Let AB be produced to meet in E a straight line through C drawn parallel to OB.

Then $B\widehat{C}E = C\widehat{B}O = O\widehat{B}A = C\widehat{E}B$.

$\therefore$ $BE = BC$, and the triangle BEC is similar to the triangle OBC.

Then on a certain scale the velocity of P when in AB is represented by BE, and on the same scale the velocity of P when in BC is represented by BC; then on a certain scale the change of P's velocity at B is represented by EC.

Hence the magnitude of the change is

$$V \cdot \frac{EC}{BE} = V\frac{BC}{OB},$$

its direction BO.

The time P takes to move from B to C is $= BC \div V$.

Dividing the change of velocity by this time, which is the interval between two successive changes in P's velocity, we get

$$V\frac{BC}{OB} \div \frac{BC}{V} = \frac{V^2}{OB}.$$

Now suppose the number of sides in the polygon to increase indefinitely, while V and OB remain the same, and the motion tends towards that of a point moving with uniform speed V in the circumference of a circle of radius $R = OB$. And in the limit the quantity $\frac{V^2}{R}$ becomes the acceleration of P in this motion, the direction being inwards along the radius vector of P.

R. F. Muirhead.

Feuerbach's Theorem.

Generally $\sum a^2(b^2 + c^2 - a^2)(b - c)^2$ is divisible by

$$\sum (b + c - a)(b - c)^2,$$

the quotient being abc.

Let a, b, c be the sides of a triangle ABC; D, E, F their middle points. The tangent from D to the in-circle is equal in length to

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$\frac{1}{2}(b \sim c)$. The in-centre I is the centroid of masses proportional to

$$4(b+c-a) \text{ at } D,\ 4(c+a-b) \text{ at } E,\ 4(a+b-c) \text{ at } F,$$

while the Nine-Point-centre is the centroid of masses proportional to

$$4a^2(b^2+c^2-a^2) \text{ at } D,\ 4b^2(c^2+a^2-b^2) \text{ at } E,\ 4c^2(a^2+b^2-c^2) \text{ at } F.$$

Hence

$$\textstyle\sum(b+c-a)(b-c)^2 = 2NI.\ 8s. \quad \text{(perp. from } I \text{ on radical axis)}$$
$$\textstyle\sum a^2(b^2+c^2-a^2)(b-c)^2 = 2NI.\ 64\triangle \quad (\ldots\ldots\ldots\ldots N \ldots\ldots\ldots\ldots\ldots\ldots)$$

or the perps. from I and N are in the ratio $64\triangle : 8\,abcs$ or $r : \frac{1}{2}R$.

Thus the radical axis of the in- and Nine-Point-circles divides externally the join of the centres in the ratio of the radii, and consequently the circles touch each other.

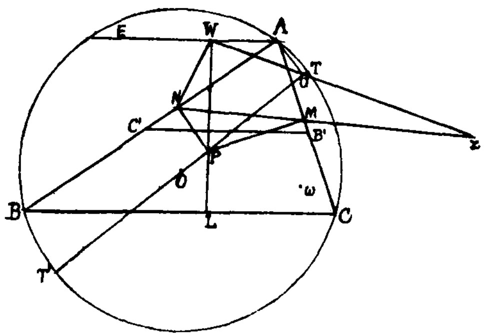
Note that $\sum(b+c-a)(b-c)^2$

$$= 2\{a^3+b^3+c^3+3abc-ab^2-ac^2-bc^2-ba^2-ca^2-cb^2\}$$
$$= 4\triangle(R-2r),$$

and that R is always greater than $2r$, except when $a=b=c$.

R. F. Davis.

Geometrical Note on the Orthopole.



Lemma.—If A, U are given fixed points; AC, AB, AE given fixed straight lines through A; and a variable circle through A, U intersects these straight lines in M, N, W respectively; then the locus x of the point of intersection of MN, UW will be a straight line parallel to AE.

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