WHERE IS THE EQUINOX ?

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1. INTRODUCTION

Within the work being carried out at Heidelberg on the establishment of the new fundamental reference coordinate system, the FK5, the determination of the location of celestial equator and the equinox form an important part. The plane of the celestial equator defined by the axis of rotation of the Earth and the plane of the ecliptic defined by the motion of the Earth about the Sun are both in motion due to various causes. The intersection of the equator and the ecliptic, the dynamical equinox, is therefore in motion. Great efforts have been made in the past to determine the location and motion of the dynamical equinox by means of observations of Sun, Moon and planets in such a manner that the dynamical equinox can serve as the origin of the right ascension system of a fundamental catalogue. The results have not been satisfactory, and we have some important evidence that the catalogue equinox of the FK4 is not identical with the "dynamical equinox". Moreover, is has turned out that the difference $\alpha(\mathrm{DYN}) - \alpha(\mathrm{FK4}) = E(T)$ depends on the epoch of observation T. Duncombe et al. (1974) have drawn attention to the possible confusion between the catalogue equinox and dynamical equinox; they mention the difference between two Earth longitude systems, one established by the SAO using star positions on the FK4 and the other one established by the JPL using planetary positions measured from the dynamical equinox. This is undoubtedly one legitimate explanation of the difference, even if other sources of errors may also have contributed.

Obviously the location of the FK5 equinox will have to be determined together with the equator point from modern observations of members of the solar system, as far as these allow us to determine corrections to the zero points of the FK4 systems of right ascension and declination. The problems arising in this task shall be described here. This discussion shall be restricted to the location and motion of the equinox, since it has been known for some time that the equator point offers less problems. In the FK4 the equator point is determined with a mean error of ± $0\overset{''}{.}021$ and no significant secular change was found.

R. L. Duncombe (ed.), Dynamics of the Solar System, 133–143.

2. NEWCOMB'S EQUINOX AND ITS DEFICIENCIES

Newcomb (1872) has carried out a discussion of the observations of the Sun from 1756 to 1869 resulting in a determination of the equinox to which the right ascensions of clock stars were to be referred. This equinox, commonly called N_1, was adopted by Newcomb (1882) as the zero point in right ascension in his "Catalogue of 1098 Standard Clock and Zodiacal Stars". This equinox has often been used as a reference up to the present day. In his determination Newcomb followed on the lines of the pioneer work of Bessel (1830) who had made one equinox determination from Bradley's observations 1750-1762 and another one from his own observations from 1822 to 1835 at Königsberg. On this basis Bessel presented his famous "Tabulae Regiomontanae" which give the mean and apparent places of 36 "Maskelyne Stars" from 1750-1850. In fact, Bessel's "Tabulae" was used during a large part of the 18th century as a fundamental reference coordinate system. It is therefore quite natural that Newcomb presented his result of equinox determinations as a correction in the sense $E = \alpha_{Obs} - \alpha_{Bessel}$. Figure 1 shows Newcomb's corrections E obtained from 26 catalogues which had included observations of the Sun.

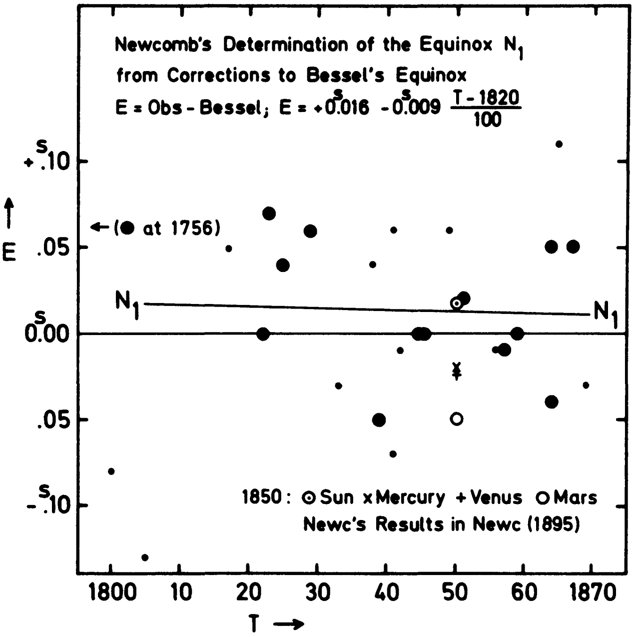

Figure 1.
Newcomb's corrections E to Bessel's equinox resulting in N_1.

The straight line represents Newcomb's equinox N_1 as the result of his least squares solution (small and large filled circles denote values of E of weight 1-2, and 3-5, respectively). The secular change is, however, not significant. The values E given for 1850 for the Sun, Mercury, Venus, and Mars are results obtained by Newcomb (1895, p. 96) from all available observations up to 1890. The most surprising feature of the figure is the smallness of the deviation of Newcomb's N_1 from Bessel's equinox.

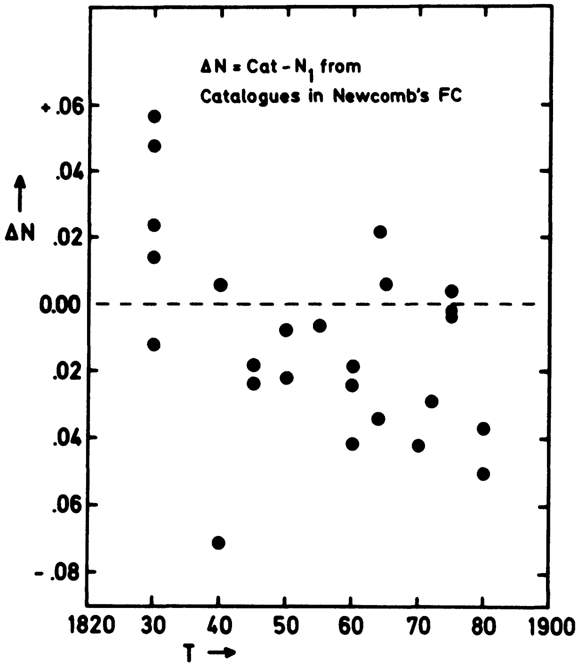

Figure 2.
Evidence of a secular variation of ΔN from catalogue comparisons with Newcomb's FC

In the formation of a fundamental catalogue Newcomb (1898) adopted the equinox N_1 as the zero point of right ascensions and the value of the general precession (Newcomb 1898) he had derived before. N_1 is therefore the catalogue equinox of Newcomb's FC. The comparison of N_1 with 28 catalogues has yielded values of ΔN from 1830 to 1880 which are shown in Figure 2. The catalogue comparisons had often to be made with a small number of bright clock stars, and in some cases the values $\Delta N = \alpha(\mathrm{Cat}) - \alpha(N_1)$ refer to catalogues which had adopted $\alpha_{(\mathrm{Dyn})}$ from observers who had established the zero point with observations of the Sun. The essential feature of Figure 2 is the secular variation of ΔN which has led Newcomb to the conclusion that there is an equinox motion which he has not taken into account in his FC.

3. EVIDENCE OF AN EQUINOX MOTION ?

Besides the evidence of a secular variation of ΔN provided by equinox determinations of the 19th century, one may obtain information on the size of the effect from an investigation of proper motions of stars given in a fundamental catalogue. For this purpose we have chosen Newcomb's FC (1899) whose basic material has exhibited the corrections ΔN given in Figure 2. The catalogue equinox is N_1, and it is the first fundamental catalogue, in which the consequences of a variation of ΔN should have become apparent. This catalogue has never before been investigated for this purpose, probably because Boss (1910a) soon afterwards had completed the PGC and determined corrections to Newcomb's precession on the basis of PGC proper motions. For the investigation of Newcomb's FC, which has very recently been completed by H. Scholl and myself, we have used the proper motions of 248 stars which occur in Fricke's (1967, 1977) basic material for the determination of corrections to Newcomb's precession. This material offers the advantage of a direct comparison with results obtained on the basis of the FK4. Our results are given in Table 1 under the headings "Newc FC" and "FK4", while the results obtained from the whole PGC by Boss (1910b) are under the heading "PGC". From the values under the heading "Fricke (1967, 1977)" the quantity Δp_1 = 1″.10 per century has been adopted in the IAU (1976) System of Astronomical Constants; and the value Δe = 1″.23 is under consideration as a correction to the FK4 proper motions in the formation of the FK5.

Table 1. Centennial values of precessional corrections from proper motions in Newc FC and from other sources

Source	Newc FC	PGC	FK4	Fricke (1967)
n	248		248	512
Δp_1	0″.53 ± ″.40	0″.88	1″.08 ± .25	1″.10 ± ″.15
Δe	1.09 .41	1.21	1.21 .24	1.23 .16

The essential result of our investigation of Newc FC is the revelation of an equinox motion Δe = 1″.09 ± 0″.41 (m.e.) in fair agreement with the result by Boss. This means that the whole amount of Δe found in the FK4 is already apparent in the proper motions based on observations made in the 19th century, as we had expected from the data of Figure 2. The result is in accordance with one obtained by Laubscher (1976) which is independent of hypotheses inherent in proper motion solutions. The question whether there is an equinox motion may thus be answered in the sense that zero point errors in right ascension have produced, indeed, a zero point error in the proper motions of the stars. We may therefore exclude the possibility of any unknown physical effect being responsible for the equinox motion.

It may be noted that Newcomb (1898) derived Δe = 0″.30 in his famous determination of precession, and that this result has often been misinterpreted, even by Boss (1910). It has no other meaning than that it indicates errors in two equinox determinations, namely, from Bradley's observations at mean epoch 1755 and from Airy's at 1860. Newcomb's work was based on the proper motions of Auwers' catalogue (1888) of Bradley Stars, and these were derived from systems of positions at 1755 and 1860. For more detailed information, reference is made to a rediscussion of Newcomb's determination of precession by Fricke (1971). One may be amazed that such a small value of Δe resulted, and it is not surprising that Newcomb has commented on this result in stating that (with respect to his system N_1) the value Δe = + 1″.00 is "legitimately worthy of discussion".

4. MAGNITUDE EQUATIONS IN POSITIONS AND PROPER MOTIONS

Systems of positons and proper motions can be strongly distorted by magnitude equations, i.e. by systematic errors depending on the magnitude. Küstner (1902) was the first to prove that if a transit circle is equipped with a micrometer with fixed threads and observations are made with full aperture, the transits of bright stars are observed too early and of faint stars too late. He found that magnitude equations can be avoided by the use of an impersonal micrometer with moving threads (hand- or motor driven) and by the use of a set of screens in front of the objective for reducing the light of bright stars to the magnitude of well observable faint stars. While every observer knows this now, there has been little concern so far about the consequences of the effect on equinox determinations of the 19th century.

From measurements of the magnitude equation in observations made at Lund from 1943 to 1945 with fixed threads and full aperture, Reiz (1951) has derived

$$\Delta\alpha_m = \alpha_{FK3} - \alpha_{Obs} = 0\overset{s}{.}004 - 0\overset{s}{.}0152\ (m - 4.5)$$

for his observations from $m = 2.5$ to about 8.5. This is a typical "personal equation"; other observers may find smaller or even greater effects. In Figure 3 this equation reduced to the FK4 is given as a dashed line. The full line in Figure 3 represents the magnitude equation of the NFK with respect to the FK4. The NFK (Neuer Fundamentalkatalog) is the second catalogue in the FK series. It was formed on the basis of 19th century observations by Auwers and completed by Peters (1907); it is the first fundamental catalogue in which the authors have taken magnitude equations into account whenever possible. The full line resulted from a comparison between the NFK and Küstner's observations made with imper-

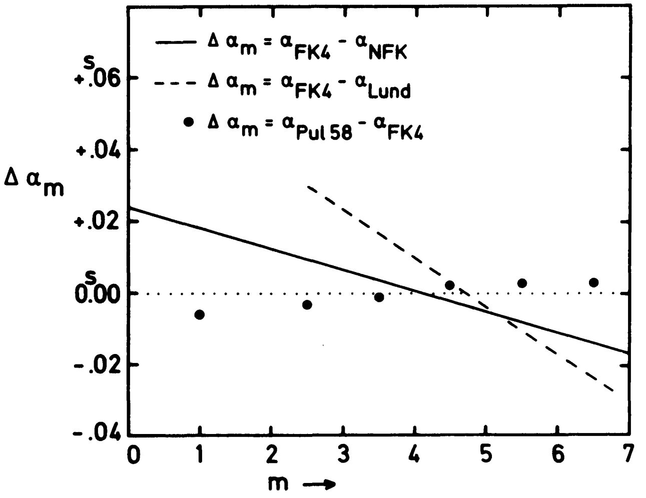

Figure 3.
Magnitude equations
in right ascension

sonal micrometer and screens and a reduction to the FK4 (the reduction was made in using $\Delta\alpha_m = \alpha$(FK3 - NFK) by Nowacki (1935) and the table $\Delta\alpha_m$ (FK4 - FK3) given in the FK4). In fact, the FK4 is very near to Küstner's observations as far as a magnitude equation is concerned. The full circles in Figure 3 represent the results of recent observations by Nemiro et al. (1977) presented in the catalogue "Pulkovo 58" which is claimed to be free of magnitude effects. Since the latest result obtained at the US Naval Observatory are similar to those of "Pulkovo 58" one may conclude that the FK4 is still affected by a small magnitude equation as was to be expected. In declinations no general trend arising from magnitude equations was found.

From the existence of magnitude equations in all older observations of right ascension we conclude that the equinox determinations before 1900 were affected correspondingly, such that the right ascensions of the Maskelyne Stars ($\bar{m} = 2.0$) were measured too small by $0^{\mathrm{s}}.02$ to $0^{\mathrm{s}}.03$ and of Airy's clock stars ($\bar{m} = 3.9$) by at least $0^{\mathrm{s}}.005$ to $0^{\mathrm{s}}.010$. From 1830 to 1900 there was no significant change in the techniques of observation with transit circles, but there was a considerable change in the choice of the clock stars from very few stars brighter than m = 1, then to about 30 stars of mean magnitude 2.0, and finally to about 200 stars of mean magnitude 4.0. Figure 4 shows the distribution in magnitude of the clock

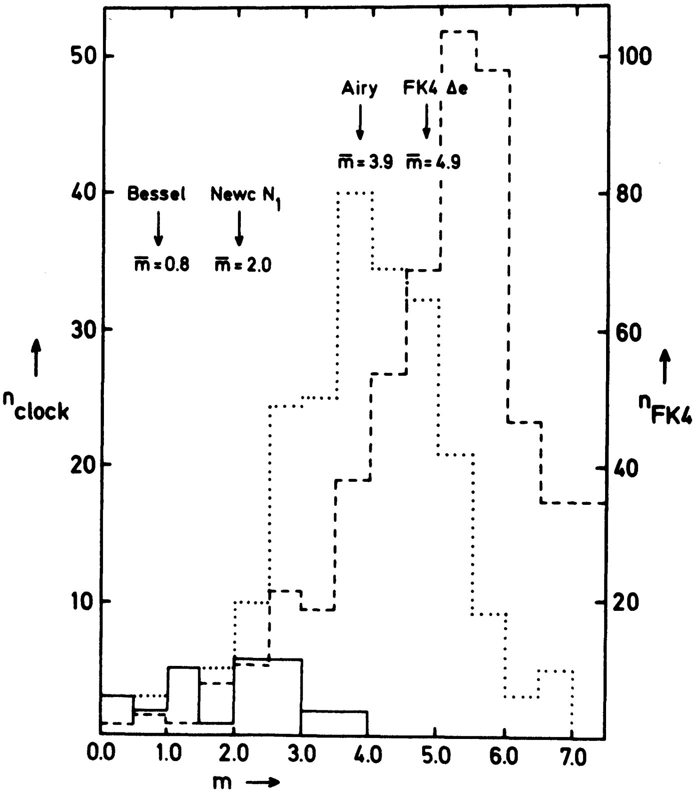

Figure 4.
Magnitude distribution in lists of clock stars and in the determination of Δe from the FK4

stars used in the determination of Bessel's equinox and of Newcomb's N_1; the Figure contains the distribution of Airy's clock stars and of the FK4 stars, which Fricke (1977) used in the determination of precession and of Δe.

One may wonder whether proper motions of stars in the FK4 system are affected by a magnitude equation such that the equinox motion Δe determined by Δk according to $\Delta k = \Delta n \cot\varepsilon - (\Delta\lambda + \Delta e)$ differs for bright and faint fundamental stars (Δn, $\Delta\lambda$ are the corrections to the general precession in declination and to planetary precession, respectively, and ε the obliquity at mean epoch). H. Scholl and the author have investigated this question in using the proper motions of the 518 FK4/FK4 Sup stars applied by Fricke (1967, 1977) in determining precession. The same procedure as in his determination has been used with some simplifications (equal weight for μ_α, μ_δ; no distances taken into account). Table 2 shows our results for Δn and Δk from solutions for equal numbers N of stars with mean magnitudes $\overline{m} = 3.95$ and 5.86, respectively. For comparison the result of our simplified procedure is given for the whole set of 518 stars.

Table 2. Solutions for Δn, Δk from proper motions of bright and fainter FK4/FK4 Sup stars. Units: arcsec per century. Errors: standard deviations.

N	256	256	512
m	0.3 to 5.2	5.4 to 7.7	0.3 to 7.7
<m>	3.95	5.86	4.90
Δn	+ .43 ± .09	+ .46 ± .10	+ .45 ± .07
Δk	- .25 .10	- .18 .12	- .21 .08

Table 2 shows the satifactory result that the corrections do not differ for bright and faint stars, and, hence, that the equinox motion in FK4 does not depend on the magnitude within the prevailing accuracy. Unfortunately, it not practicable to carry out the same analysis for Newcomb's FC.

5. MODERN EQUINOX DETERMINATIONS FROM OBSERVATIONS OF THE SUN AND PLANETS

The catalogue equinox of the FK3 determined by Kahrstedt (1931) is $N_1 - 0^s\!.05$ for 1913; it is a constant in FK3 for every given epoch. It resulted from determinations based on observations of the Sun, Mercury, and Venus around 1900. In the FK4 no change of the equinox was made because no significant correction appeared to be possible: For the GC, Boss (1937) adopted the equinox $N_1 - 0^s\!.04$, and Morgan (1952) adopted the same in N30 as a constant for every epoch.

Most of our present knowledge on the location of the equinox is based on the determinations which are given in Figure 5; this is a plot

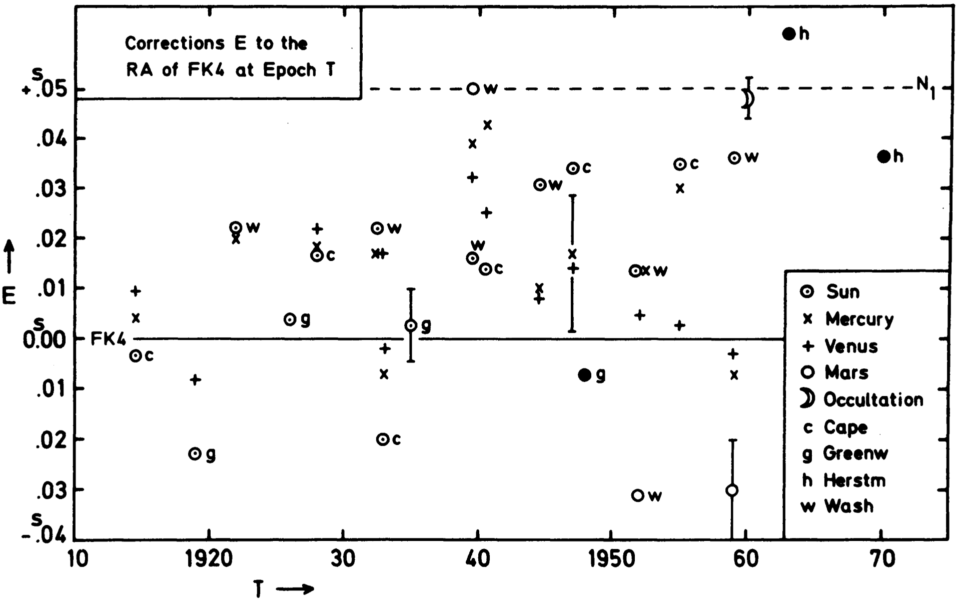

Figure 5. Corrections E from observations of different objects

of corrections E to the FK4 either as given directly by the observers or reduced to the FK4. The dashed line is Newcomb's N_1. Included in Figure 5 are the results from observations of the Sun, Mercury, Venus, Mars, and of an equinox determination by van Flandern from lunar occultations between 1950 and 1969. Filled circles are the results of observations of the Sun at Greenwich and Herstmonceux as given by Blackwell (1977) without details of the analysis. Error bars indicate typical mean errors of results from the various objects. The problems arising in the discussion of this material are the following: (a) Various results need corrections for the variation in the rate of rotation of the Earth; (b) some of the results require corrections for differences in RA observations made in day-light and at night, and (c) the relation between the observations and the FK4 has to be verified in some cases. As a rule, observers state too lightly a system to which their results are referred, and in rare cases only can this statement be confirmed. Another problem is the averaging of the results from different objects. One may, as Newcomb did, consider the Sun, Mercury, and Venus as the most suitable objects for equinox determinations and in this case average the individual results with due weights; in general, this procedure is not allowed. As can be seen in Figure 5, the observations of Mars at mean epoch 1940 (made with the 9" TC in Washington) have yielded a result which is discordant with those for the mean epochs 1952 and 1959 (obtained from observations with the 6" TC in Washington). For reasons given in the next section, it may well be advisable to use the observations of Mars only in combination with observations of minor planets. A comprehensive review of the methods for the determination of zero points and periodic errors of star catalogues including the various types of equations of condition has been given by Duma (1974). In considering the corrections determined at different epochs from the Sun alone, and in taking into account the variation of the rate of the Earth's rotation and the difference between day and night observations, Blackwell (1977) found an increase of E by about $0^{s}.05$ in the observations of the 20th century.

6. EQUINOX DETERMINATIONS FROM OBSERVATIONS OF MINOR PLANETS

Reference is made to Jackson's (1967) analysis of observations of minor planets for the purpose of determining the equinox. This work must be considered as an important manual in this field, in particular, since the criteria to be fulfilled by the observations either were not sufficiently known from the beginning or were not seriously considered. The criteria are: the observations have to be made in a well-defined system whose relation to a fundamental system must be known; a program must have observations of several minor planets because there are rather high correlations between the corrections of the individual elements of a minor planet orbit and corrections of the Earth's orbit and the coordinate system; the observations must be made at several oppositions and extending as far from opposition as possible.

From observations of Ceres, Pallas, Juno, and Vesta that fulfilled the criteria in an optimal way, Jackson obtained the correction $E = +0^{s}.002 \pm 0^{s}.015$ to the FK4 at epoch 1950.0. This result is equal to the average of Blackwell's value $E = -0^{s}.007$ at 1948 from Greenwich observations of the Sun and the value $E = +0^{s}.010$ at 1952 from Washington 6" TC observations of the Sun, Mercury and Venus by Adams et al. (1964); even if one takes $E = +0^{s}.013$ for 1952 from observations of the Sun alone, the average remains surprisingly good. In our opinion this is the only result so far known to us from minor planets fulfilling the conditions for corrections to the FK4.

Nikol'skaya (1972) has proposed a method for solving ill-conditioned equations in making use of a matrix calculus developed by Gavurin (1962). Duma and Koval (1977) have applied this method to the normal equations with 12 unknowns for each of the first four minor planets with the effect that, for example, from Vesta observations the value $E = -0^{s}.079 \pm 0^{s}.028$ obtained by the method of least squares is changed into $E = +0^{s}.005 \pm 0^{s}.002$. From a study of the method H. Scholl and I have come to the conclusion that there are no objections against Gavurin's matrix calculus but that it has been misinterpreted in the applications. From the matrix of the normal equations a matrix of Eigenvalues can be formed, and the Eigenvalues will greatly differ when strong correlations exist between the unknowns. In this case the matrix of Eigenvalues is split up into two parts, one matrix with large Eigenvalues and another one with small. Then the solution for the unknowns is the orthogonal sum of two vectors, where one is significantly determined and the other is highly insignificant. Nothing can be said against this. The authors, however, in considering the significantly determined part as a refined solution, in our opinion forget that they present that portion of the values of the unknowns which is significant, and this may be a very small part. There is apparently no way out of the difficulties in a solution of a problem, if one is dealing with unsuitable observations.

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DISCUSSION

Aoki: Have you already corrected the reference system for the change of precession ($\Delta P_1 = 1\overset{''}{.}1$/cent), when you have the Δe from the planetary observations?

Fricke: No.

Aoki: I think that we must change the system by introducing the corrections to precessions and equinox motion, simultaneously, for the introduction of a new fundamental system.

Fricke: You are quite right.

Aoki: Then, in order to have the equinox motion from planetary observations, it is necessary to have a system already corrected for the correction to the precession. Is it not true?

Fricke: This is not true, because a change in precession alone (without taking Δe into account) does not change the result of precession reductions; in other words, in a precession reduction an error in precession is in practice completely compensated by the corresponding error in the proper motions.