in this way the number of known families has been considerably increased.

Široký has studied the gaps in the ring of minor planets and concludes that they are cosmogonical and not mechanical in character; he examines various laws analogous to those of Bode-Titius and Mohorovičič.

Milankovitch has devoted much attention to possible connections between the glacial ages and the secular variations of the elements of the Earth's orbit. Michkovitch has revised some of the work of Leverrier on the secular variations. They consider that more work on the subject is necessary.

Roure has sought to simplify the calculation of ephemerides of minor planets by combining the method of the Hill-Brown lunar theory with Bohlin's group theory.

Reznikovsky has applied the theory of periodic and non-periodic solutions by Liapunov-Duboshin's method to the secular perturbations of Mercury.

Chazy has studied the advance of the node and perihelion of a planet under the action of a circular ring of matter, on the hypothesis of a small correction to Newton's law, proportional to the product of the Newtonian attraction by the derivative of the radius with respect to the time.

Melchior has published a series of papers on canonical variables and the representation of planetary distances.

Brendel proposed to use as one of the angular arguments in the planetary theory the angle at the disturbed planet between the lines to the Sun and the disturbing planet, instead of the angle at the Sun, as is usual. Behrens has applied the principle to one of the minor planets.

Elenevskaya has worked out a development of the disturbing function with the eccentric anomaly as independent variable, according to powers of the eccentricities without the use of Newcomb's operators.

Duboshin gave his method of developing the principal part of the disturbing function in series, and applied it to the cases of a round material ring, the Gaussian elliptic ring, a flat two-dimensional ring, and an ellipsoid with arbitrary density distribution.

The U.S. Naval Observatory and the Yale University Observatory, with the support of the Office of Naval Research, have been collaborating with each other and with the Watson Scientific Computing Laboratory, in several studies of the motions of the principal planets. One of the principal results so far obtained is a calculation of the co-ordinates of the five outer planets from 1653 to 2060 at 40-day intervals, by simultaneous numerical integration with the Selective Sequence Electronic Calculator of International Business Machines Corporation. The results, particularly in the case of Saturn and Uranus, agree much better with observations than do the tables of Hill and Newcomb: appreciable errors in the general theories of Jupiter, Saturn, Uranus, and Neptune, increasing in magnitude with the time from 1850, are indicated. The first-order general perturbations of Mars by Hansen's method have been calculated by Clemence; appreciable errors in Newcomb's theory are indicated.

The meridian observations of Mercury have been thoroughly discussed by Clemence, those of the Sun by Morgan and Scott, and those of Mars (not yet published) by Scott. Other discussions are under way: for Venus by Duncombe, and for Jupiter by Hertz. The relativity effect in the motion of the Earth has been detected observationally, and that of Mercury is confirmed to a high degree of accuracy. These discussions, supplemented by similar discussions of the motions of selected minor planets, will eventually lead to a good determination of the constant of precession.

Brouwer and Wylie, from a study of the perturbations produced by Pluto in the motion of Neptune, conclude that Pluto's mass is not greatly different from the mass of the Earth. Kourganoff, in connection with a study of the prediscovery observations of Uranus, comes to the same result. These dynamical determinations of Pluto's mass, combined with Kuiper's determination of the diameter, lead to a value of the density that seems inadmissibly large.

At the Institute of Theoretical Astronomy of the Academy of Sciences of the U.S.S.R. are being developed the following problems: (*a*) Improvement of the analytical methods in the planetary theory. (*b*) Applications of periodic orbits to the minor planets. The case of the 2 : 3 commensurability was worked out in detail, and the motion of Hilda represented satisfactorily for 75 years. (*c*) The perturbations of Pluto by Jupiter, Saturn, and Uranus have been developed analytically, and it is planned to start work soon on the perturbations by Neptune. (*d*) The general perturbations of Ceres by all the major planets has been completed to the first order. The second-order perturbations by Jupiter are being calculated. (*e*) The motion of Jupiter's eighth satellite as studied by numerical integration from 1908 to 1950, and work was begun on a new analytical theory.

B. A. Orlov has applied the Delaunay-Hill method to the 3 4 group of minor planets.

Herget has applied modern punched-card calculating machines to several numerical problems of considerable magnitude. He has completed precise numerical integrations of Ceres, Pallas, Juno, and Vesta, from 1920 to 1960, also the Jupiter-perturbations by Hansen's method of special perturbations for twelve minor planets. At present he is engaged in calculating precise ephemerides of the Sun and Venus from 1800 to 2000, using the tables of Newcomb.

Hertz has extended Brown's ideas for a theory of the Trojan asteroids, with a numerical application to Nestor, as yet not published.

Satellite Theory. E. W Brown and D. Brouwer completed the second part of the theory of Jupiter's eighth satellite, by the method of Brown, using the true orbital longitude as independent variable. The theory was recently compared with observations by Proskurin, who finds that large residuals must be ascribed to incompleteness of the theory and errors in the elements. The choice of independent variable leads to practical difficulties in comparing with observations.

Grosch has executed an extensive precise numerical integration of the motion of Jupiter's eighth satellite.

Sharpless found an appreciable secular increase in the orbital mean motion of the inner satellite of Mars. Later, van Woerkom found a similar effect in the motion of Jupiter's fifth satellite.

A. A. Orlov developed a theory of periodic orbits of arbitrary inclination and applied it to the satellite-system of Uranus.

Woolard, by an extension of the classical theory, has partially reconciled the observational and theoretical values of the ratio between the motions of the poles of the orbits of the satellites of Mars.

Duboshin studied the general properties of the motion of a material point in a certain gravitational field and in a resisting medium, and obtained precise conditions for stability (in Liapunov's sense). The results were applied to the problem of the stability of Saturn's rings, considering a particle in the ring as moving under the influence of the planet (taken as ellipsoidal with ellipsoidal distribution of density), of the remaining particles of the ring, and of all the satellites of Saturn. Taking into account the resistance of the medium, Duboshin showed that under given definite values of the parameters of the system, stable circular motions in the plane of the ring are possible for the particle under consideration.

Duboshin also studied the analytical theory of the satellites of Saturn strictly according to the methods of Liapunov, which make it possible to obtain absolutely convergent series for the Saturnicentric co-ordinates of the satellites, under the action of the ellipsoidal planet, its rings, the other satellites, and the Sun. He employs rectangular and cylindrical co-ordinates. He and his collaborators are now engaged in calculating the actual series that give the co-ordinates of the satellites.

Armellini has continued his work on Hill's lunar theory.

Attraction between bodies of finite size. Brouwer made a study of the motion of a particle with negligible mass under the gravitation attraction of a spheroid. This led to an extension of the problem to the motion of a system containing two rigid bodies. It was shown that the results obtained may be applied to close binary systems, although the analysis may prove difficult.

Evrard has studied the figure of equilibrium of double stars in the case where periods of rotation and revolution are equal, and derives equations for both homogeneous and heterogeneous fluids. Zagar has studied the dynamics of binary and triple systems.

Merlin has studied the attraction between an ellipsoid and an exterior point.

Harold Jeffreys, from a study of the Earth's constitution, concludes that the long-outstanding discrepancy between the values of the mass of the Moon, as determined from the observed lunar inequality and from the observed constant of nutation, must be attributed to inadequacy of the classical theory of the nutation. He stresses the importance of observational determinations of the terms of short period, and of the principal coefficients in longitude and obliquity independently of each other.

Sekiguchi has re-developed the classical theory of precession and nutation, assuming three unequal principal moments of inertia; he concludes that the effects of the difference between the equatorial moments are too small to be appreciable to observation, but asserts that methods hitherto in use are inadequate.

Elliptic motion. Melchior has published a series of papers on canonical elements and representation of planetary distances.

Široký makes certain modifications in the Lagrange-Wilkens method of determining orbits.

Petr has made a new derivation of certain equations of elliptic motion.

Bazhenov has proved the convergence of iterative processes in the determination of orbits.

Miscellaneous work. Eckert and Brouwer developed a new method for the differential correction of orbits, using rectangular co-ordinates; this method is especially useful in connection with numerical integrations that provide the rectangular co-ordinates directly.

Porter devised a short method for approximate differential correction of orbits of comets and minor planets, using a few new observations, by separating the elements in such a way as to reduce the labour of calculation considerably.

Kuiper derived a new law of planetary and satellite distances, consistent with a hypothesis regarding the origin of the solar system, which satisfies the actual system considerably better than the Bode-Titius law. Svestka shows that with Kuiper's law, the validity of the Bode-Titius law is accidental.

Kourganoff has made an extensive and delicate study of the role played by celestial mechanics in the discovery of Pluto; he concludes that Lowell's calculations, based on the departures of Uranus from its ephemeris, effectively bounded an area of the sky, in which the planet was found.

Merlin has made a study of the problem of two bodies with decreasing mass.

Swings and Goffin have studied planetary perturbations under certain quasi-Newtonian laws of gravitation.

Armellini has examined the variation of eccentricity of the orbit in the problem of two bodies of decreasing mass, and concludes that the eccentricity can increase in the case where the product of the time by the mass of the system approaches zero as the time approaches infinity. He has also examined the effects of certain small changes in Newton's law, calculating the resulting secular changes in the elements.

Whipple has formulated a theory of the constitution of comets, under which the observed secular change in the mean motion of certain comets may be explained by the evaporation of material when they are near the Sun.

G. M. Clemence
President of the Commission

Rapport Supplémentaire

BRIEF SUMMARY OF WORK CARRIED OUT IN THE U.S.S.R. (1932–51)

Works on celestial mechanics were carried on at the Institute of Theoretical Astronomy (Leningrad), the Sternberg Astronomical Institute (Moscow), the Kharkov Astronomical Observatory, the Lvov Astronomical Observatory, the Geophysical Institute (Moscow), and at some other institutes.

This summary does not include the works within the competence of other Commissions (Moon, Minor Planets, Comets and Satellites).

Institute of Theoretical Astronomy of the Academy of Sciences of the U.S.S.R.

This Institute was established in 1943 to replace the previously existing Leningrad Astronomical Institute. The problems being worked out at this Institute may be noted as follows:

(1) The development of analytical methods for determining planetary perturbations for the purpose of making them more effective, as well as to clarify various questions concerning the convergence of applied expansions. In particular, the question of employing other independent variables instead of the time in the expansions was studied. (M. F. Subbotin, N. S. Yakhontova.)

The question of the practical significance of Wilkens's method for the expansion of the perturbative function was studied by N. A. Bokhan.

(2) The study of periodic solutions of the restricted problem of three bodies (Hill's problem) and the utilization of these solutions for the study of minor planets during long periods of time. By means of expedient construction of a resulting solution new classes of periodic solutions have been obtained in which it is possible to dispose of four parameters, whereas in the Poincaré and Schwarzschild solutions it is possible to dispose of only two parameters. (G. A. Merman.)

The question of the construction of such periodic solutions by numerical methods, which would be useful for the study of the motion of minor planets in cases of close commensurability. has been investigated at length. The case of commensurability of 2 : 3 has been especially studied in great detail. With the aid of a periodic orbit constructed for this case it was possible to represent quite satisfactorily the motion of Hilda during seventy-five years, that is, from the time of its discovery up to the present time. (G. A. Chebotarev.) The possibility of a further application of this method both to minor planets and to comets is now being studied.

(3) The construction of the analytical theory of the motion of Pluto. The determination of the perturbations of Pluto from Jupiter, Saturn, and Uranus has been completed. In the immediate future, the perturbations from Neptune will be determined and final improvement of Pluto's elements will be carried out. (Sh. G. Sharaf.)

(4) Construction of the analytical theory of the motion of Ceres. The determination of perturbations of the first order from all major planets according to Hill's method has been completed. The computation of perturbations of the second order from Jupiter and the preparation for the comparison of the constructed theory with observations for the final determination of the integration constants are being carried out. (V F Proskurin.)

(5) The motion of satellite VIII of Jupiter was studied in two different ways. On the one hand, the motion of this satellite from 1908 to 1950 was represented by the numerical integration method, and a final determination of the elements of its orbit prepared with the aid of all available observations. (D. K. Kulikov.) On the other hand, the inadequacy of Brown's analytical theory for a more or less satisfactory representation of the motion of the satellite was elucidated and the construction of a new analytical theory of its motion was begun. (V F. Proskurin.)

(6) Methods for the correction of orbits of minor planets and comets were improved. (M. F. Subbotin, N. S. Yakhontova, and K. D. Kulikov.)

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(7) The improvement of the methods for the computation of orbits from three observations was the object of works by P. Sh. Mesis and A. V Purtskhvanidze.

(8) The processing of all the observations of the path of Mercury about the disk of the Sun in order to find the relative secular acceleration of the perihelion of Mercury. This acceleration resulted to be equal to $42''.65 \pm 0''.60$, which agrees very well with the theoretical value. (G. A. Chebotarev.)

State Sternberg Astronomical Institute

The basic scientific work of the Department of Celestial Mechanics consisted of the following two problems:

(1) The development of methods for the qualitative analysis of the characteristics of motion.

(2) The development of methods for a strict analytical qualitative analysis.

In both problems the application of a method being developed to the concrete questions of celestial mechanics was also specified.

In the field of the first problem, Prof. N. D. Moiseiev has given a series of observations applicable to the theory of the trajectory contacts characteristics: (i) with curves of a given auxiliary family, (ii) to the theory of orbital stability of trajectories, (iii) to the theory of longitudinal and lateral stability of phase trajectories, (iv) to the theory of motion stability with terminal declinations, etc.

Moiseiev and his pupils (N. F. Rein, G. K. Badalyan, A. N. Chibisov, V T. Kondurar, I. P Tarasashvili and others) have published a number of works on the concrete applications of these methods of qualitative analysis to various particular cases of the restricted problem of three bodies and other problems of celestial mechanics.

In the field of celestial mechanics N. D. Moiseiev was mainly engaged in the development of the theory of secular and long-period perturbations. He developed a certain systematization of the so far existing simplified tables of the problems of celestial mechanics, pursuing the object of selecting secular or long-period perturbations (Gauss tables, Gauss-Fatou table, and Hill-Delaunay table). N. D. Moiseiev proved that all these known tables may be considered as a result of a certain perturbative function averaged in an appropriate manner. When developing and generalizing the averaging methods used, M. D. Moiseiev showed a series of averaged tables of problems of celestial mechanics, which may be successfully applied in the study of secular and long-period perturbations as well as for the construction of intermediate orbits (once-averaged inner variant of the problem of three points, a generalized once-averaged problem of three points of Hill-Delaunay, averaged variants of the problem of three points obtained by interpolation, etc.). An important new variant of the once-averaged elliptic problem of three points was shown by N. F. Rein.

Considerable results were obtained by A. A. Orlov in the field of the analytic theory of motion in averaged problems of celestial mechanics. He developed the theory of periodic orbits of arbitrary inclination and applied this theory in the investigation of the motion of the satellites of Uranus. The theory of periodic or non-periodic solutions, which are obtained through the Lyapunov-Duboshin method, was applied by P. T. Reznikovsky in the study of secular perturbations of the motion of Mercury on the basis of Moiseiev's averaged tables.

Between 1932 and 1940, Prof. G. N. Duboshin studied mainly the theory of motion of a material point in a certain gravitation field and in the resisting medium. The general characteristics of motion were examined with the aid of the methods of the qualitative theory of differential equations. The most important problem in this field was considered to be the problem of stability in Lyapunov's sense of circular solutions, which may exist when certain specific conditions are carried out.

The conditions for the existence of circular solutions were obtained and it is established that these conditions can be effectively carried out in most urgent astronomical cases.

In order to investigate the stability of these solutions the general theorems of Lyapunov's second method were applied, by means of which the exact stability conditions were obtained. The investigation referred to was applied to the problem on the stability of Saturn's ring. The latter problem was as follows: One of the infinite numbers of particles forming the ring of Saturn was examined. Considering that such a particle moves under the effect of the planet's attraction, which was considered as an ellipsoid body with an ellipsoid distribution of densities, under the effect of the attraction of all remaining particles of the ring, and under the effect of the attraction of all satellites of Saturn, G. N. Duboshin, making an allowance for the resistance, determined that at certain values of basic parameters of Saturn's system, for the particle being examined, circular motions in the ring's plane are possible and that these circular motions are really stable in A. M. Lyapunov's sense.

From 1940 to 1950, G. N. Duboshin studied mainly the analytic theory of the satellites of Saturn.

This theory, in contradistinction to well-known classical theories, was constructed on the basis of A. M. Lyapunov's strict methods which make it possible to obtain absolutely convergent series representing the saturnicentric co-ordinates of the satellites, with the consideration of the effect of the attraction of the ellipsoid planet, its ring, perturbative effect of other satellites, and the perturbative effect of the Sun.

Here G. N. Duboshin rejects the application of Keplerian co-ordinates and prefers to use ordinary rectangular or cylindrical co-ordinates. In this consists the other difference of G. N. Duboshin's theory as compared to other known classical theories.

For a possibly better calculation of the forces affecting a satellite moving around Saturn, it is was also necessary to examine certain problems in potential theory.

G. N. Duboshin gave his method of the expansion of the inverse distance into an infinite series and on the basis of the general expansion resulting he obtained convenient and simple expansions of force functions of the material circular ring, Gauss's elliptic ring, flat two-dimensional ring, and an ellipsoid with an arbitrary distribution of densities.

At the present time, Duboshin and his collaborators are occupied with the factual calculating of series determining the co-ordinates of the satellites, for which Struve's classical observations carried out by him on a 30 cm. diameter refractor in Pulkovo at the end of the last century are utilized.

In the sector of celestial mechanics of the Lvov Astronomical Observatory (N. B. Yelenevskaya) investigations on the expansion of the perturbative function in the case of great mutual inclination of orbits were carried out. In particular, a new method for the expansion of the perturbative function into a trigonometric series in multiples of the mutual inclination of the orbits has been developed. Yelenevskaya also worked out in detail the question of the expansion of the perturbative function in multiples of eccentric anomaly and powers of the eccentricity without the aid of Newcomb's operators.

In the section of celestial mechanics of the Kharkov Observatory, G. M. Bazhenov studied in detail the question of utilizing matrices in celestial mechanics. He was also the first to give proof of the convergence of iterative processes in the problem of determining orbits. Also a series of improvements were introduced into the solution of this problem. K. N. Savchenko developed a theory of motion of bodies with variable masses.

At the Geophysical Institute of the Academy of Sciences of the U.S.S.R. the final motions in the problem of three bodies and in the problem of n bodies were studied. On a particular example, worked out by O. Yu Schmidt, N. N. Parysky. and G. Khilmi, it was determined that in the problem of three bodies, in spite of certain results by Chazy, capture is possible. G. F. Khilvi found several general criteria of the formation and dispersion of stable subsystems in the system of gravitating bodies.

Prof. M. F. Subbotin