

**VII THEORETICALLY AND EMPIRICALLY ORIENTED APPROACH TO
SPECTRAL CLASSIFICATION**

Discussion Leader, Helmut Abt

SPECTRAL CLASSIFICATION FROM A THEORETICAL POINT OF VIEW

Erika Bohm-Vitense

University of Washington

ABSTRACT

We study the conditions under which a two-dimensional spectral classification according to T_{eff} and n_e is possible and when 3- or 4-dimensional spectral classification is necessary because the classification criteria are unavoidably influenced by the metal and helium abundances. For late type stars, where molecular absorption becomes important for the continuum a seven- or more- dimensional classification may be necessary. At this point spectral classification may not be useful any more. We find generally that a derivation of the gravitational acceleration from the spectroscopically determined electron density is only possible if the helium abundance is known.

1. INTRODUCTION

I would like to come back to the question of Dr. Feast: Is there still any use for spectral-typing? When looking through the literature it seems that even the experts in spectral classification feel on the defensive. In fact Dr. Morgan's note read to us by Father McCarthy said: The best thing would be to compute the whole spectrum and to compare it point by point with the observed high dispersion spectrum.

The discussion in the literature about the use of spectral classification centers around the question: Do we need a detailed spectrum analysis in order to determine the basic stellar parameters luminosity, L and temperature, T and possibly mass, M , or can we determine these quantities from a few criteria used in spectral

classification? Clearly if we want all the element abundances we have to analyze the whole spectrum or make at least a 92 dimensional classification.

The spectrum is formed in the stellar atmosphere which does not know M or L , only T and gravity $g \propto M/R^2$. Only if we have an independent method to determine M or R^2 can we determine L from T and g . For supergiants the spherical effects, probably important in Dr. Fehrenbach's discussion of the LMC supergiants (as suggested by Dr. Code earlier) may provide this additional information to determine the radius, R from the spectrum. For single stars this appears to be the only possibility. The atmosphere never knows about M .

Actually when looking at the luminosity criteria they do not indicate g either but rather the electron density n_e . For the determination of T and L we then actually have to consider three steps

1. classification according to T and n_e
2. determination of $g(T, n_e)$
3. determination of R or M in order to find $L(g, T)$

It should not surprise us that we find a different L for a given spectral type and luminosity class since additional parameters are involved. In the following discussion we shall be concerned with steps 1 and 2 only.

2. GENERAL THEORY OF LINE FORMATION

Spectral classification is usually based on lines or possibly discontinuities. As Unsold (1948) has shown, the line depth R of a weak line is given by

$$R_\lambda = \frac{F_c - F_\lambda}{F_c} \propto \frac{\kappa_L}{\kappa_c} \frac{d \ln B_\lambda}{d \tau_c} \Bigg|_{\tau_c = 2/3} \quad \text{for } \frac{\kappa_L}{\kappa_c} \ll 1 \quad (1)$$

Here F_c is the radiative flux on the continuum adjacent to the line

- F_λ is the flux in the line at the wavelength λ
 κ_c is the continuous absorption coefficient at the wavelength λ
 κ_L is the line absorption coefficient at the wavelength λ
 τ_c is the optical depth in the continuum at the wavelength λ

B_λ is the source function at the wavelength λ which in local thermodynamic equilibrium equals the Planck function.

If the line is not optically thin the R_λ for the center of the line still appears as the abscissa in the curve of growth and the equivalent width of the line will be a monotonic function of this R_λ .

It is the factor

$$\frac{d \ln B_\lambda}{d \tau_c} = \frac{d \ln B_\lambda}{dT^4} \cdot \frac{dT^4}{d\bar{\tau}} \cdot \frac{\bar{\kappa}}{\kappa_c} \quad (2)$$

which makes many theoreticians believe that spectrum synthesis is necessary in order to determine T_{eff} since the accurate determination of $\frac{dT^4}{d\bar{\tau}}$ requires the whole theory of stellar atmospheres and the complete knowledge of all the abundances. The rare elements heavier than the iron peak elements generally do not influence temperature stratification and the abundances of the more abundant elements appear to be well correlated except perhaps sometimes the abundances of C, N and O. So we actually have to deal only with the abundance parameters

$$Y = \frac{n(\text{He})}{n(\text{H})}, \quad Z(\text{Met}) = \frac{n(\text{Met})}{n(\text{H})}$$

describing the total metal abundance and perhaps the abundances of CNO which may vary independently. If the other metal abundances are not well correlated we shall call the star peculiar and not consider it here.

There may be other parameters influencing the atmospheric structure such as magnetic fields or rotation. The turbulence presumably is a function of T , n_e and metal abundance Z and will not introduce a new parameter.

In this context I shall call magnetic stars peculiar and shall not discuss them here. Dr. Slettebak talked about possible effects of very rapid rotation which we cannot take into account either because we do not know $\sin i$. We therefore ignore very rapidly rotating stars also. We will be dealing with so-called normal stars only.

3. DEPENDENCE OF THE TEMPERATURE STRATIFICATION ON THE METAL ABUNDANCES.

In the grey case $\frac{dT}{d\tau}$ is independent of the abundances. For temperature stratification element abundances become important only due to nongrey effects. These cause a rise in temperature in deeper layers called backwarming, and a decrease of the temperature in high layers called surface cooling. If the nongrey effects are mainly due to line absorption then the rise in temperature at $\tau = 2/3$ the ΔT due to backwarming can be estimated to be

$$\Delta T \sim T_{\text{eff}} \cdot 1/4 \cdot \eta \quad (4)$$

where η , the fraction of the total flux absorbed in the lines, was assumed to be $\eta \ll 1$.

We have computed $\eta(\tau)$ for supergiants with $T_{\text{eff}} < 9000$ K. In Fig. 1 (Bohm-Vitense 1975) we reproduce the variation of η_λ in a wavelength interval of about 200 Å. It was found

$$\eta_{4200} = \eta_0 + \alpha \log Z \quad \text{with } \alpha = 0.07 \text{ for } T_{\text{eff}} \sim 6000\text{K} \quad (5)$$

and $\alpha = 0.11$ for $T_{\text{eff}} \sim 4300\text{K}$.

A similar relation holds for the total line absorption η . If for a given star we consider Z to be uncertain by a factor of 10 then η will be uncertain by about 0.1 and T will be uncertain by about 2.5%. If we are willing to accept such an uncertainty in the temperature we do not have to consider the backwarming effect. When judging what this means we have to remember that the uncertainty in the solar temperature, the best known star, is still about 1%,

$$T_{\text{eff}}(\text{Sun}) = 5810 \pm 30^\circ, \text{ and for } \alpha \text{ Lyrae we have}$$

$$T_{\text{eff}}(\alpha \text{ Lyrae}) = 9500 \pm 200^\circ \text{ or } \pm 2\%.$$

These values come from the best possible spectral analysis.

The surface cooling effects cannot be estimated so easily. In our computations we did not find very large changes in the surface temperatures for stars for different Z . But the computations are not reliable at the surface. For spectral classification it would be safer to avoid lines originating in very high layers such as Na or molecular lines in solar type stars.

Next we have to consider the factor $\bar{\kappa}/\kappa_c$ in equation (2). If only one atom or ion is responsible for the continuous absorption in the high flux spectral region then $\bar{\kappa}/\kappa_c$ will be independent of Z .

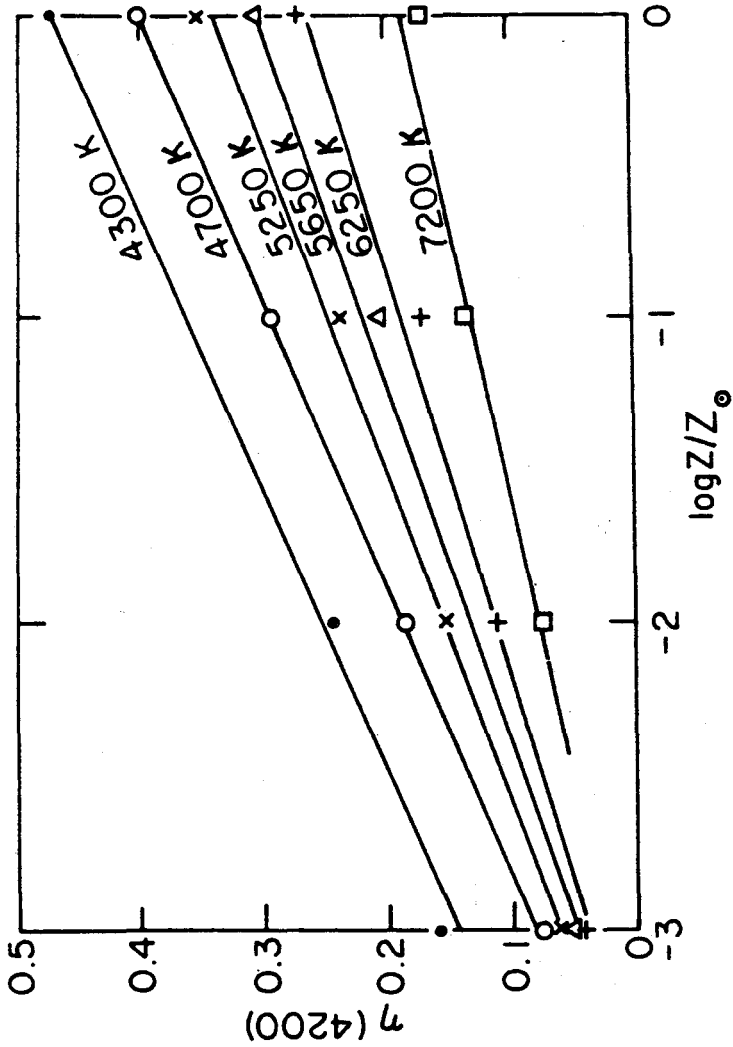


Fig. 1. Copied from Bohm-Vitense (1975). It shows the dependence of the line absorption η at $\sim 4200 \text{ \AA}$ on the metal abundance Z . η depends linearly on $\log Z$.

However, if with changing metal abundance a transfer occurs for instance from H^- absorption to H absorption as may happen for F stars, the κ/κ_c may change.

In Fig. 2 I have compared the $\bar{\tau}/\tau_{5000}$ for F stars with different Z . Even for these stars the Z dependence is rather small. (However the effects discussed by Dr. Feast for late type supergiants of the SMC may possibly be due to a transition from H^- absorption to Rayleigh scattering with decreasing Z .)

In Fig. 3 a comparison is given for $T(\bar{\tau})$ for F stars with different Z . At a given $\bar{\tau} > 0.1$ changes of T are of the order of a few hundred $^{\circ}K$ as estimated above.

Our computations deal with rather low temperatures only. I am not aware of similar studies for hot stars. Generally we do not expect large changes in Z for hot stars except perhaps in external galaxies.

From these discussions I would conclude that presently uncertainties introduced by ignoring the influence of Z on $\frac{d \ln B_{\lambda}}{d \tau_c}$ are tolerable considering the accuracies of the best temperature determinations that can be achieved now.

Uncertainties in Y could become important for hot stars if $Y > 1$, and the continuous κ of helium becomes more important than that of hydrogen. The He lines should, however, show large helium abundances.

In cool stars the continuous κ could change from H^- to H_e^- for large helium abundances as expected for carbon stars and some white dwarfs. Clearly these have to be considered separately for temperature classification.

With these precautions we then have to consider the factor κ_L/κ_c only for spectral classification.

4. CLASSIFICATION BY MEANS OF HYDROGEN LINES AND BALMER DISCONTINUITY

The first factor, namely κ_L/κ_c , on the right hand side of equation (1) generally determines the classification criteria. A two dimensional classification according to T and n_e , independently of Y and Z is possible from the hydrogen lines and the Balmer discontinuity as long as they are measurable.

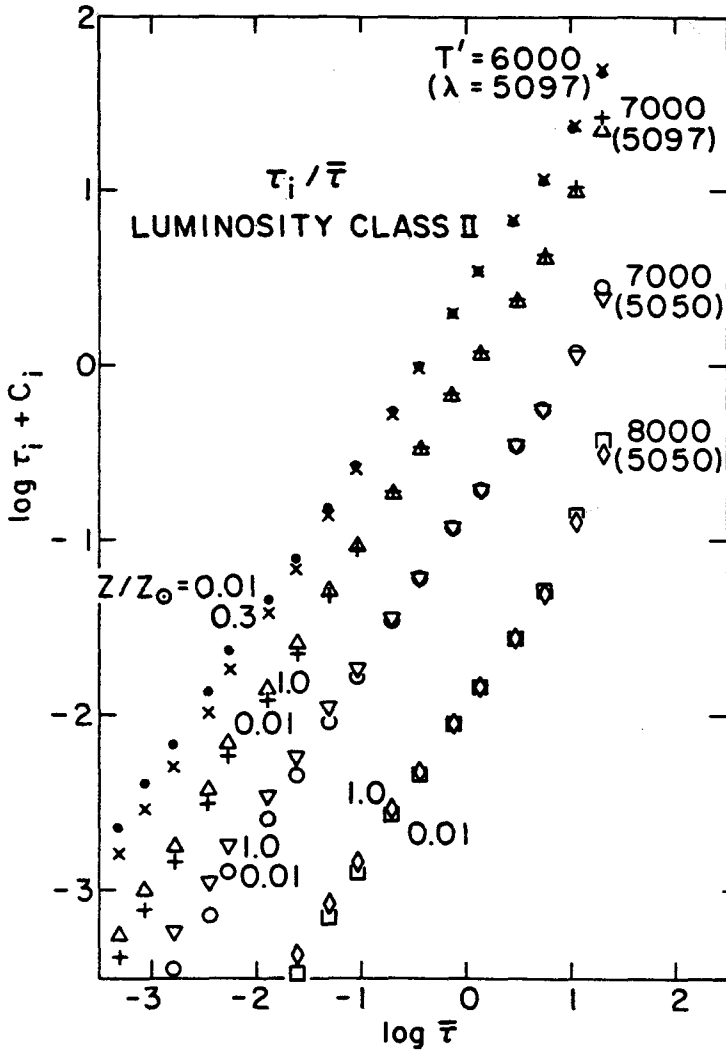


Fig. 2. A comparison of $\tau_{\lambda}(\tau)$ for $\lambda = 5050 \text{ \AA}$ or 5090 \AA for stars with different metal abundances Z in the temperature range $6000 \text{ K} \leq T_e \leq 8000 \text{ K}$. For $\tau > 0.1$ $\tau_{\lambda}(\tau)$ changes very little with metal abundance Z .

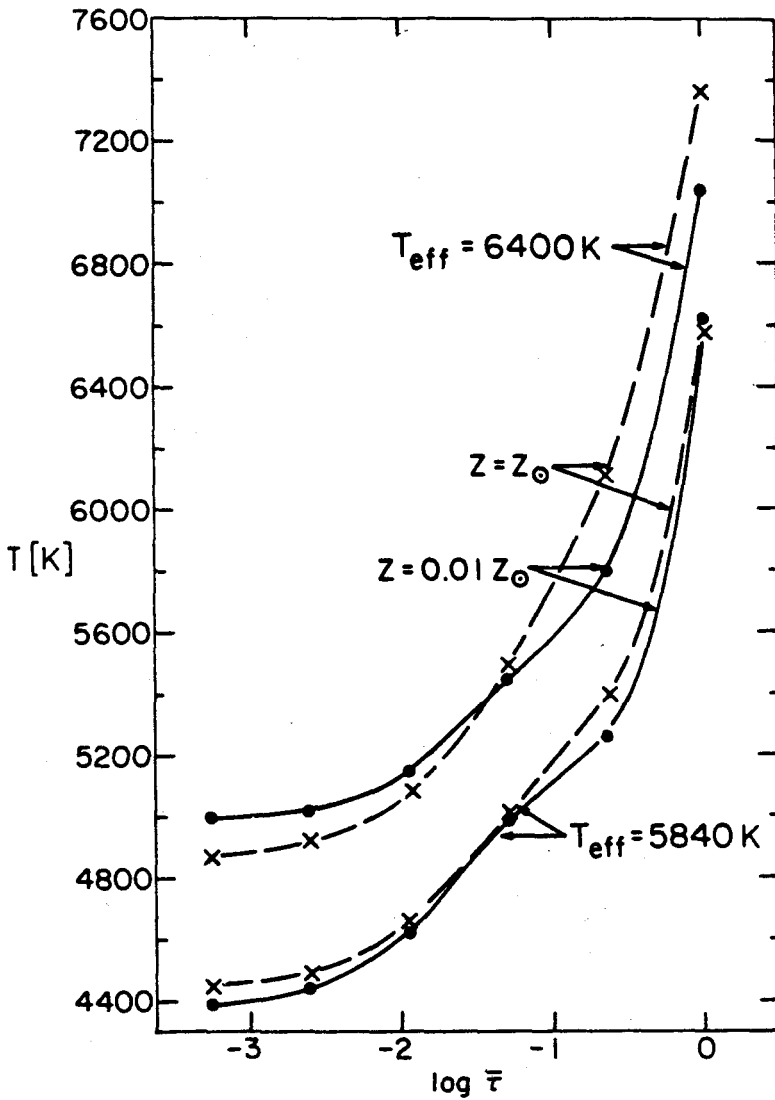


Fig. 3. Copied from Bohm-Vitense (1975). A comparison $T(\bar{\tau})$ is given for stars with nearly equal T_{eff} but with metal abundances differing by a factor 100.

4.1 High T

For these high temperatures we find

$$\frac{\kappa_L}{\kappa_c} \propto \frac{n(H,2) \cdot n_e}{n(H,3)} = f(T, n_e) \quad (6)$$

for the Balmer lines.

The Balmer jump D depends on the ratio of the continuous κ on both sides of the discontinuity which is

$$\frac{\kappa(3674^-)}{\kappa(3647^+)} \propto \frac{n(H,2)}{n(H,3)} = f(T) \text{ only} \quad (7)$$

(If electron scattering is also important D depends on n_e . NLTE effects could introduce an additional dependence on n_e). Here $n(H,2)$ = number of hydrogen atoms in the second quantum level. $n(H,3)$ = number of hydrogen atoms in the third quantum level.

4.2 Low T

For lower temperatures the continuous κ in the visual is mainly due to H^- and we find: For the Balmer lines

$$\frac{\kappa_L}{\kappa_c} \propto \frac{n(H,2) \cdot n_e}{n(H^-)} \propto \frac{n(H,2) \cdot n_e}{n(H,1) \cdot n_e} = f(T) \text{ only} \quad (8)$$

This of course is the reason why Dr. Garrison determined the solar temperature from $H\beta$ alone. For the Balmer jump we find

$$\begin{aligned} \frac{\kappa(3647^-)}{\kappa(3647^+)} &= \frac{n(H,2) \cdot \kappa_2 + n(H,1) \cdot n_e \cdot \kappa(H^-)}{n(H,1) \cdot n_e \cdot \kappa(H^-)} \\ &= 1 + \frac{n(H,2) \cdot \kappa_2}{n(H,1) \cdot n_e \cdot \kappa(H^-)} = f(T, n_e) \end{aligned} \quad (9)$$

κ_2 = continuous absorption coefficient per hydrogen atom in the second quantum level

$\kappa(H^-)$ = continuous absorption coefficient per electron and neutral hydrogen atom.

For both temperature regions we have two independent equations to determine T and n_e . This of course, is the theoretical background for the Chalonge Divan (1952) system of D, λ_1 classification

and for the Strömgren (1963) $H\beta, c_1$ system.

5. ADDITIONAL CLASSIFICATION CRITERIA

For hot stars this system is not very temperature sensitive, since

$$\frac{n(H,2)}{n(H,3)} \propto e^{\Delta E/kT} = e^{1.5 \text{ eV}/kT} \quad (10)$$

The spectral type B covers a range in which the temperature changes by a factor of 3. On the cooler side a similar change in temperature extends over five spectral classes. Clearly a better subdivision is needed for B stars as pointed out by Dr. Zekl.

We obviously find higher temperature sensitivity if we use effects influenced by larger energy differences ΔE . Since the temperature usually enters with the factor $e^{-\Delta E/kT}$, large values of ΔE will usually lead to weak lines. This can partly be compensated by using lines of elements with large abundances. Also lines in the UV are generally stronger because of the larger $d\ln B_\lambda/d_c$. Using the far UV is therefore of advantage for temperature classification except if there is very little flux in this region, as for the cool stars, or if $\bar{\kappa}/\kappa_c$ gets to be very small as in the A and F stars. For hot stars classification by satellite far UV spectra appears to offer advantages. In the visual we would mainly consider He lines as long as they can be recognized. If we can use the intensity ratio of HeII to HeI lines we do not need to know the helium abundance since according to the Saha equation $\frac{n(\text{He}^+)}{n(\text{He})} = f(T, n_e)$ only. Here the ΔE involved in the line intensity ratio is of the order of 50eV, and gives excellent temperature sensitivity. Clearly n_e has to be evaluated simultaneously. The Stark effect broadened He or H lines provide the additional relation for the determination of n_e . If we use the He lines for this purpose the helium abundance Y enters as a third parameter. We will need a three dimensional classification.

This third parameter also enters if we consider step 2, namely the relation between n_e and gravity g as we will see below.

For somewhat lower temperatures i.e. $T < 30,000\text{K}$ the HeII lines cannot be used any more. We have to use other ionization equilibria which are independent of Z if we use two ions of one element, and if we do not use spectral regions where κ_c is influenced by different absorption mechanisms for the two stages of ionization, (κ_c has to cancel out in intensity ratios), i.e. lines should be close in the wavelength λ . For the B stars the

H lines are still rather T insensitive but they become increasingly better n_e indicators.

For $T \leq 10000$ K the HeI lines cannot be used any more. The hydrogen lines are near maximum intensity and are not very temperature sensitive. Metallic lines are generally weak, but ionization equilibria of abundant heavy elements like CNO or Fe can be used as temperature indicators. If line intensity ratios of ions of the same element are used the metal abundance does not enter as a new parameter in the T, n_e classification.

For $T < 8000$ K the hydrogen lines and the Balmer discontinuity, as long as it can be measured, appear to be the best classification criteria. As seen above we can indeed make a one dimensional temperature classification from the hydrogen line wings as long as they can be seen. Ionization equilibria of any one given element can be used in addition without introducing new parameters.

Two dimensional spectral classification according to T and n_e does indeed seem to be possible for all temperatures if the criteria can be chosen appropriately. However, additional parameters enter when we consider the relation between n_e and g , i.e. step 2 in the introduction.

6. THE RELATION BETWEEN ELECTRON DENSITY AND GRAVITY.

For hot stars when H and He are completely ionized, the electron pressure P_e equals 1/2 the gas pressure P . The gas pressure P is determined by the hydrostatic equation which reads

$$\frac{dP}{d\tau_c} = \frac{g \cdot \rho}{\kappa_c} \quad (11)$$

where ρ = density and κ_c = continuous absorption coefficient per cm. The equation of state gives $\rho = \rho(P, \mu, T)$, where μ is the mean molecular weight which for complete ionization is

$$\mu = \frac{1+4Y}{2(1+Y)} \quad \text{with } Y = \frac{n(\text{He}) + n(\text{He}^+)}{n(\text{H}) + n(\text{H}^+)} \quad \text{assuming } n(\text{He}^{++}) \ll n(\text{He}^+) \quad (12)$$

This gives $1/2 < \mu < 2$ for $0 < Y < \infty$

and $1/2 < \mu < 5/4$ for $0 < Y < 1$

The integration of equation (11) yields

$$p^2 \propto p_e^2 \propto g \cdot \frac{1+4Y}{2(1+Y)} \quad (13)$$

We therefore have to determine Y in order to determine g from n_e and T .

A classification according to T and g has to determine Y simultaneously and therefore needs to be three dimensional. If Y is undetermined g remains uncertain by a factor of up to 4.

For high temperatures Y can be determined from the helium lines.

For $T < 30000$ K helium is mainly neutral and

$$\mu = \frac{1+4Y}{2+Y} \text{ so } 1/2 < \mu < 5/3 \text{ for } 0 < Y < 1 \quad (14)$$

$$\text{and we find } n_e \propto \sqrt{g \cdot \frac{1+4Y}{2+Y}}$$

For $7000 < T < 10000$ K hydrogen is only partly ionized but the free electrons are essentially all supplied by the hydrogen. If the κ_c is mainly Paschen and Balmer continuum we find

$$n_e \propto \sqrt{g \frac{1+4Y}{1+Y}} \quad (15)$$

If the continuous κ is due to H^- we derive

$$p^{3/2} \propto g(1+4Y) \cdot \sqrt{1+Y} \text{ and } p_e^2 \propto p/1+Y \quad (16)$$

We still need a three dimensional classification but we have no possibility to determine Y except perhaps in the future by determining $n(H)$ from the van der Waals broadening of Stark effect insensitive lines. The ratio of $n_e/n(H)$ would depend on Y .

For $4000 < T < 7000$ K the situation becomes more complicated since now the number of free electrons enters into the continuous κ which is due to H^- . The free electrons are now mainly supplied by the metals. We then have

$$n_e = n(M^+) \text{ and } \kappa_c = \kappa(H^-) \propto n(H) \cdot n_e \quad (17)$$

For ionized lines one obtains now

$$\frac{\kappa_L}{\kappa_c} \propto \frac{n(M^+)}{n(H) \cdot n_e} = \frac{1}{n(H)} \quad (18)$$

The metal abundance does not occur explicitly but only implicitly in the number of hydrogen atoms per cm^3 $n(\text{H})$, which by means of the hydrostatic equation does depend on Z (Bohm-Vitense 1975). Under these conditions the integration of the hydrostatic equation yields for the electron pressure

$$P_e \propto \sqrt{g \cdot Z} \sqrt{\frac{1+4Y}{1+Y}} \quad (19)$$

The relation between n_e and g now contains a fourth parameter, the metal abundance Z .

We now need a four dimensional classification if we want to classify according to T and g . The same values of T , n_e for different stars does not imply that they have the same g . Z of course can be determined from the strengths of the metallic lines. The ionization equilibrium of the ions of one element is of course still a function of T and n_e only, but n_e depends on Z and therefore the ratio $\frac{n(\text{M}^+)}{n(\text{M})}$ depends on Z for a given value of T and g . There is no way to determine Y from the spectrum and therefore even when Z is known the gravities will always be uncertain by about a factor 2.

For still lower temperatures molecules may possibly contribute to the continuous κ . Their abundances will therefore enter into the hydrostatic equation, and thereby into the relation between n_e and g . Unfortunately the abundances of C, N and O enter decisively into the molecular abundances, which means that now these three abundances, which appear to vary independently, enter as new parameters into the relation between n_e and g . We would need a 7 dimensional classification, sensitive to $n(\text{H})$. For cool giants, when H^- still determines κ_c a four dimensional classification should be all right, but when the molecules become important for κ_c 7 dimensions would be needed unless the CNO abundances are well correlated with the other metal abundances. It takes more courageous and optimistic people than me to attempt a 7 dimensional classification at the cool end of the spectral sequence.

This would conclude the main topic of this talk.

7. A POSSIBLE CLASSIFICATION CRITERION FOR EARLY A STARS

The preceding discussion has shown that an accurate temperature classification is especially difficult for early A type stars with temperatures around 9000 K. For these stars we may have a much better way to classify them by their far UV spectra.

In Fig. 4 I have copied some energy distributions from early A stars from the ESA catalogue (Jamar et al 1976). The spectral types given are those from the catalogue. I have not traced their origin and not checked them since I have never done any spectral classification myself. For a given spectral type a large variety of UV energy distributions is found especially around 1800 Å probably demonstrating the uncertainty of the classification. The energy distribution around this discontinuity at 1780 Å (probably due to F₂) appears to be a much better way to classify these spectra as suggested in Fig. 5. Clearly many more studies are needed before this can actually be used.

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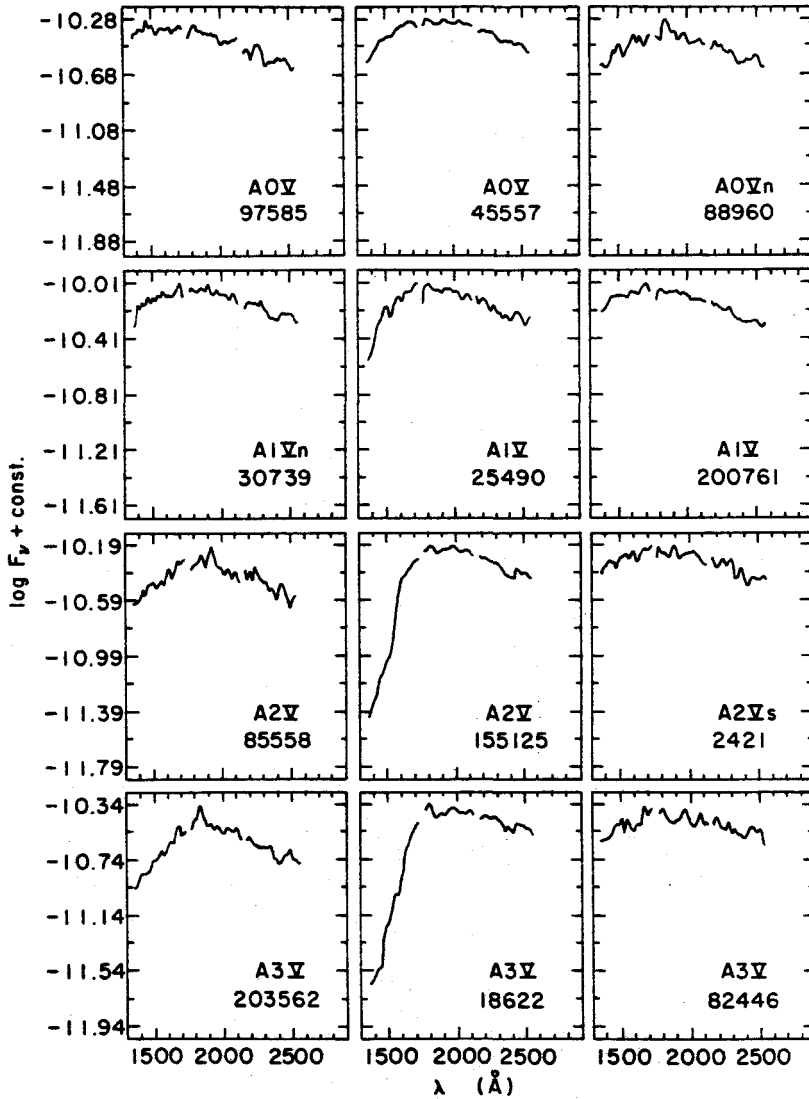


Fig. 4. Shows the mismatch between spectral types and discontinuity at 1780 \AA in many cases, probably indicating that the spectral types at early A stars are not good temperature indicators. The discontinuity at 1780 \AA may be a much more sensitive T indicator.

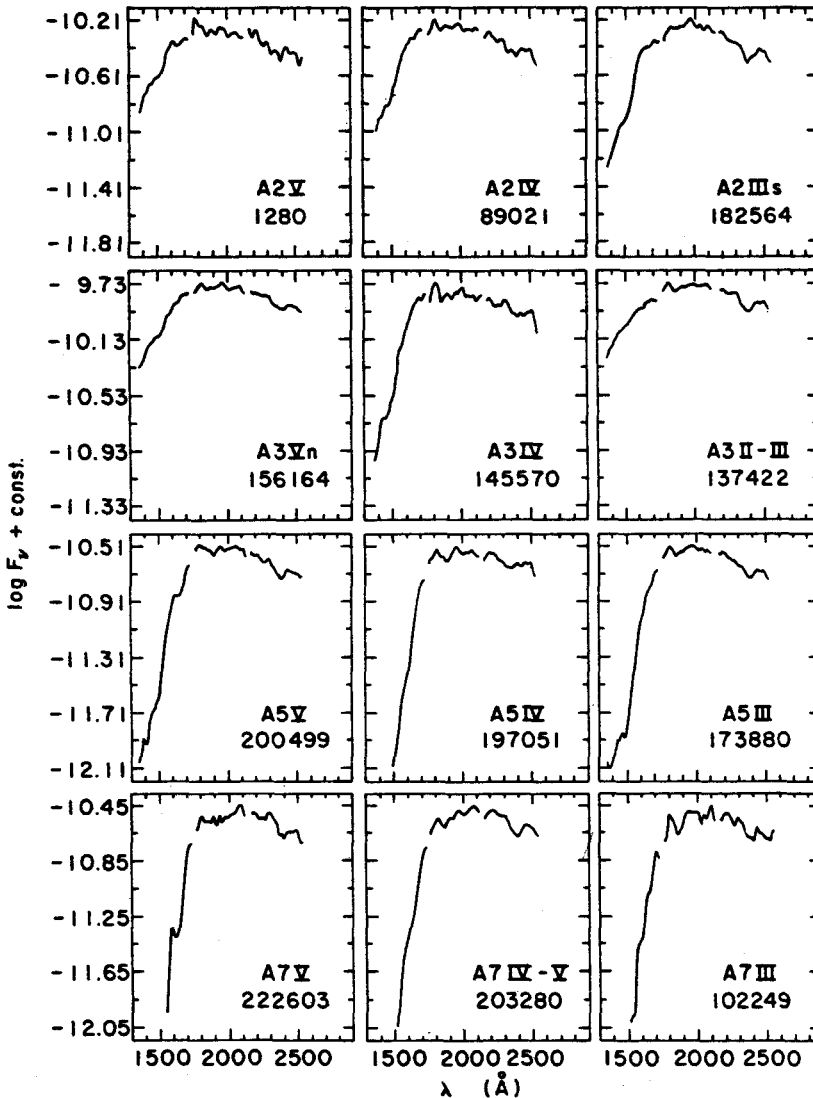


Fig. 5. Shows the dependence of the discontinuity at 1780 Å on spectral type. A very strong T dependence is seen, but no obvious luminosity dependence. The energy distributions are taken from Jamar *et al.* 1976.

DISCUSSION

Jaschek: With regard to your last slide on the A stars we shall hear more this afternoon, and an Atlas of ultraviolet spectra will be distributed in which this sharp decline toward shorter wavelengths is being used.

Böhm-Vitense: I am sorry I did not know that.

Keenan: You discussed gravity only in terms of hydrostatic equilibrium. Could you say something about the situation when the atmosphere is distorted by turbulence or other mechanical means?

Böhm-Vitense: As long as the turbulence velocities are smaller than the sound velocity I think the gas pressure cannot be changed by more than a factor of 2. I have had discussions with Vardya about this question, but I think that this also holds if there is locally a rather large gradient of the turbulent velocity. If the velocities are supersonic then one would have to study the depth dependence of the turbulence in order to judge the effects.

Nandy: When does the non-LTE effect become important and in what temperature range?

Böhm-Vitense: Non-LTE effects may become important for early B and for O type stars according to present knowledge. I did not discuss them because they will not introduce a new dimension at the present state of the art. They will, however, change the temperature calibration. Only if the changes of the far UV radiation field due to NCO or Ne continua are important will a new dimension have to be introduced. To the best of my knowledge this has not yet been computed.

I suspect that non-LTE effects may turn out to be important also for the A stars if this large discontinuity at 1800 Å is taken into account for the ionization of the metals.

Divan: You mentioned that for high temperatures the Balmer jump was a function of T only. But empirically we find that even for O stars there is a clear variation of the observed Balmer jump with luminosity class. How do you interpret this?

Böhm-Vitense: For reasons of simplicity I have left out of consideration the electron scattering contribution to the continuous absorption, which will be of increasing importance for higher temperatures. The electron scattering will cause a dependence of the Balmer jump on the gravity.

Cowley: One of the reasons for the great success of the MK system is that many of the factors which conceivably might vary, in fact do not vary very much, or those variations which occur are correlated with one another so that nature has reduced the number of variables that we have to deal with. This appears to be the case for stars on the lower main sequence, and for many of the late-type giants. With the later stars we can exclude a small number of objects as peculiar, and proceed in our classification with good success. In the upper-main sequence stars it is my opinion that abundances are no longer well correlated with one another, and for these objects, if one looks at different spectral regions, or with high resolution within the MK domain, difficulties immediately arise. The percentage of stars that can be excluded as abnormal rises rapidly, and becomes comparable to the total number of upper main sequence stars. I cannot overemphasize the difference between the upper and lower main sequence insofar as our ability to describe them in terms of a minimum number of parameters is concerned.

Vardya: Can we use the He infrared triplet at $\lambda 10830$ for the study of helium abundance?

Böhm-Vitense: This is formed in the chromosphere and we do not understand the formation well enough yet. If we do in the future we can certainly study the helium abundance from this triplet.

Code: In the earlier A stars there is a strong depression in the ultraviolet continuum as well as the discontinuity, and FeII is certainly a major contributor. The effect of this ultraviolet line blanketing does effect the visual region of the spectrum so that the Paschen continuum corresponds to a higher temperature than either the lines, the UV continuum or the empirical effective temperature. This must be rather sensitive to abundance differences. Perhaps the metallic line A variable stars are examples of this redistribution of energy.

Böhm-Vitense: I have looked at the variations in $\alpha 2CVn$. In the visually brighter phase the energy distribution in the visual corresponds to a lower temperature. I therefore feel that the increased brightness cannot be due to an increased backwarming effect.